PHILOSOPHICAL TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXLVIII.

PART I.

LONDON:

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MDCCCXLVIII.

ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions,* take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions,* that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body,

upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

The Meteorological Journal hitherto kept by the Assistant Secretary at the Apartments of the Royal Society, by order of the President and Council, and published in the Philosophical Transactions, has been discontinued. The Government, on the recommendation of the President and Council, has established at the Royal Observatory at Greenwich, under the superintendence of the Astronomer Royal, a Magnetical and Meteorological Observatory, where observations are made on an extended scale, which are regularly published. These, which correspond with the grand scheme of observations now carrying out in different parts of the globe, supersede the necessity of a continuance of the observations made at the Apartments of the Royal Society, which could not be rendered so perfect as was desirable, on account of the imperfections of the locality and the multiplied duties of the observer.

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ROYAL MEDALS.

HER MAJESTY QUEEN VICTORIA, in restoring the Foundation of the Royal Medals, has been graciously pleased to approve the following regulations for the award of them:

That the Royal Medals be given for such papers only as have been presented to the Royal Society, and inserted in their Transactions.

That the triennial Cycle of subjects be the same as that hitherto in operation: viz.

1. Astronomy; Physiology, including the Natural History of Organized Beings.
2. Physics; Geology or Mineralogy.
3. Mathematics; Chemistry.

That, in case no paper, coming within these stipulations, should be considered deserving of the Royal Medal, in any given year, the Council have the power of awarding such Medal to the author of any other paper on either of the several subjects forming the Cycle, that may have been presented to the Society and inserted in their Transactions; preference being given to the subjects of the year immediately preceding: the award being, in such case, subject to the approbation of Her Majesty.

The Council propose to give one of the Royal Medals in the year 1848 for the most important unpublished paper in Astronomy, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1845, and prior to the termination of the Session in June 1848.

The Council propose also to give one of the Royal Medals in the year 1848 for the most important unpublished paper in Physiology, including the Natural History of Organized Beings, communicated to the Royal Society for insertion in their Transactions after the termination of the Session in June 1845, and prior to the termination of the Session in June 1848.


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Amoy
Corea Str.
JAPAN Is.
Loo-choo Is.
Bashee Is.
Phillippines
BORNEO
Petropaulowsky
Kamtchatka
Kurile Is.
Aleutian Is.
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Kadiak I.
Sitka
Observatory Inlet
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Realejo
Nicoya
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Arica
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Valdivia
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PHILOSOPHICAL TRANSACTIONS.

I. The Bakerian Lecture.—*Researches on the Tides. Thirteenth Series. On the Tides of the Pacific, and on the Diurnal Inequality.*

By the Rev. W. Whewell, *D.D., F.R.S., &c.*

Received November 11,—Read December 16, 1847.

1. In 1833 the Royal Society did me the honour to publish, in its Transactions, a memoir of mine, entitled "Essay towards a First Approximation to a Map of Cotidal Lines;" and, in subsequent years, a number of further communications on the subject of our knowledge of the tides, as deduced from observations of those phenomena. These later "Researches" have modified my first views,—a result which I from the first contemplated as probable, as I intended to imply by entitling my memoir "An Essay towards a First Approximation," and as I expressed more fully in the memoir itself. I have also obtained from various persons, since my last communication to the Society, a considerable amount of recent tide observations, made in various quarters of the globe; and I am desirous of pointing out the general bearing of these additional materials of knowledge. I wish especially to bring under the consideration both of mathematicians and of navigators, the problem of the tides of the Pacific Ocean. When I wrote my first memoir on the subject, our knowledge of the tides of that ocean was so imperfect, that I did not even venture upon a first approximation to the cotidal lines. And I have since seen reason to believe that, not only for that ocean but for all large seas, the method of drawing cotidal lines which I formerly adopted, is very precarious.

2. There is another leading feature of the tides, which has been brought clearly into view in the course of these researches, which is of great interest and importance to the navigator, as well as to the mathematician, and of which I have assigned the laws in a general manner, and with an accuracy sufficient for most practical purposes; I mean the Diurnal Inequality which makes the common or semidiurnal tides differ alternately in excess and in defect. I have already examined various series of

tide observations in which this diurnal inequality prominently appears; but I have now the means of showing it to be much more extensively distributed and larger in amount than has been supposed.

These two points, the Cotidal Lines, and the Diurnal Inequality, will be the subject of the present memoir.

Of Cotidal Lines.

3. Great light was thrown upon the form of the cotidal lines by very extensive series of observations made for that purpose in June 1834 and June 1835, by the Preventive Service at all their stations on the coasts of England, Ireland and Scotland, and by naval officers at many points of North America, Spain, Portugal, France, Belgium, the Netherlands, Denmark and Norway. These observations, made at my suggestion, by the kindness of the authorities of that time, were so numerous and exact as to determine with considerable accuracy the form of the cotidal lines in the neighbourhood of the oceanic coast of Europe. One main feature, very prominent in all these lines, was that they meet the shore at a very acute angle, and follow its flexures at a little distance with an almost parallel course; and that consequently, the tide-wave which runs up a channel is, in the middle of the channel, very much in advance of its place at the sides of the channel*. This form of the cotidal lines is also easily shown to be in harmony with the laws of the motion of fluids†; and it cannot be doubted that those lines must affect such a form to a much greater extent than was assigned to them in my First Approximation.

This character of the cotidal lines must prevail to such an extent that I conceive all attempts to draw such lines *across* a wide ocean by means of observations on its shores, must be altogether worthless. This applies beyond doubt to the Pacific Ocean, and probably, taking other reasons into account, to the Atlantic also.

4. This conclusion is further confirmed by our finding that if we do draw "cotidal lines" across wide oceans, as for instance, the Atlantic, they do not agree with tides observed at islands in the mid-ocean, without ascribing to the lines such flexures as deprive them of all simplicity, and make them require further evidence.

5. Again, it is found that, for the most part, the tides in the mid-ocean isles are very small; and this circumstance again, makes the assumed oceanic continuity of cotidal lines very doubtful.

6. Further: if the tides in the Atlantic and Pacific be conceived to be brought by a progressive wave, which the scheme of cotidal lines assumes, they must be conceived to be brought from some part of the ocean where such a wave can travel round the globe of the earth so as to follow the moon, or at least, to be connected with such a part of the ocean: and such a supposition was accordingly involved in

* See the Charts of the British Isles and of the Coasts of Europe in the Sixth Series of Tide Researches, Philosophical Transactions, 1836, Part II.

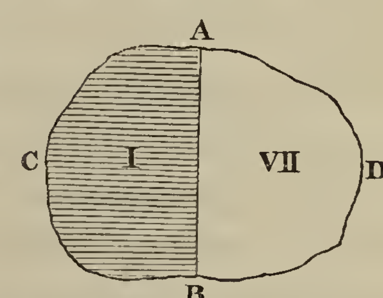
† See Mr. Airy on Tides and Waves, Art. 359, in the Encyclopædia Metropolitana.

the attempts to draw the cotidal lines of the Atlantic and Pacific. But it appears unlikely that this supposition rightly represents the mode in which the waters of the Atlantic and Pacific obey the action of the sun and moon.

7. If it be asked what other mode of operation of the lunar and solar forces upon the ocean can be conceived, different from this progressive wave which is expressed by means of cotidal lines; an answer immediately suggests itself, that a *stationary undulation*, corresponding in its period with the period of the moon's apparent revolution, from meridian to meridian (that is, a lunar half-day), is a possible mode of motion for a fluid under such circumstances. By a "stationary undulation," I mean a motion such as that which takes place in a vessel of water, when one side is suddenly lifted from rest, and then set down again. When this is done, the water oscillates, the surface rising alternately on the raised side and on the other, and the middle line of the surface neither rises nor falls. In this case the oscillation is *free*, depending on the dimensions of the fluid only: but if the fluid were subject to periodical forces, such as an attracting body passing over it at equal intervals of time, it might perform *forced* oscillations of the same kind; and in this case, the period of the oscillation would necessarily, in the ultimate condition of the fluid, be the same as the period of the forces.

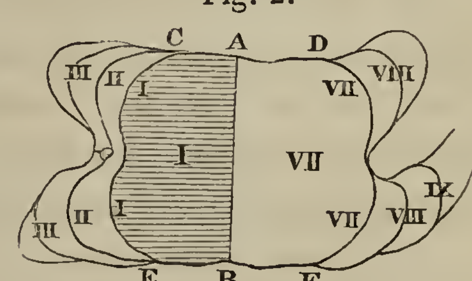
8. The lunar attraction passes over every wide ocean once in every lunar half-day; and it is conceivable that such an ocean, under the influence of the lunar forces, should perform, every lunar half-day, such a stationary oscillation as has been described. On this supposition, we should have a regular tide at its eastern and western shore, but no tide in the middle part; and in such a case there would be no cotidal lines. The ocean would be divided into two portions or areas by a *line of no tide* (AB); and these two areas would each have the tide over the whole area at the same time, the two times differing by six lunar hours. For instance, if the time of high water on the eastern shore, ACB, were one o'clock, the time of high water on the western shore, ADB, would be seven o'clock. This might be expressed by distinguishing the two spaces by shading, and marking them I. and VII. respectively. (See fig. 1.)

Fig. 1.



9. But this cannot be a representation of the state of the tides of oceans generally, at least as to their *littoral* spaces. For we know by observation that, along large tracts of the shores of all seas, the tide does travel progressively, in such a manner that its course in those parts may be represented by a series of cotidal lines. And if the shore be broken by shallow inlets and bays, the tide must, by the laws of fluids, travel progressively up these recesses. In such cases, we may properly represent the state of the tides by such a diagram as before, bordered with a series of cotidal lines representing the course of the tide into the

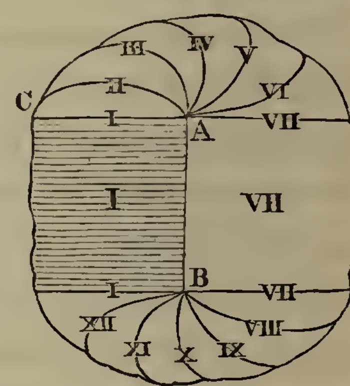
Fig. 2.



recesses of the coast; which will be expressed by drawing the cotidal lines of two hours, three hours, &c. (II. III., &c.) belonging to the tides derived from the oceanic tide I.; and by drawing the cotidal lines of VIII. IX., &c., derived in like manner from the oceanic tide VII. (See fig. 2.)

10. But, moreover, if CD be the shore transverse to the line of no tide AB, it is very possible that the tide may not be absolutely simultaneous from C to D, but may vary continuously along this shore, although the tide in the central oceanic parts be of the nature of a stationary undulation, as already supposed. In this case, the state of the tides will require that, in a map, we should place the extremity A of the line of no tide at some distance from the shore, and there will be a series of cotidal lines I. II. III. IV. V. VI. VII., which will revolve about the point A, and will carry the tide in succession to all the points of the coast CD. These lines will have the general form of cotidal lines which we have described; and will be determined in their details by the shore along which the progressive tide-wave travels. (See fig. 3.)

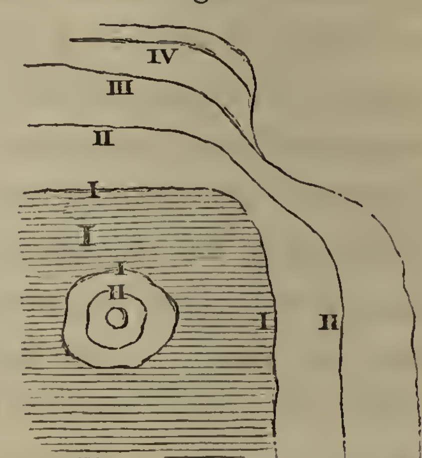
Fig. 3.



11. But there is yet another modification which the cotidal spaces of the oceanic tides undergo, in passing into the cotidal lines of littoral tides. The oceanic tides produced by a stationary undulation, with its midline of no tide, give tides necessarily differing *six* lunar *hours* from each other on the opposite sides of the ocean. It is high water on one side when it is low water on the other. Now this cannot be the case all over an ocean which is of different breadths in different parts; nor, in fact, can it be the case in any part of it. For that motion of the parts of the fluid which a stationary undulation requires, cannot take place on a shore shallow in proportion to the depth of the oceanic spaces. Near the shore, we shall have a tide which is progressive from the oceanic space towards the land. Hence, even when the tide occurs at the same time along a great extent of shore on one side of an ocean, we cannot assume that it is directly produced by the oceanic tide. It may be a tide which is later than the oceanic tide, and which may be represented by a cotidal line bordering the space which the oceanic tide occupies. (See fig. 4.)

12. And this may be the case with regard to a detached island, as well as to an extensive coast; especially if the island be the summit of an extensive part where the ocean grows shallower. In such a case the island may have about it cotidal lines in the form of rings. (See fig. 4.)

Fig. 4.



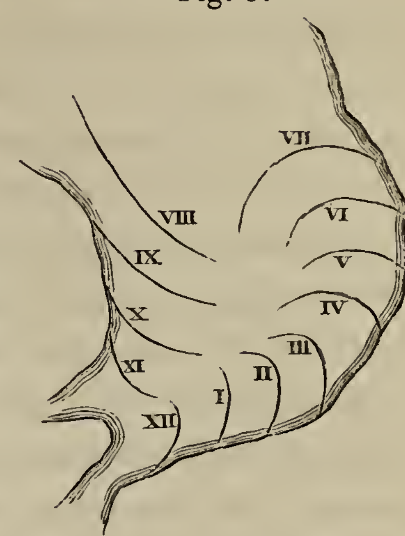
13. From these considerations, it appears that it must be very difficult to determine the time of high water in the oceanic spaces. All that can be certainly known is, that the time must be earlier than the earliest neighbouring littoral tides. And such an oceanic tide being assumed, the

sequence of the tide-hours along the shore may be rightly represented by cotidal lines drawn nearly parallel to the shore, according to the data which observation supplies.

14. It is to be observed, however, that the oceanic tide necessarily includes two different areas, in which the times of high water differ by six lunar hours; and therefore that which is assumed as the oceanic space, must be so situated that the tides on its opposite sides differ by about six lunar hours. The oceanic space thus occupied by the stationary undulation must, further, be so situated that the tides in its middle parts are small or disappear.

15. But a stationary undulation, such as has been supposed to occupy the oceanic space, is not the only mode in which we may conceive large littoral tides combined with small central tides. Such a combination may be produced by cotidal lines revolving round a fixed centre, as in the upper part of figure 3. Nor is such a state of things imaginary only. The course of the tides on the opposite coasts of England and the Netherlands is such as not to be intelligible in any other manner than by supposing such a rotation of the tide-wave, as may be seen in my Sixth Series of Tide Researches. And the smallness of the tide in the central parts, which this view of the subject implies, has been verified by the observations of Captain Hewett and others. A case somewhat of the same kind occurs on the coast of Ireland. At Courtown on the east coast, there is no lunar tide*, though there is a progressive tide-wave on the opposite coast of Wales.

Fig. 5.



16. It may be thought, therefore, that we shall find it impossible to decide whether the tides of an ocean in which the central tides are small and the littoral tides progressive, are to be represented by a revolving wave or by a stationary oceanic undulation with bordering cotidal lines. But this difficulty is not of much real consequence; for one of these hypotheses passes gradually into the other, as may be seen in fig. 3. And the result of both the one and the other is, that we cannot pronounce anything certain about the time of high water in the oceanic space, till we have been enabled, by numerous observations, to draw the littoral cotidal lines with considerable accuracy; and when this is done, the nature of the oceanic movement will probably show itself upon the face of our chart.

17. On these grounds, I am now disposed to retract parts of what I have said with regard to the form of the cotidal lines of the Atlantic in my "Essay." I do not think it likely that the course of the tide can be rightly represented as a wave travelling from south to north between Africa and America. We may much better conceive the state of things by means of a stationary undulation, of which the middle space is between Brazil and Guinea, in which region the tides are very small, as at St. Helena

* See Mr. Airy in the Philosophical Transactions, 1845, Part I.

and Ascension; while at Tristan d'Acunha, a detached island like the others, but removed out of this medial space, the rise is eight or nine feet. This would explain also many of the circumstances which made it so difficult to give any possible form to the cotidal lines in the Atlantic; for instance, the tide occurring nearly at the same time all the way from the Cape of Good Hope to the Congo. This is accounted for if we suppose that the South Atlantic is mainly occupied by the oceanic tide, and that this area is bordered by cotidal lines nearly parallel to the shore. I must confess, however, that at present I am unable to carry this hypothesis into comparison with the general body of the facts. Such a comparison would require observations much more numerous than we as yet possess; and, as I have already said, even with numerous observations, we can only hope to draw the cotidal lines in the neighbourhood of the shores.

18. For the same reason I shall not attempt to determine the general course of the tides in the Pacific; but I will remark that the view now given of the distribution of the tides in an ocean explains several of the features of the Pacific tides which were before very perplexing. If we suppose an ocean tide from the borders of which proceed tides having their progress marked by cotidal lines, we can easily draw the lines so as to include the following facts of observation:—

(1.) The *easterly* motion of the tide-wave round Cape Horn, which is established by Capt. KING's observations*, and which, as I formerly observed†, is difficult to reconcile with the supposition of a tide revolving from west to east round the south pole. This is explained by its being a tide proceeding from the oceanic tide.

(2.) The tide being at nearly the same hour along a large portion of the coast of South America, namely, from the western extremity of the Straits of Magellan for twenty or thirty degrees northward. This shows that the cotidal line is nearly parallel to the shore.

(3.) The very small tides or no tides at the islands in the centre of the Pacific, Tahiti, and the Sandwich Islands. These belong to a central portion of the ocean, where the rise and fall of the surfaces nearly vanishes.

19. I shall now proceed to give such materials for a knowledge of the tides of the Pacific as I have been able to collect, in addition to those which I had under my notice in my former Essay. They are for the most part derived from tide observations made under the direction of naval officers employed on expeditions of survey and discovery. The series of observations at each place was often necessarily brief and inexact; and therefore it is no reflection upon the skill and care of the officers and men to whom we owe them, to say that they are often wanting in correctness.

20. There are, moreover, two other sources of inaccuracy in tide observations; namely, the want of a clear understanding as to the thing to be observed, and the irregularity or complexity of the facts themselves. With regard to the former point, I hope that several misapprehensions formerly prevalent among navigators are now

* Sailing Directions. † Philosophical Transactions, 1833, p. 192.

no longer common, such as confounding the time of high water with the time of turn of the tide-stream. But there is probably still some unnecessary difficulty produced by regarding, as a cardinal point in the observation, the "establishment," as vulgarly understood, namely the hour of high water *on the day of new or full moon*; for, in fact, the hour of high water on this day is of no more importance than the hour of high water on any other day, except in so far as it gives the means of knowing the hour on other days. And it does not afford the means of doing this, any more than the hour of high-water for any other given age of the moon does. For just as much inaccuracy as, from whatever cause, there is, in deducing the time of high water at all ages of the moon from the time at a given age, just so much inaccuracy is there, from the same causes, in deducing the time of high water for all ages of the moon, from the time for full or new moon. And if the time at which the tide follows the moon on two or three successive occasions, be greatly and irregularly different, the observations are equally of little value, (either for drawing cotidal lines, or for predicting tides, or for any other purpose,) whether any of the observed tides fall on the day of new or full moon, or do not. If the tides are regular and the observations good, the common "establishment" may be obtained from the observations of any one day; although to give much value to this deduction, the tides should be observed for a fortnight. And if such observations be made for a number of very distant places, the common "establishment" does not represent a corresponding fact at different places. In some places it means the time of high water one day after the highest tide; in some, the time two days after the highest tide; in some, three days; for the "age of the tide" is different at different places, and the tide which corresponds to the new or full moon comes after the new or full moon by one, two, or three days. Hence, in order that we might compare the tides of distant places by means of a fact which had the same meaning in all of them, I proposed, in my former Essay, instead of taking this common Establishment, to take what I then called *the corrected Establishment*, namely the *mean* of all the lunitidal intervals, that is, of the intervals by which the tide follows the moon's transit. And this corrected establishment I used in the discussion of the extensive series of observations made in 1836. In general, the corrected establishment is about thirty minutes less than the common establishment. It has been used by Admiral LÜTKE in his discussion of the tides of the Pacific. As however the common establishment is still the one familiar to navigators, and as no material error will result from the use of it, I shall make it the basis of my remarks on the tides of the Pacific. But it may be useful to bear in mind what I have said, that this "establishment" may be deduced from observations not made at the new or full moon*.

A very simple and convenient way of recording the result of a tide observation on any day, would be to state the *time of moon's transit* (which is given by the tables

* I have here said that in cases where the tides follow the common laws, we may deduce the time of high water on one day from the time on another: I might have said the same thing of the heights.

without observation) and the time at which the high water follows the moon's transit (*the lunitidal interval*); for the comparison of such intervals in successive half-days would immediately show whether the observations give any consistent result: and if they do not, they must be of little value. This test I have accordingly applied to the observations hereafter quoted. Where the lunitidal intervals on successive half-days differ very much (by two or three hours, for instance), or follow some progression inconsistent with the usual laws of the tides, the results of those observations can be of no immediate value, either for drawing cotidal lines, or for any other purpose.

21. But besides the irregularities in the observed tides arising from inaccurate or confused processes, there are others arising from the phenomena themselves. For instance, there are some places where the tides are regulated by the sun as much as they are by the moon, or even more. At these places we should obtain no intelligible result by referring them to the moon alone. Again, in other places the lunitidal interval is affected by a large inequality which makes it alternately greater and less: for instance, in successive tides it is, (in minutes,)

30, 150, 60, 180, 30, 210,

and so on. Now what shall we say is the "establishment" in such a case? If we take the "corrected establishment," as I have defined it (the mean interval), it is 110^m; but this is not an approximate value for any single tide. If we take the three smaller intervals, which may happen to be the day tides, we have, for the mean, 40^m; if we take the three others, we have 180^m, or three hours. And if we use the common "establishment," the confusion is still greater. The fact is, that in such cases the term "establishment" loses all definite meaning, as I have already observed*. It cannot be of use, either in expressing the laws of the tides at any one place, or the mode of transmission of the tide from one place to another. The former subject, the law of the diurnal inequality of the tides at a particular place, I have been able to assign for several places, and in some degree, in general. The latter, the mode in which the diurnal and the semidiurnal wave are transmitted and continued, is a more difficult question. I shall however add a few words on each of these points.

22. Before I quit the subject of cotidal lines, I may remark, that Capt. FitzRoy has pointed out the difficulties which attend the representation of the tides of the Atlantic and Pacific by means of cotidal lines, and has suggested a "libration or oscillation" of the ocean as better explaining the phenomena†. I had already, in 1836‡, pointed out other difficulties which belong to the case of the Atlantic, according to the series of observations then made.

We must recollect, however, that there are other ways besides the two which I have noticed (a stationary undulation and a revolving tide-wave) which will give us

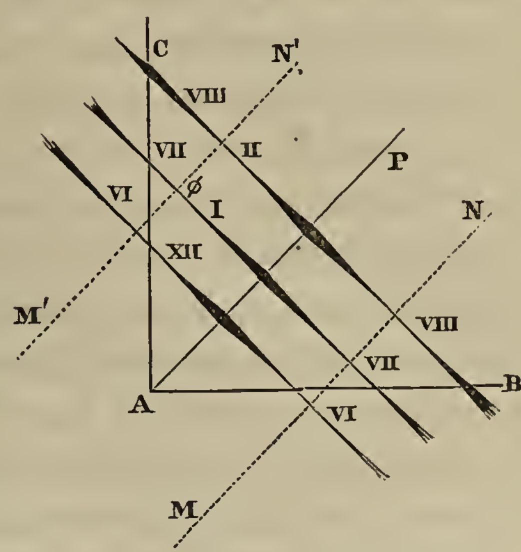
* Philosophical Transactions, 1840, Part I., p. 164.

† Voyages of the Adventure and Beagle. Appendix, p. 277.

‡ Philosophical Transactions, 1836, Part II., p. 304.

the result of a sea with considerable tides in some parts, and with spaces of no-tide. This may happen by the interference of two different tides; for instance, if two tide-waves differing six hours in the hour of high water arrive at any place, the two tides will destroy each other, and there will be no tide. And this may occur in a more complex manner. Thus, as Mr. Airy has remarked*, if two equal rectilinear waves, transverse to each other, travel across the same ocean, in directions AB and AC, the result will be that compound waves will travel in the direction AP which makes equal angles with the two directions AB and AC. But in this case there will be certain lines MN, M'N', parallel to AP, along which there will be no tide. And though the lines which mark the compound waves may be considered as the cotidal lines, these lines will be discontinuous, and the parts outside of the space MN, M'N' will be separated by six hours from the parts within that space. This example shows also how partial and limited cotidal lines may be, and how precarious must be all inferences on that subject from a few points to wide oceans.

Fig. 6.



23. The complex cases of tides appear at first sight to be interpreted in a different manner when we draw complex cotidal lines, as in Article 15, and when we suppose a combination of simple undulations, as in the last Article. But this difference of interpretation does not necessarily exist, if we conceive the cotidal lines to be mere *geometrical diagrams*, not lines marking the progress of a wave by motions of the particles perpendicular to the line of the wave. With this extension of the notion of a cotidal line, such lines may still be used to represent, in the first instance, the results of tide observations made at a series of places in the same seas; nor does it appear that there can be at present devised any better method of bringing tide observations into geographical combination, than this method of drawing cotidal lines. The case of the tides of the German ocean, for instance, where the order of the tide-hours had led me to draw the cotidal lines as converging to a centre of *no-tide* (as in Article 15), has been differently explained by Mr. Airy†. He conceives that these tides arise from the combination of a tide running along the eastern coast of England to the south, with a tide coming through the Straits of Dover to the east. But the combination of these two tides would, in fact, produce such cotidal lines as I have drawn, with a point of no-tide where I had predicted it, and where it was found by Capt. Hewett. It is true, that on this supposition the point of *no-tide* would not be a point of rest of the ocean; nor did I ever suppose that it would be so. There is, at the point of no-rise-and-fall of the surface, a considerable stream of tide alternately north-east and south-west, as Capt. Hewett found. And the same will

* Tides and Waves, Art. 367. † Ibid. 526.

probably be the case in other instances where the cotidal lines affect the same converging form, and where the tide in the central part of the sea vanishes.

Materials for a Tide Map of the Pacific.

24. The materials which I formerly employed in my "Essay towards a First Approximation to a Map of Cotidal Lines," were principally the following:—the collection of the facts then known, given in the fourth volume of LALANDE's Astronomy; Books of Astronomy and Navigation, and Sailing Directions, as NORIE's Epitome of Navigation, PURDY's Memoir on the Atlantic Ocean, his Memoir on the Ethiopic or South Atlantic, and his Columbian Navigator: Nautical Surveys, as the Surveys of the Australian coasts by Captains FLINDERS and KING, and of Patagonia by the latter officer; and the Survey of the Pacific by Captain BEECHEY; also foreign materials of the same kind, as MALASPINA's Voyage, and ROUSSIN's Survey of Brazil: to which may be added the "Remark Books" of various ships, which I was allowed to consult at the Admiralty, Memoirs relative to particular places, and other miscellaneous sources of information.

25. I have more recently collected other materials of the same kind, which I will briefly describe. They are partly gathered from books: for instance,—

Voyage autour du Monde sur l'Uranie et la Physicienne, pendant les années 1817, 1818, 1819, 1820, par M. L. DE FREYCINET. Paris, 1826.

One volume of this work, consisting of 356 pages, is entirely filled with tide discussions; and it is mortifying to see so much zeal and mathematics thrown away from the want, so general even yet, of a knowledge of the best modes of dealing with tide observations. The observations were only made at four places: at Rio Janeiro, sixteen days; at the Isle de France, twenty-seven days; at Rawak, fifteen days; at Guam, nine days. With the whole of the discussion employed, little or nothing of value is obtained; for even the resulting "establishments" are too vague to be depended on.

In the Voyage of the Astrolabe under the command of M. J. D. D'URVILLE, 1826 to 1829, I do not find any tide observations recorded.

26. Voyage autour du Monde sur la Fregate la Venus, 1836 à 1839, par ABEL DU PETIT-THOUARS.

This navigator gives the "establishment" and rise of tide at fifteen places, principally on the western coast of America and in other parts of the Pacific. These results will be used in the present memoir, along with others; but I will insert here the general remarks which M. DU PETIT-THOUARS makes upon the subject, after giving his results.

"When we see from this table that the tide rises only one-fourth as high at Acapulco as at la Magdalena; when we remark differences of two hours and a quarter, and of four hours and a half between the tide-hours of ports at small distances from each other, and situate on a coast on which the ocean has free range; when

we note that the interval of three hours intervenes between high water at Payta and at Callao; no one can maintain that the question of the tides is exhausted, or even that there is not a great deal to be done, in order to decide in what manner invisible obstacles, as the inequalities of the bottom of the sea, act upon the velocity of propagation of the tide-wave and upon its height. In the age in which we live, to propose a scientific question in a distinct form is half its solution."

I may remark that the object of the present memoir is mainly to give additional distinctness to the problem thus proposed, in order that further observations and calculations may help us on to the solution.

27. In Captain FitzRoy's "Narrative of the Surveying Voyages of H.M.S. Adventure and Beagle, between 1826 and 1836," there are in the Appendix, two articles on the tides; one, containing the time and rise of the tide at a great number of places, the other containing some general remarks, to which I have already referred. Captain FitzRoy notices the features in the tides of the Pacific partly as Admiral Du Petit-Thouars does. Thus he says (p. 281),—

"It is high water at Cape Pillar and at Chiloe, including the intermediate coast, almost at one time: from Valdivia to the Bay of Mexillones (differing eighteen degrees in latitude) there is not an hour's difference in the time of high water: from Arica to Payta the times vary gradually as the coast trends westward: from Panama to California, the times also change gradually as the coast trends westward: and from forty to sixty north, high water takes place at one time."

Captain FitzRoy combines these and many other facts respecting the tides into a hypothetical general view of the movement of the ocean, which may be of service in provoking further inquiry, though it can hardly be considered tenable in detail, as perhaps the present memoir may serve to show.

28. In the Philosophical Transactions for 1840*, I mentioned and discussed certain remarkable tide observations made at Petropaulofsk in Kamtschatka, which I had received from Admiral Lütke. That officer has since combined several tide observations of his own and others, in a "Notice sur les Marées Périodiques dans le Grand Ocean Boreal et dans la Mer Glaciale," and has, in a chart of the North Pacific attached to this notice, drawn the cotidal lines belonging to that ocean. I will place here an extract from this Notice, pointing out the grounds on which he proceeds, and the difficulty in which he finds himself involved.

"In examining the order of sequence of the tide-hours on the west coast of America and in the Aleutian Isles, we see clearly that the tide coming from the south runs along this shore to the north-west; and then to the west along the chain of the Aleutians to the coast of Kamtschatka, employing twelve hours in passing from San Blas to Petropaulofsk. Proceeding from this point, we can no longer follow it with the same certainty, having no data for the tides at the Kurile Isles and on the eastern coast of Japan. Further to the south we again have some obser-

* Part I., p. 161.

vations (Loo Choo, Bonine, Rawak, Guahan) which show well enough the position of the cotidal lines of IX^h., X^h. and XI^h. And continuing these lines with those more to the north, we recognise here, with sufficient probability, the same undulation which, stopped by the coast of Kamtschatka, takes a southerly direction and reaches those shores. The difference noticed above between *the age of the tide* at Rawak and at the stations more to the north, agrees very well with this supposition, according to which the tide would make three-fourths of the circuit of this sea in eighteen hours.

"But here we are stopped by a dilemma very difficult to resolve, at least without the help of new observations. About 200 leagues to the south-east of the line of XI^h., we should, according to what we have hitherto seen, expect to find the line of XII^h., or thereabouts. But we find here at two or three places (Ualan, Radak, &c.), the tide-hour four hours or five hours, and we ask ourselves what can it be which retards the propagation of the tide-wave, so as to make it employ six hours in traversing a space which under ordinary circumstances it would pass over in one hour."

It will appear, I think, from the whole of what has preceded, that this difficulty is one which occurs in every part of the ocean, and can only be explained, or indeed its nature ascertained, by drawing the cotidal lines on a larger mass of observations.

29. Capt. Sir E. Belcher's voyage has also supplied numerous observations of the tides in various parts of the Pacific, which he has kindly placed at my disposal, and which have been discussed by Mr. D. Ross of the Admiralty. In like manner Mr. Ross has discussed tide observations supplied to me by Capt. Sir James Clarke Ross during his last voyage ; also some tide observations by Capt. Stokes, Capt. Kellet, and some others which may be noticed hereafter.

30. I may take the liberty of remarking that many of these observations, being given as they were originally recorded, show how very imperfect is the accuracy which can be obtained on such occasions, when the series of observations is short and the apparatus necessarily rude. They thus show that the "establishment" deduced from such observations cannot be considered as entitled to much confidence. I will, for the sake of example, explain further what I mean. The "establishment" is the interval of time by which the tide follows the moon, either at new and full moon, which is the *vulgar establishment*, or its mean value, which is my *corrected establishment*; and whichever be taken, the term has no meaning, and observations made to determine the establishment have no value, except so far as this interval is definite in itself, and is determined by the observations. If the tides be regular and the observations good, this interval differs from day to day, sometimes as much as ten or twelve minutes on successive days, (in virtue of the *semimensual inequality*,) but on the whole fortnight, little more than an hour (its most usual value is $1^h\ 20^m$). If therefore this interval, as observed in successive tides, differ by two or three hours, there must be some cause of irregularity either in the tides or in the mode of obser-

vation, which deprives the results of their value. We should not be justified in disturbing any previous determinations of the "establishment," on the ground of new observations of this kind; although it is true, that for aught we know, the previous determinations may rest on no better data. To show the extent to which this may go, I will give the values of the lunitidal intervals as they result from some recent observations on the west coast of America.

Lunitidal intervals observed.

Place.	Lat. S.	Long.	Interval.		Range.		Observed.
			Greatest.	Least.	Greatest.	Least.	
	° ′		h m	h m	ft.	ft.	
Callao	12 3	......	7 25	3 0	2	½	12 tides.
Puna Island	3 37	......	6 16	3 24	13	11	15 tides.
	North.						
Panama	8 57	......	4 0	1 48	15	9	17 tides.
Nicoya	9 56	......	3 50	1 24	10	6	40 tides.
Reafejo	12 28	......	3 20	1 10	13	1	31 tides.
Acapulco	16 50	......	3 0	1 12	1½	½	10 tides.
Magdalena Bay	24 38	......	9 30	5 24	6	3	13 tides.

I have added the range of the tide, that it may be seen whether the case was one in which much accuracy was to be expected. It is evident that at such places as Callao, where the tide ranges only from half a foot to a foot and a half, or two feet, it must be very difficult, especially by any rude apparatus, to determine with any accuracy the time of high water; accordingly we see that the interval, as observed, varies nearly four hours and a half. But at a place where, as at Panama, the tide ranges from nine to fifteen feet, it is more surprising to find the lunitidal interval varying as much as $2^h\ 12^m$. I am not however disposed to question the correctness of these observations; for the *diurnal inequality*, added to the semimensual inequality, may make the difference as great as this: and it will be observed that the mean result, $2^h\ 54^m$, would give an "establishment" about $3^h\ 24^m$; not differing much from the establishment given by Mr. Lloyd*, namely $3^h\ 20^m$.

31. I shall now proceed to give the tide-hours for the coasts of the Pacific, according to the best accounts which I find, judging them in the manner I have described. After noticing the course of the tide near Cape Horn, I shall follow it along the whole western coast of America, till, in the north, we reach the Aleutian Isles; and then, following this chain of islands, to the shores of Kamtschatka. I shall then consider the islands in the central parts of the Pacific, and proceed from them westward, according to my materials.

I have already, in my first Essay, shown that round Cape Horn, the tide-wave has an *easterly* motion. Thus, as I have there said, according to Captain King†, at Cape Pillar it is high water at 1^h on the day of full and change; at York Minster, 5° of longitude to the east, it is at 3^h; at Cape Horn, 3° further east, it is at $3\frac{1}{2}^h$; in Good

* Philosophical Transactions, 1830.

† Sailing Directions, p. 96, and Table, pp. 13, 14.

Success Bay, in Strait le Maire, the hour is 4; on the east side of Strait le Maire it is 5^h. It appears also from Captain King's observations*, that the tide travels in the same direction along the coast, that is, to the northward, on the eastern shore of Patagonia. This direction appears by Capt. FitzRoy's Tables† to continue as far northward as latitude 40°, the wave employing about twelve hours in travelling from latitude 50° to 40° south. Along this coast the tides are very large; at Gallegos River, in latitude 52°, they rise forty-six feet. This circumstance might lead us to imagine that they are the result of accumulated waves converging from the north as well as the south; and this is probably the case. Yet it is remarkable, especially when considered in connection with this view, that in the great estuary of the Plata there is no perceptible tide‡.

I shall not, however, dwell at present upon the tides of the Atlantic, and shall proceed to those of the western coast of America.

Those observations which I now take account of for the first time are marked with an asterisk (*).

West Coast of America (South).

	Lat. S.	Long. W.	Tide-hour.	Greenwich time.		
	° ′	h m	h m	h m	ft.	
Cape Pillar	52 46	5 0	1 0	6 0		
Chiloe	41 52	4 57	11 30	5 0	6	King's Table, p. 15.
———	......	......	12 30	......	12	Heron, R. B.
Valdivia	39 50	4 56	11 30	3 26	...	Norie—Purdy.
———	......	......	......	3 30	...	*FitzRoy, p. 284.
Conception	36 49	4 53	10 0	2 53	...	Malaspina.
Valparaiso	33 2	4 45	9 25	2 10	...	Malaspina.
———	......	......	9 40	......	4	*Du Petit-Thouars.
Coquimbo	29 54	4 45	9 20	2 5	...	Remark Books.
———	......	......	......	2 0	...	*FitzRoy, p. 285.
Iquique	20 0	......	......	1 30	...	*FitzRoy, p. 285.
Arica	18 28	4 40	......	......	...	*FitzRoy, p. 285.
Callao	12 4	5 7	6 15	......	...	Malaspina.
———	......	......	6 0	11 7	2½	*Du Petit-Thouars.
Payta	5 3	5 24	3 18	......	6	*Du Petit-Thouars.
Guayaquil.						
Puna Island	3 27	5 19	5 0	......	12	*Capt. Sir E. Belcher.
Punta Piedra	3 30	5 19	6 10	......	10	*Capt. Kellet.
Galapagos.						
Charles Island	1 15	6 2	2 10	8 12	...	*FitzRoy, p. 84.
Chatham Island	1 0	5 54	3 30	......	...	Purdy, E. M.
Ile Charles	......	......	3 19	......	6	*Du Petit-Thouars.
	North.					
Cocos	5 34	5 15	2 10	7 35	...	Purdy, E. M., p. 50. Vancouver.
———	......	......	4 0	......	...	Purdy, E. M., p. 47. Colnett.
———	......	......	......	8 0	...	*FitzRoy, p. 285.
Panama	8 57	......	3 20	......	...	Lloyd, Phil. Trans., 1830.
Panama Bay	......	5 18	4 0	......	13	*Capt. Kellet.
———	......	......	3 36	8 54	15	*Sir E. Belcher.

* Sailing Directions, p. 17. † App. p. 66. ‡ FitzRoy, App. p. 280.

West coast of America (North).

	Lat. N.	Long. W.	Time, high water.	Greenwich time.	Rise.	
	° ′	h m	h m	h m	feet.	
Nicoya	....	....	....	8 0	....	*FitzRoy.
Island S. Lucas	9 56	5 42	3 0	8 42	6	*Sir E. Belcher.
Realejo	12 28	5 48	3 0	8 48	6	*Sir E. Belcher. Great irregularities.
Acapulco	16 50	6 39	2 41	9 20	1	*Sir E. Belcher.
————	....	....	3 5	9 44	2	*Du Petit-Thouars.
Magdalena Bay	24 38	7 28	8 30?	....	6	*Sir E. Belcher. Very anomalous.
————	....	....	7 37	3 5	10	*Du Petit-Thouars.
San Blas........................	21 32	7 1	8 5	3 6	....	Mem. on South America.
————	....	....	....	3 0	....	*FitzRoy.
————	....	....	9 41	4 42	....	*Beechey.
Mazatlan........................	23 0	7 10	9 50	5 0	....	Beechey.
King George's Sound	35 2	7 52	11 30?	....	3	*Sir E. Belcher. Very anomalous.
Monterey	36 36	8 6	9 42	5 48	....	Beechey.
————	....	....	9 52	....	7	*Du Petit-Thouars.
San Francisco	37 48	8 9	12 30?	....	6	*Sir E. Belcher. Very anomalous. Diurnal inequalities, high water and low water, large in height and times.
————	....	....	10 52	....	....	Beechey and Malaspina.
————	....	....	10 33	6 42	6	*Russian navigators (Lütke's notice).
Port Bodega	38 19	8 11	11 41	7 52	....	*Russian navigators.
Columbia River	46 16	8 16	1 0	9 16	8	*Sir E. Belcher.
————	....	....	1 30	....	....	Vancouver.
Straits of Juan de Fuca........	48 0	....	12 30	....	8	*Capt. Kellet.
Nootka Sound	49 36	8 25	12 30	8 55	....	*Cook.
Sitka	57 2	9 2	10 40	....	12	*Sir E. Belcher. Great diurnal inequality.
————	....	....	12 33	9 35	12	*Lütke.
F. Nicolaefsky (Cook's River)	60 15	10 6	3 49	1 55	28	*Wrangell.

From this point the coast turns westward, and the stations are arranged according to longitude without regard to their latitude.

	Lat. N.	Long. W.	Time, high water.	Greenwich time.	Rise.	
	° ′	h m	h m	h m	feet.	
American Coast.						
*F. Nicolaëfsky (Cook's River)............	60 15	10 6	3 49	1 55	28	*Wrangell.
Harbour of St. Paul (Kadiak Island) ...	57 46	10 8	0 30	10 38	10	*Russian navigators.
Harbour 3 hiérarq..........................	57 8	10 12	0 19	10 31	10	*Russian navigators.
Nouchagak Bay..............................	58 31	10 34	2 14	0 48	12	*Wrangell.
Aleutian Isles						
*St. Paul Island..............................	57 10	11 20	3 47	3 7	4	*Russian navigators.
Atkha Island	52 25	11 36	0 20	11 56	5	*Russ. navigators: doubtful.
Attou Island	52 57	12 28	0 48	1 16	22	*Russ. navigators: doubtful.
Kamtschatka.						
Petropaulofsk..............................	53 1	13 26	3 38	5 4	4	*Lütke in 1827.
————	....	....	3 43	5 9	....	*Lütke in 1828, diurn. ineq.
————	....	....	3 54	....	3	*Du Petit-Thouars.

32. Looking at the general assemblage of the numbers which occur in the column marked "Greenwich time," it is evident that the tide-wave of the hour VIII. which

is at Cocos Island and at the Galapagos about eight o'clock, comes to the continent at Nicoya and Realejo about 10° and 12° north latitude at about three quarters of an hour later; while the tide is at hours later than this, both to the northward and to the southward. Proceeding first southward, we find the line of XI. not far from Callao, that of II. near Coquimbo, or Valparaiso, and that of III. and a half, near Valdivia: and further south, we have the line of V. at Chiloe, and of VI. at Cape Pillar, whence the wave moves to the eastward round Cape Horn, as already stated. Considering these points as fixed, it is easy to interpolate the other cotidal lines along this coast. The observed hour at Guayaquil is later than its position would give, a result which we should expect, since the tide will occupy some time in travelling up the gulf in which Guayaquil is situated.

Again, proceeding from Nicoya and Realejo to the northward, we find a like progression of tide-hours. The line of X. is not far from Acapulco, according to the data here collected. But the tide at Acapulco is small, and hence the accuracy of the result is doubtful. Perhaps the smallness of the tide is an indication that the point of divergence of the tide-wave, which occurs on this part of the American coast, is not far from Acapulco. It appears that the line of III. passes near San Blas, and also near the Bay of S. Magdalena on the coast of California. At Mazatlan, somewhat within the Gulf of California, the time is an hour or two later, as we should expect.

When we reach Monterey and S. Francisco, the hour is about VI. according to Capt. Beechey's observations. The more recent ones are too anomalous to proceed upon. At Port Bodega, in lat. 38°, we have the VIII. tide line; and at Nootka Sound, Cook's observations, which give $12^h\ 30^m$ (whence Greenwich IX^h. nearly), are confirmed by Capt. Kellet's observations in the Straits of Fuca, south of Vancouver's Island. The next point is the Russian settlement of New Archangel, in the island of Sitka, where the tides exhibit very curious features, as I have already stated, from the observations of Admiral Lütke*, and as I find further confirmed by the observations of Sir E. Belcher. The line belonging to Sitka appears to be IX. and a half.

From this point, we depend upon Russian observations which are given by Admiral Lütke in his "Notice." These enable us to see that the cotidal lines bend, as usual, deep into the head of the bay in which is Cook's river in lat. 60°. The coast here trends to the west, and the wave follows it and pursues its course along the chain of the Aleutian Isles, where it has been traced by Admiral Lütke and the navigators of the Russo-American Company. It appears that the lines of XI., XII., I., II., fall near this chain; and that the line of V. is near the coast of Kamtschatka. It is not difficult to arrange the cotidal lines so as to conform to these data.

Admiral Lütke has observed the tides at other places on the Asiatic coast, as far north as 65°, but I shall not attempt to arrange them.

* See Philosophical Transactions, 1840, Part I. p. 107.

33. Our next attempt must be to arrange the tides of the oceanic isles of the Pacific, taking in the first place those south of the Equator.

Isles of the Pacific (South).

	Lat. S.	Long. W.	Time, high water.	Greenwich time.		
	° ′	h m	h m	h m	feet.	
Easter Island	27 9	7 20	2 0	9 20	......	Noric.
Gambier's Group	23 0	9 0	1 50	10 50	......	Beechey.
Lagoon Island	18 0	9 18	12 30	9 48	......	Cook, Phil. Trans. 1772.
———	....	....	11 15	....	......	Lalande.
Marquesas.						
Resolution Bay	9 53	9 20	5 7	2 27	6	*Du Petit-Thouars.
Low Archipelago.						
Bow Island	18 6	9 23	6 30	3 53	1	*Belcher.
Tahiti	....	....	Anomalous.			*Belcher.
Tonga Isles.						
Annamooha	20 15	11 40	6 0	6 20		
Eooa	....	11 40	7 0	7 20		
Tongataboo	21 8	11 41	6 50	7 19		
Wallis Island	13 26	11 44	5 0	4 44	......	Zebra, R. B.
Feejee Islands.						
Nakulau	18 9	12 6	8 30	8 36	5	*Belcher. Diurn. ineq.
Mathuaha	....	....	5 30	....	6	*Wilkes(U.S.Ex.111.322).
Ooolau	....	....	6 10	....		
New Hebrides.						
Tanna, Port Resolution	19 32	12 41	5 45	6 26	......	Lalande.
New Caledonia	20 17	13 2	6 30	7 02	......	Norie.
Norfolk Island	29 0	12 48	7 45	8 33	......	Norie.
New Zealand, Tobago Bay	38 22	12 57	6 0	6 7	......	Cook.
New Ireland.						
Carteret's Harbour	4 39	13 50	Anomalous.		......	*Belcher.

We have also the following observations given north of the equator.

Isles of the Pacific (North).

	Lat. N.	Long. W.	Time, high water.	Greenwich time.		
	° ′	h m	h m	h m	feet.	
Sandwich Isles.						
Honoloulou	21 18	10 32	3 35	2 7	2	*Du Petit-Thouars.
———	....	....	2 55	....	......	*Kotzebue.
Caroline Isles.						
Ualan	5 15	13 7	3 35	4 42	5	*Lütke.
Ladrone Isles.						
Guahan	13 32	14 20	8 23 ?	10 43	......	*Freycinet.
Bonin Isles	26 52	14 29	6 43	9 12	3	*Lütke.
———	....	....	6 3	....	......	*Beechey.
Loo Choo Isles	26 30	15 28	6 28	9 56	6	*Beechey.
Sand Isle, Samboango	6 55	15 52	7 36	11 28	4	*Belcher.
Bashee Group.						
Baton Isle	22 0	15 50	....	....	......	*Belcher.
Corean Archipelago	34 17	15 51	4 49	8 40	13	*Belcher. Anomalous.
Patchusan	26 20	15 41	6 36	10 17	5	*Belcher. Diurn. in.
Hong Kong	22 12	16 23	9 37	2 0	6	*Belcher. Diurn. in.
Amoy Harbour	24 16	16 8	12 52	5 0	18	*Capt. H. Smith.
Santubon	1 48	16 39	4 21	9 0	12	*Belcher.

These observations, especially those south of the equator, appear to imply a general motion westward of the tide-wave; but I conceive that they are much too few and too unconnected to justify me in drawing cotidal lines; besides which, the smallness of the tides in the central parts of the ocean makes the observations more than usually doubtful, and is accompanied by some circumstances inconsistent with the notion of a simple progressive wave as the representation of the tidal phenomena of those seas. I will consider those circumstances for a moment.

Tides of the Central Pacific.

34. The tides over a great portion of the central part of the Pacific are so small, that we may consider the lunar tide as almost vanishing. Thus at Bow Island it is stated as only one foot; at Tahiti it is hardly more; at the Sandwich Islands it is two feet; and even at New Ireland, where we are no longer in the central space, but among the larger islands to the west of it, the tide is only about two feet. But moreover at some at least of these places the tide, small as it is, is not the *lunar* tide following its ordinary laws. At Tahiti, for instance, the time of high water appears never to deviate from noon by more than a certain difference, although Sir E. Belcher has shown that it varies from about nine o'clock in the forenoon to three in the afternoon*. At Bow Island there appears reason to believe that the limits are much the same, and perhaps at Carteret's harbour in New Ireland. Now it will easily be seen that such a result as this would follow, if we were to suppose the tidal influence of the sun and of the moon to be equal. On this supposition, it is plain that the high water would always occur half-way between the sun's transit and the moon's transit. Hence at new moon the high water would be at noon; as the moon went away to the eastward of the sun, the tide would be later and smaller; till, when the moon was at 6^h distance from the sun, the tide would be at 3^h; but would in fact vanish. After this point, the tide would reappear at 9^h A.M., or a little later, the inferior transit of the moon now taking the place of the superior one, in determining the tide; and from this time the tide would be gradually later and larger, till, at full moon, it would again be at noon; and so on. This appears to agree pretty well with the phenomena of the tides at Tahiti, as determined by Sir E. Belcher.

35. A more minute examination of the tides in these regions will enable us to pronounce more decidedly whether the law of the phenomena is that which has been just stated. And if it appear that the phenomena do follow this law, we shall have further to consider how such a motion of the sea in those parts is to be combined with the very different movements which occur in other places, and what is the general movement of the ocean which they indicate; whether, for instance, they are best explained by looking upon the solar and lunar parts of the tide as produced by two separate waves, which may increase and diminish separately, and may start from different epochs in their motions. I shall not now pursue this point further; nor

* Philosophical Transactions, 1843, Part I.

shall I further examine how far the phenomena approach to the cases of fluid motion already described, in which there is a marked wave at the outskirts of the mass, and an approximate quiescence of the surface in the central parts; namely, the case of a stationary undulation, and of a revolving undulation*. I may remark, however, that the latter supposition, that of a revolving undulation by which the tide is carried from California northwards along the American shore and to the coast of Kamtschatka, while the cotidal lines converge to some central point in the North Pacific, would explain the smallness of the tides at the Sandwich Isles.

36. When we proceed westward from the central parts of the South Pacific to New Zealand and Australia, we again find the feature which we have already noticed in the tides; namely, that the cotidal lines run nearly parallel to the shore in its neighbourhood, but that we cannot easily infer the oceanic from the littoral tides; for the tide lines of VI., VII., VIII., IX., X., XI. succeed each other along the coast of New Zealand, and apparently double round its northern and southern extremities, as we should expect from the laws of fluids. But yet the line of X. recurs again on the coast of Australia, and is succeeded by later hours as we proceed northward and southward from about lat. 30° south. Cotidal lines may be drawn to accommodate themselves to these data; but of these lines the parts which occupy the ocean between New Zealand and Australia must be very doubtful.

I have been favoured by Sir James Ross with about a month's observations of the tides in the Bay of Islands in New Zealand, which I may refer to hereafter; but my means of tracing the *progress* of the tides along the coast of New Zealand still depend upon Captain Cook's statements†. The longitude of New Zealand is so nearly 12^h that the local tide-hour may be considered as coincident with the Greenwich hour. At Tobago Bay, near the most easterly point of these islands, the time is 6^h. In proceeding to Mercury Bay and the Bay of Islands on the north-east coast, it becomes $7^h\ 30^m$ and 8^h respectively. And in proceeding southward from Tobago Bay, we have also a retardation. At Queen Charlotte's Sound and Admiralty Sound, in Cook's Strait, which separates the two islands, it is $9^h\ 30^m$ and 10^h, the strait producing a considerable retardation. At Dusky Bay, the southern point of the island, the time is $10^h\ 57^m$.

Tides of Australia.

37. With regard to the coast of Australia, I have been furnished with a considerable quantity of tide observations, resulting principally from the surveys of Captain Blackwood and Captain Stokes. These are now to be added to the observations formerly collected from Captains Cook, Flinders and King. The earliest tides, as I formerly observed, appear to occur between the latitudes 24° and 35° south. I shall therefore proceed from this part southwards and northwards.

* Or rather a revolving cotidal line. See Article 23.

† Philosophical Transactions, 1772.

In the case of the recent observations I have given, not the "establishment," but the original observations, from which, of course, the establishment is to be deduced. It is much more satisfactory to have the observations themselves than a conventional mode of expressing the result; but the attempt to combine the two classes of data shows the difficulty into which we are led by this convention of an "establishment" in the vulgar sense. For instance, at Port Bowen, the time of high water at new moon is $9^h\ 46^m$; but the interval at which the tide follows the moon varies from $8^h\ 37^m$ to $11^h\ 9^m$. The mean of these, the *corrected establishment*, is $9^h\ 53^m$; and in general the vulgar establishment is, as I have said, about 30^m greater than this, and therefore would here be $10^h\ 23^m$. The great discrepancy of the "establishment" obtained in these two different ways, arises from an anomalous course of the lunitidal intervals which appears in these observations, the mean interval being considerably removed away from the days of new and full moon. But the same discrepancy appears more or less in most cases. I combine the old and new observations as fairly as I can.

Australia (North side).

	Lat. S.	Long. E.	Time, high water.	Greenwich time.		
	° ′	h m	h m	h m	feet.	
Botany Bay	34 0	10 4	8 0	9 56	....	Cook, Flinders.
Hervey's Bay	24 40	....	8 0	....	....	Flinders.
Bustard Bay	24 30	....	8 0	....	....	Cook.
Port Curtis	23 52	....	8^h to 9^h	....	....	Flinders.
Keppel Bay	23 8	....	9 30	....	....	Flinders.
Port Bowen	22 28	10 3	10 0	11 57	15	Flinders.
Strong-tide Passage	....	....	10 0	....	....	*Blackwood.
Shoalwater Bay	....	....	10 30	....	....	Flinders.
Thirsty Sound	22 6	....	10 45	....	....	Flinders.
Broad Sound	....	....	11 0	....	....	Flinders.
Percy Islands	21 19	....	11 0	....	....	Flinders.
Cumberland Isles	....	....	11 0	....	....	King.
Cape Upstart	19 40	9 51	10 30	12 39	....	*Blackwood.
Endeavour River	15 27	....	9 30	....	....	Cook.
Princess Charlotte's Bay	14 20	....	8 0	....	....	King (ii. 251).
Endeavour Strait	10 37	....	1 30	....	....	King (ii. 251).
Murray's Island, Torres Straits	9 55	....	10 30	....	....	Flinders.
Endeavour Straits	....	9 25	1 30	4 5	....	Cook.
Liverpool River and Goulburn Island	....	....	6 0	....	....	King (ii. 309).
Port Essington	11 22	8 48	4 48	8 0	....	*Sir Gordon Bremer.
Saint Asaph's Bay	....	....	5 45	....	....	King (ii. 237).
King's Cove	....	....	5 15	....	....	King (ii. 237).
Vansittart Bay	....	8 22	9 15	12 53	....	King (ii. 324).
Montague Bay	....	....	12 0	....	....	King (ii. 324).
Canning Bay	....	8 20	12 0	3 40	....	King (ii. 324).
Prince Regent's River	....	....	12 0	....	....	King (ii. 324).
Roebuck Bay	....	8 8	....	....	30	King, Dampier.

The tides at Port Essington will be the subject of more special consideration hereafter. But I shall first consider the progress of the tide to the south side of Australia.

Australia (South side).

	Lat. S.	Long. E.	High water. Greatest interval.	High water. Least interval.	Mean.	Low water. Greatest interval.	Low water. Least interval.	Mean.	Greatest range.	Least range.	Diurnal inequality.
	°	h m	h m	h m		h m	h m		ft.	ft.	
Cape Upstart	20	9 50	11 0	9 0	10 0	6 30	2 40	4 35	10	2	Not observed. *Belcher.
Port Bowen	22½	10 3	10 40	9 0	9 50	4 30	2 30	3 30	15	1	High water, 3 ft.; low water, 1 ft. *B.
Bass's Straits.											
Kent's Group	39½	9 49	11 50	9 40	10 45	5 40	4 0	4 50	6½	6	Low water? *Stokes.
Port Refuge	39	9 45	11 36	10 50	11 13	5 14	4 28	4 51	6	5½	High water? low water, 2 ft. *Stokes.
Western Port	38½	9 41	1 30	1 0	1 15	7 40	7 0	7 20	6	5	High water, 2 ft.; low water, 2½ ft. *S.
Port Phillip	38	9 38	12 48	11 0	11 54	7 10	4 50	6 0	3	2	Low water? *Stokes.
Preservation Island	40½	9 52	11 12	10 40	10 56	5 30	4 50	5 10	7	4	Low water, 2 ft. *Stokes.
Swan Island	41	9 52	10 20	9 40	10 0	4 50	3 12	4 1	6	3	
Port Dalrymple	41	9 47	12 14	11 24	11 49	7 36	5 40	6 38	9	6	High water? low water, 3 ft. *Stokes.
Circular Head	41	9 41	12 0	11 38	11 49	6 0	5 14	5 37	9	7	High water? low water, 2½ ft. *S.
Three Hummock Island	40½	9 36	12 20	11 30	11 55	6 30	4 50	5 40	6	5	High water, 1 ft.; low water, 1 ft. *S.
Adelaide	35	9 14	6 24	3 24	4 54	13 0	9 30	11 15	7	3	High water, 3 ft.; low water, 1 ft.
King George's Sound	35	7 52									
Swan River	32	7 43	10 48	7 0	8 54	8 48	6 30	4 39	3	1	Single day tides. Flinders. King.
Latitude Island	28¾	7 35	11 0	9 12	10 6	5 30	3 0	4 15	2	1	Di. in. of *times*, high and low water. *S.
East Wallaby	28¾	7 35	11 30	8 0	9 45	6 10	2 30	4 20	2	1	Di. in. of t. and h. H.W. and L.W. *S.
Depuch Island	20½	7 50	12 0	10 12	11 6	5 30	4 12	4 51	14	6	High water, 2 ft.; low water, 2 ft. *S.
Port Essington	11	8 48	3 30	2 0	2 50	10 12	9 0	9 36	8	2	High water, 2 ft.; low water, 8 ft.

Supposing the "establishment" of a place on this coast to be about 30^{m} more than the mean lunitidal interval (that is, than the *corrected establishment*), we can connect them with the preceding observations. Thus we shall have

	Long. West.	Time of high water.	Greenwich time.
Port Phillip	$14^{h}\ 22^{m}$	$12^{h}\ 24^{m}$	$2^{h}\ 46^{m}$
Adelaide	14 46	5 24	8 10
Port Essington . . .	15 12	3 20	6 32
Swan River	16 17	9 24	1 41

At Adelaide I have a considerable series of observations which deserve special attention.

38. I have now put together all the principal materials which I have procured for determining the course of the tides of the Pacific. But it is apparent from what has been said, that the materials are insufficient to give us any complete or consistent view of the tidal movements of the waters of that ocean and the neighbouring seas.

39. I may observe, moreover, that there appears to be little chance that our knowledge of these tides will ever be much increased by observations made in voyages principally directed to other objects. Although, in the surveying and exploring voyages since Captain King's, many tide observations on the coasts and at the islands of the Pacific have been made, and many of them with care and skill, we have scarcely any material fact added to our knowledge; and the cotidal lines for the shores of America, New Zealand and Australia, as I drew them in 1833, remain with scarcely any alteration. Cook's observations at New Zealand, for instance, are for this purpose, better than any since made, because they are connected (being made by the same navigator and in close succession) and extend along a continuous shore. It is only by observations thus connected and having some degree of geographical continuity, that we can hope to trace the course of the tides.

I add a very slight and imperfect sketch of the cotidal lines of the Pacific as they result from the materials just examined. (See the Plate.)

The Diurnal Inequality.

40. There is a feature in the tides, very important both in its practical effect and in its bearing upon the theory, which has long been, in some degree, known to navigators, but of which they do not seem sufficiently to appreciate the generality, and of which they have commonly mistaken the law. I speak of the difference of the two tides on the same day, which I have termed the *Diurnal Inequality*. It was noticed at Plymouth and at Bristol as long ago as the time of NEWTON, and was, to a certain extent, explained by him. It was shown by LAPLACE to exist in the tides at Brest. It was found by Captains COOK, FLINDERS and KING, to be very large on the coasts of Australia. Its amount at Sincapore is enormous. Admiral LÜTKE found it to obtain in all his observations in the North Pacific. It appears in most parts of the Atlantic; and it is very considerable at the Falkland Isles, at Cape Horn, at New Zealand and at Kerguelen's Island. Indeed the cases where it does not occur are the exceptions, and commonly belong to shores where two tides are combined, as in the east coast of England, where it is nearly obliterated.

41. I have said that the laws of this inequality have commonly been mistaken. Thus navigators have spoken of the difference of *day* tides and *night* tides. Captains COOK, FLINDERS and KING, say that on the coast of Australia the night tides are *always* greater than the day tides. I have shown* that the seeming truth of this assertion was occasioned by the time of year at which those navigators respectively made their observations. Lieut. WILKES† says that on the coast of the Nisqually Indians in the Oregon Territory, the day tides during his observations were two feet higher than the night tides. This also was only a temporary rule. No reference of the inequality to day tides and night tides, or morning tides and evening tides, can express its law. It depends upon the moon's declination, and changes to alternate tides when the moon's declination changes from north to south, and *vice versâ*. Its rule is expressed in the following form‡:—

For moon's N. declination. { *Add* to the tide following moon's South transit,
Subtract from the tide following moon's N. transit,

For moon's S. declination. { *Subtract* from the tide following moon's S. transit,
Add to the tide following moon's N. transit,

the quantity added or subtracted being greater as the declination is greater; and the declination being taken for one, two, or three days previous to the tide. According to this law, the inequality has been introduced into the Tide Tables for Liverpool, Bristol and Plymouth.

It may be worth while to show that the rule of the diurnal tide which has just

* Philosophical Transactions, 1833, p. 221. † United States Exploring Expedition, iv. 417.

‡ Philosophical Transactions, 1837, p. 84.

been given may, *for some months*, produce the effect of making the afternoon tides greater than the morning tides, or *vice versâ*. Suppose the place to be one where the tide happens (in general terms*) soon after the moon's (*south* or *superior*) transit; then, beginning from new moon, the afternoon tide for a fortnight follows the south transit of the moon. Supposing that during this fortnight the moon has north declination; then the diurnal inequality is additive by the rule, and therefore the afternoon tide is, during this fortnight, the highest. Now at the end of a fortnight of north declination, the declination changes to south. But at the end of a fortnight, the afternoon tide begins to be that which follows the *north* or *inferior* transit of the moon; and therefore again, by the second part of the rule, the inequality is still additive, and the afternoon tide is still the greater. And this will continue to be the case till the points of no lunar declination are shifted away from the syzygies by the motion of the moon's nodes relative to the sun. But if the declination pass from north to south, or the reverse, at a different period from that which transfers the afternoon tides from one transit to the opposite one, we shall no longer have this apparent constancy in the relation of morning and afternoon tides. If, for instance, the tide-hour being such as has already been supposed, the change of declination, north and south, takes place when the tide is at four, five, six or seven o'clock; the afternoon tide will then (or rather one or two days later) change from being the greater to being the less, or *vice versâ*. Or if the tide-hour be six o'clock, the tide being (in general terms) six hours after the moon's transit, the afternoon tide will follow a south transit of the moon from the time when the moon is six hours west of the sun to the time when she is six hours east of him, and then change and follow a north transit; and so on alternately. Hence, if in this case the moon's ascending node be at six hours west from the sun, the declination will be north while the afternoon tide follows a south transit, and therefore the afternoon tide will be the greater for the whole lunation. But if, in this case, the node be in conjunction with the sun, the afternoon tide will change from smaller to larger, or the reverse, at the syzygy, that is, when the tide is at six o'clock; or rather, as I have said, a day or two later.

42. This last-mentioned circumstance, that the change in the features of the tides takes place a day or two, or perhaps longer, after the astronomical configuration by which it is determined, is common to all the empirical laws of the tides, as I have repeatedly remarked in the memoirs on this subject already published in the Transactions. It has recently been shown by Mr. Airy† that this is a result which follows from supposing the tidal motions of the sea to be affected by friction. The amount of this retardation of the phenomena for each place, or, as we may term it, the "age of the tide" relative to this particular phenomenon, the diurnal inequality, is different for different places; and must, for each place, be learnt from observation.

43. The account which I am now giving of the diurnal inequality supposes it to depend upon the moon alone; and so, for most purposes, it does almost entirely, as

* I leave out of consideration, in this explanation, the semimensual inequality.

† Tides and Waves, 452.

to its law, and in a very great degree as to its amount. The sun also will of course produce a diurnal inequality, which will depend upon his motions by laws similar to those which have been mentioned, and which will combine with and modify the lunar diurnal inequality. The determination of the solar, as well as of the lunar effect, is requisite, both for the construction of accurate Tide Tables for each place, and for the comparison of the tides with their theory. But this regard to the solar effect is not needed, either to make up such a general view of the progress of the tides as that at which I here aim, or to predict the general course of the tides so as to know whether the morning or afternoon tide will be the greater.

44. In stating that the lunar diurnal inequality alone appears in a conspicuous form in the facts, I do not rest merely upon theoretical views or upon a few cases; but am able to show it to be so, by numerous, distant, and extensive series of observations. These observations also enable me to trace the course and modifications of the diurnal inequality along the greater part of the shores of the Pacific; and I shall state their general results for that purpose.

I shall make this statement in words and figures, without offering to the reader any of the diagrams by which the results have been obtained. I am still of opinion that by far the best method of discussing tide observations is to lay down the original observations (both the heights and the lunitidal intervals) as the ordinates of curves. When this is done, the eye perceives at once several of the leading features of the case; the diurnal inequality and the semimensual inequality, especially. It perceives also where the glaring anomalies are; and we can often thus discern what amount of correction of the observations, on the ground of their general tendency, is allowable; and whether, as sometimes happens, the anomalies are so great that the observations are worthless.

45. The mode in which the diurnal inequality shows itself when tide observations are thus laid down in curves, has already been presented repeatedly in the Transactions; especially in the Sixth and Seventh Series of these Researches*. The inequality of heights appears in the *zigzag* form of the line drawn through the summits of the ordinates which represent the heights of successive tides. This zigzag structure is sometimes of a moderate degree of abruptness, as in the tides of the coast of North America, and of Portugal† and those of Plymouth‡, and sometimes extremely abrupt, as the heights of low water at Sincapore§. In this latter case, the diurnal inequality sometimes makes a difference of no less than *six feet* between the height of the morning and afternoon tide; the whole rise of the mean tide being only seven feet at springs, and the difference of mean spring and neap tides not more than two feet.

In the new observations which I have before me, there are other cases, quite as extraordinary as that of Sincapore. I shall now begin to state the general aspect of the diurnal inequality, as given by my new materials.

* Philosophical Transactions, 1836, Part II.; 1837, Part I.

† Ibid. 1836, Plate XXVII.

‡ Ibid. 1837, Plate II.

§ Ibid. Plate III.

The Diurnal Inequality from Observations.

46. I shall follow the same order of place which I have employed in stating the tide-hours. The amount of the diurnal inequality given is the greatest difference of the two successive tides.

FALKLAND ISLES. *Port Louis.*—An excellent series of observations by Sir JAMES C. ROSS, beginning May 10, 1842. Nine semilunations complete; and the dependence of the diurnal inequality on the moon's declination throughout regular.

High water heights: diurnal inequality = 2 ft. Difference of mean springs and neaps = 2 ft.

Low water heights: diurnal inequality = 2 ft. Difference of mean springs and neaps = 2 ft.

The age of the diurnal inequality is about a day and a half.

CAPE HORN. *St. Martin's Cove.*—Observations by Sir J. ROSS. Three semilunations, tolerably regular. Sir J. ROSS himself remarks*, that the diurnal inequality† "is as great here as at the Falkland Isles, and at first seemed to present most unaccountable irregularities; but the limited period of observation did not admit of thorough investigation."

High water heights: diurnal inequality = 2 ft. Difference of mean springs and neaps = $1\frac{1}{2}$ ft.

Low water heights: diurnal inequality = $1\frac{1}{2}$ ft. Difference of mean springs and neaps = $1\frac{1}{2}$ ft.

High water times. There is a large diurnal inequality in the lunitidal intervals, amounting sometimes to $2\frac{1}{2}$ hours, while the semimensual inequality also appears to amount to two hours.

The following results are derived from the new observations already quoted:—

CALLAO.—No definite result. See before.

GUAYAQUIL. *Puna Is.*—Apparently diurnal inequality in high water heights.

PANAMA.—No apparent diurnal inequality.

NICOYA.—High water heights: diurnal inequality = 1 ft. Low water heights: diurnal inequality = $\frac{1}{2}$ ft.

REALEJO.—High water heights: diurnal inequality = 1 ft. Low water heights: diurnal inequality = $\frac{1}{2}$ ft. Apparently diurnal inequality in the high water and low water times, but irregular in the observations.

ACAPULCO.—Apparently diurnal inequality, but small and doubtful.

MAGDALENA BAY.—High water heights: diurnal inequality = 1 ft; low water heights, not apparent. High water times: diurnal inequality = $2\frac{1}{2}$ hours: but these results doubtful, from the shortness of the series and its irregularity.

COLUMBIA RIVER.—Diurnal inequality not apparent.

KING GEORGE'S SOUND.—High water heights: diurnal inequality = $1\frac{3}{4}$ ft.

* Voyage, ii. 313.

† He says "the semidiurnal inequality," but it is plain that he means our diurnal inequality.

San Francisco.—High water heights: diurnal inequality=2 ft. Low water heights: diurnal inequality $= 3\frac{1}{2}$ ft. High water times: diurnal inequality = 2 hours. Low water times: diurnal inequality = 2 hours?

Sitka.—High water heights: diurnal inequality $= 2\frac{1}{2}$ ft. Low water heights: diurnal inequality = 4 ft. High water times: diurnal inequality = 2 hours. Low water times: diurnal inequality $= \frac{1}{2}$ hour: irregular. North of this point the observations are too imperfect to show diurnal inequality till we come to

Kamtschatka. *Petropaulofsk.*—High water heights: diurnal inequality = 1 ft. Low water heights: diurnal inequality = 4 ft. High water times: diurnal inequality = 4 hours. Low water times: diurnal inequality = 1 hour.

The tides of Sitka and Petropaulofsk have already been noticed in the Philosophical Transactions*. I there pointed out the large diurnal inequality of times at high water, and of heights at low water, which these places exhibit;—also the curious fact, that apparently the maximum and zero of the diurnal inequality of high water do not coincide in time with those of low water, but rather alternate with them.

47. I proceed to the islands of the Pacific.

Bow Island.—There appears to be a diurnal inequality of the time of high water, but necessarily doubtful, from the smallness of the tide.

Tahiti.—Here also there appears, in a portion of the tides, to be an alternation of late and early, approaching to the nature of diurnal inequality.

Feejee Islands.—High water heights: an evident diurnal inequality of half a foot.

New Ireland.—Something of the nature of diurnal inequality in the high water times, but very doubtful.

Corean Archipelago. Black Island.—High water heights: diurnal inequality = 2 ft. High water times: diurnal inequality = 2 hours.

New Zealand. Bay of Islands.—A series of observations including two semilunations by Sir J. C. Ross.

High water heights: diurnal inequality $= \frac{1}{2}$ ft.: very regular.

High water times: diurnal inequality $= 1\frac{1}{2}$ hour: regular.

We now come to Australia, and have the following results for the *heights*:—

	Diurnal inequality.	
Cape Upstart	Not observed.	
Port Bowen	High water, 3 ft.	Low water, 1 ft.
Bass Straits.		
Kent's Group		Low water?
Port Refuge	High water?	Low water, 2 ft.
Western Port	High water, 2 ft.	Low water, $2\frac{1}{2}$ ft.
Port Phillip		Low water?
Preservation Island . .		Low water, 2 ft.

* 1840, Part I. p. 161.

	Diurnal inequality.	
Port Dalrymple . . .	High water?	Low water, 3 ft.
Circular Head . . .	High water?	Low water, $2\frac{1}{2}$ ft.
Hummock Head . .	High water, 1 ft.	Low water, 1 ft.
Adelaide	High water, 3 ft.	Low water, 1 ft.

48. For Port Adelaide I have been furnished with twelve months' observations made by Mr. Bealton, Tide Surveyor at that Port.

Of these I have thrown the high water heights of ten semilunations into curves, and the result is very striking to the eye. There is a diurnal inequality which, for the months of December and January 1839, amounts to no less than four feet. In the succeeding months it is smaller, but equally regular as to its law. It follows the moon's declination with great regularity, at an interval of two days, altering from tide to alternate tide as the declination alters from north to south, and reversely. Its amount when greatest is nearly an inch for each degree of lunar declination. Probably a further discussion of these observations would show why the diurnal inequality for January, February, March 1840, is less than that for the two preceding months: but my present purpose does not require this labour. We may hope that these tides will receive a full discussion from some local mathematician.

49. At King George's Sound, near the south-west point of Australia, there is a large diurnal inequality of the times, which sometimes reduces the two daily tides to one*. Proceeding onwards to the western coast of Australia, and the settlement of Swan River, we find the diurnal inequality assuming new forms. The islands called Houtman's Abrolhos, in latitude 29° S., have been surveyed by Captain Stokes, and tide observations made at East Wallaby Island; of which the following is the result:—

East Wallaby Island.—High water heights: diurnal inequality $= 1\frac{1}{2}$ ft. Low water heights: diurnal inequality $= \frac{1}{2}$ ft. High water times: diurnal inequality $= 5$ hours. Low water times: diurnal inequality $= 2$ hours.

As the rise of the surface at this point is only about two feet altogether, there may be some doubt of the times; but the observations are remarkably regular and consistent.

The observations made at Rat Island, another of the same group, agree in the general character of the results.

If we proceed to the north along this coast, and pass round the N.W. Cape of Australia to Depuck Island, in latitude $20^\circ\frac{1}{2}$ S., longitude $7^h\ 51'$ E., we have (still from Captain Stokes's observations) a marked diurnal inequality of the heights, both high and low water, amounting to two feet in a tide of $14\frac{1}{2}$ ft.; but we discern no regular diurnal inequality of the times.

As apparently more connected with the South of Australia than with any other continental coast, I may here notice Kerguelen's Island in latitude 49° S. longitude $4\frac{3}{4}^h$ E. It appears by the observations of Sir J. Ross in 1840, that here also there is a large

* Philosophical Transactions, 1837, Part I. p. 83.

diurnal inequality, apparently amounting to one or two hours, in the time of high water, but no conspicuous diurnal inequality in the heights. The inequality or irregularity of the times is so great, that it is difficult, at this place, to speak of a tide-hour. The lunitidal interval varies from $11\frac{1}{2}$ hours to $16\frac{1}{2}$ hours.

50. I shall not entangle myself in the seas broken by innumerable large and small islands which extend from Torres Straits to the coasts of India, Arabia, and Africa; but I may observe that in these Indian seas the diurnal inequality is very marked in general, as I have already noticed for several places. I will refer back to these.

Phil. Trans. 1839, Part I. p. 164. Bassadore, at the entrance of the Persian Gulf.—A very large inequality of the times, amounting in some instances to more than *two hours*.

Surat and Gogah, in the Gulf of Cambay.—A large diurnal inequality in the heights of high water, amounting to not less than seven or eight feet. The age is two days.

Phil. Trans. 1833, p. 221. The River Hoogly.—The night tides highest from November to February: the day tides highest from March to October.

Phil. Trans. 1833, p. 224. Tonquin.—One tide in twenty-four hours.

Phil. Trans. 1837, Part I. p. 78. Sincapore.—A very large diurnal inequality, high water heights, diurnal inequality $= 1\frac{1}{2}$ ft. Low water heights, diurnal inequality $= 6$ ft.

The age of the diurnal inequality is a day and a quarter.

This very large diurnal inequality in the low water heights of Sincapore was, so far as I know, a new fact, when it was thus extracted from the observations, and was probably looked upon by most persons as a singular case. There is however good reason to believe that the same fact extends over the whole of the sea in that region. At least I have the means of showing that it prevails on the north coast of Australia, namely, at Port Essington.

51. Port Essington.—I am supplied with a considerable series of observations, somewhat interrupted, made by Sir Gordon Bremer, of which I have thrown five semilunations into curves. The observations indicate a diurnal inequality in the high water heights, but not very regularly. But at low water there is a diurnal inequality, of which the magnitude is not less than four feet, and this appears to be regular. The observations however are not exact enough to determine the age of this diurnal inequality.

I will add, as apparently connected with this, a reference to the peculiarities of the tides in the Gulf of Carpentaria. Flinders found only one ebb and one flood in the day, as I have noticed*.

I may remark, that the phenomena involved in the diurnal inequality are very curiously distributed to some of our principal colonies in Australia. At Adelaide, on the south, there is a large diurnal inequality in the *heights* of *high* water: at Port Essington, on the north, this inequality falls mainly upon the *low* water; while at

* Philosophical Transactions, 1833, p. 225.

Swan River, on the west, it principally affects the *times*. And the diurnal inequality which alters the low water four feet at Port Essington, and six feet at Sincapore, affects the high water to a still greater extent in the Gulf of Cambay, and disturbs the times at the entrance of the Persian Gulf.

Where the diurnal inequality affects the heights so much, it probably affects materially the time of tide in all places, as we know it does in some. And therefore over the whole of the Indian seas, the terms "establishment" and "tide-hour" must, without further explanation, be very ambiguous. And as we cannot know what facts are designated by such terms in books of navigation, all attempts to found upon such statements a coherent view of the tides of those seas, must be very precarious*.

52. Upon the occasion of the great series of tide observations made in 1835, I was led to trace the progress of the diurnal inequality wave along the coast of Europe. This I attempted in the Eighth Series of these Tide Researches†. The difficulty of separating the diurnal from the semidiurnal wave was there in some degree overcome. But even in that case, the problem was very imperfectly solved; for the coasts of the Pacific, and even for the coasts of Australia, our materials are too scanty and disconnected to give us any hope of success at present, if we were to attempt the same problem. With due materials, the diurnal inequality appears plainly to be separable from the semidiurnal tide; for it sometimes affects high water most, sometimes low water; sometimes the heights, sometimes the times; and is large sometimes when the semidiurnal tide is small, and sometimes exceeds the semimensual inequality.

53. It was remarked also, on the occasion of the observations of 1835‡, that the diurnal inequality on the coast of North America followed the changes of the moon's declination almost instantaneously; while on the coasts of Portugal, Spain and France, the changes of lunar declination were represented in the diurnal inequality two or three days later; and at the Cape of Good Hope, about the same time. I have already noticed that this feature throws great difficulty in the conception of that motion of the waves by which the tides are produced. It suggests the necessity of some new mode of conceiving that motion; a subject which I shall not here pursue.

* Philosophical Transactions, 1833, p. 200. † Ibid. 1837, Part II. ‡ Ibid. 1836, Part II. p. 302.

Trinity Lodge, Cambridge,
Nov. 8, 1847.

II. *On the Solution of Linear Differential Equations.* By Charles James Hargreave, Esq., B.L., F.R.S., *Professor of Jurisprudence in University College, London.*

Received June 10,—Read June 17, 1847.

I. *General Theorems of the Calculus of Operations.*

If the operation of differentiation with regard to the independent variable x be denoted by the symbol D, and if $\varphi(\mathrm{D})$ represent any function of D composed of integral powers positive or negative, or both positive and negative, it may easily be shown, that

$$\varphi(\mathrm{D})\{\psi x.u\}=\psi x.\varphi(\mathrm{D})u+\psi' x.\varphi'(\mathrm{D})u+\frac{1}{2}\psi'' x.\varphi''(\mathrm{D})u+\frac{1}{2.3}\psi''' x.\varphi'''(\mathrm{D})u+\ldots \quad . \quad (1.)$$

and that

$$\varphi x.\psi(\mathrm{D})u=\psi(\mathrm{D})\{\varphi x.u\}-\psi'(\mathrm{D})\{\varphi' x.u\}+\frac{1}{2}\psi''(\mathrm{D})\{\varphi'' x.u\}-\frac{1}{2.3}\psi'''(\mathrm{D})\{\varphi''' x.u\}+\ldots \quad (2.)$$

and these general theorems are expressions of the laws under which the operations of differentiation, direct and inverse, combine with those operations which are denoted by factors, functions of the independent variable.

It will be perceived that the right-hand side of each of these equations is a linear differential expression; and whenever an expression assumes or can be made to assume either of these forms, its solution is determined; for the equations

$$\varphi(\mathrm{D})\{\psi x.u\}=\mathrm{P} \text{ and } \varphi x.\psi(\mathrm{D})u=\mathrm{P}$$

are respectively equivalent to

$$u=(\psi x)^{-1}\{\varphi(\mathrm{D})\}^{-1}\mathrm{P} \text{ and } u=\{\psi(\mathrm{D})\}^{-1}((\varphi x)^{-1}\mathrm{P}).$$

The formulæ (1.) and (2.) indicate true propositions whenever they are interpretable; that is, whenever $\varphi(\mathrm{D})$ and $\psi(\mathrm{D})$ are capable of being expressed in integer powers of D. In conformity with recognized principles of reasoning, when the subjects of the process are regarded merely as symbols, we may assume that these propositions are true generally; and we shall therefore not hesitate to pronounce any interpretable result derived from the free use of these theorems true, although the intermediate steps of the process are not capable of a rational interpretation.

Bearing these remarks in mind, it will be seen, by an inspection of the above equations, that if in (1.) D be written for x, and $-x$ (or t) be written for D, we obtain

$$\varphi t.\psi(\mathrm{D}')u=\psi(\mathrm{D}')\{\varphi t.u\}-\psi'(\mathrm{D}')\{\varphi' t.u\}+\frac{1}{2}\psi''(\mathrm{D}')\{\varphi'' t.u\}- \ldots$$

D′ denoting the operation $\frac{d}{dt}$. This equation is identical in form with (2.), and is

therefore true; and the correctness of the result thus derived from this interchange of symbols leads to the inference, that if in any linear differential equation capable of being reduced to the form (1.) or (2.), and its symbolical solution, x be changed into D and D into $-x$, we shall obtain another form accompanied by its symbolical solution. Possibly the form so obtained and its solution may not be interpretable; but in every case in which they are interpretable, they will be found to be true; and if, by any transformation, meanings can be attached to those forms which appear to be unintelligible, they also will be found to be true.

It is an essential condition to be observed in all operations in which this process is used, that the solutions are to be preserved in a symbolical form; or, in other words, that the operations are not to be performed or suppressed. It would manifestly be a source of error to write zero for $(x^{-1}).0$, if in a subsequent stage x is to be converted into D.

The process may be conveniently exemplified by applying it to the general equation of the first order,

$$\varphi x.\mathrm{D}u+\psi x.u=\mathrm{X};$$

of which the solution, (the processes being preserved,) is

$$u=\varepsilon^{-\int\frac{\psi x}{\varphi x}dx}\{\mathrm{D}^{-1}(\varepsilon^{\int\frac{\psi x}{\varphi x}dx}(\varphi x)^{-1}\mathrm{X})\}, \text{ or } u=\varepsilon^{-\chi x}\mathrm{D}^{-1}(\varepsilon^{\chi x}(\varphi x)^{-1}\mathrm{X}).$$

If we make the proposed conversions, we have for the solution of

$$-\varphi(\mathrm{D})\{xu\}+\psi(\mathrm{D})u=\mathrm{X} \quad \text{. . . . . (3.)}$$

$$u=-\varepsilon^{-\chi\mathrm{D}}\{x^{-1}\varepsilon^{\chi}\ (\varphi\mathrm{D})^{-1}\mathrm{X}\}.$$

But equation (3.), by (1.), is equivalent to

$$x\varphi(\mathrm{D})u+(\varphi'(\mathrm{D})-\psi(\mathrm{D}))u=-\mathrm{X} \text{ or } \mathrm{X}_0.$$

Let $\qquad \varphi'(\mathrm{D})-\psi(\mathrm{D})=\lambda(\mathrm{D}), \text{ or } \psi(\mathrm{D})=\varphi'(\mathrm{D})-\lambda\mathrm{D}.$

Then $\qquad \chi(\mathrm{D})=\int\frac{\psi(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D}=\log\varphi(\mathrm{D})-\int\frac{\lambda(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D};$

and the solution assumes the form

$$u=(\varphi(\mathrm{D}))^{-1}\varepsilon^{\int\frac{\lambda(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D}}\{x^{-1}\varepsilon^{-\int\frac{\lambda(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D}}\mathrm{X}_0\} \quad \text{. . . . . (4.)}$$

the equation to be solved being

$$x\varphi(\mathrm{D})u+\lambda(\mathrm{D})u=\mathrm{X}_0.$$

This solution was first given by Mr. Boole in the Philosophical Magazine for February 1847. It indicates in a striking manner the interchange of symbols which is here proposed as a general theory; and leads naturally to the inquiry, whether such a conversion may not be extended to other forms.

I am not prepared to assert that the considerations stated above actually establish its validity as a theoretical process; but it possesses considerable practical utility, when applied to a subject in which the value of the result, if true, is in a great measure independent of the validity of the process.

In this paper I propose to apply the formula (1.), by the aid of Mr. BOOLE's solution above given, to the discovery of soluble forms of linear equations with variable coefficients; I shall also show that by the use of the conversion of symbols, many forms of solution apparently incapable of interpretation may be made to give useful results; and I shall point out a remarkable connection between the solutions thus obtained and the solutions of the same equations in the form of definite integrals.

Amongst the cases of (4.) which are obviously and immediately interpretable, may be mentioned

$$\left.\begin{aligned} &x\varphi(\mathrm{D})u+m\varphi'(\mathrm{D})u=\mathrm{X} \\ &u=\{\varphi(\mathrm{D})\}^{m-1}x^{-1}\{\varphi(\mathrm{D})\}^{-m}\mathrm{X}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

$$\left.\begin{aligned} &x\varphi(\mathrm{D})u+m\frac{\varphi(\mathrm{D})}{\mathrm{D}}u=\mathrm{X} \\ &u=\{\varphi(\mathrm{D})\}^{-1}\mathrm{D}^{m}\{x^{-1}\mathrm{D}^{-m}\mathrm{X}\};\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

but, as will be afterwards shown, most cases are interpretable when $\varphi(\mathrm{D})$ and $\lambda(\mathrm{D})$ assume the ordinary form, and consist only of integral powers.

II. *Application of these Theorems to the Solution of Equations.*

I proceed then to apply equations (5.), in conjunction with the original theorem (1.), to the solution in finite terms of forms of linear differential equations. Commencing with equations of the second degree, we have, by (1.),

$$\{\mathrm{D}^2+b\mathrm{D}+c^2\}(\psi x.u)=\psi x.\mathrm{D}^2u+(b\psi x+2\psi'x)\mathrm{D}u+(c^2\psi x+b\psi'x+\psi''x)u$$

$$\{2\mathrm{D}+b\}\qquad(\psi x.u)=\qquad 2\psi x.\mathrm{D}u+(b\psi x+2\psi'x)u.$$

Consequently equations included under the form

$$x\psi x.\mathrm{D}^2u+((bx+2m)\psi x+2x\psi'x)\mathrm{D}u+((c^2x+bm)\psi x+(bx+2m)\psi'x+x\psi''x)u=\mathrm{X},$$

$$\text{or}\quad \mathrm{D}^2u+\left(b+\frac{2m}{x}+\frac{2\psi'x}{\psi x}\right)\mathrm{D}u+\left(c^2+\frac{bm}{x}+\left(b+\frac{2m}{x}\right)\frac{\psi'x}{\psi x}+\frac{\psi''x}{\psi x}\right)u=(x\psi x)^{-1}\mathrm{X}=\mathrm{P}, \quad . \quad (7.)$$

are readily soluble, the solution being

$$\begin{aligned} u&=(\psi x)^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{-m}\mathrm{X}\} \\ &=(\psi x)^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{-m}(x\psi x.\mathrm{P})\}.\end{aligned}$$

When X or P is zero, the solution may be reduced to the simpler form,

$$u=(\psi x)^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{-1}.0\},$$

which will be found to be

$$\begin{aligned} u=(\psi x)^{-1}x^{-m}\Big\{&k\varepsilon^{\alpha x}\Big(1+(m-1)\frac{m}{(\beta-\alpha)x}+(m-1)\frac{m-2}{2}\frac{m(m+1)}{(\beta-\alpha)^2x^2} \\ &+(m-1)\frac{m-2}{2}\frac{m-3}{3}\frac{m(m+1)(m+2)}{(\beta-\alpha)^3x^3}+\,..\Big)+k'\varepsilon^{\beta x}\Big(1-(m-1)\frac{m}{(\beta-\alpha)x} \\ &+(m-1)\frac{m-2}{2}\frac{m(m+1)}{(\beta-\alpha)^2x^2}-(m-1)\frac{m-2}{2}\frac{m-3}{3}\frac{m(m+1)(m+2)}{(\beta-\alpha)^3x^3}+\,..\Big)\Big\}\end{aligned}$$

α and β being the roots of $t^2+bt+c^2=0$.

The solution here given is finite, in those cases only in which m is an integer positive or negative. When m is fractional, the undeveloped expression involves fractional operations.

If however $b=-2c$ so that the roots are equal, the solution assumes the form

$$u=(\psi x)^{-1}\varepsilon^{cx}(kx^{-2m+1}+k')$$

without any restriction upon the values of m.

The form (7.) deserves particular attention, as it will be found to include the most remarkable of the equations of the second order, which have heretofore been integrated by artificial methods.

Thus, if $\psi x=\varepsilon^{ax}$ we have the solution of

$$\mathrm{D}^2u+\left(b+2a+\frac{2m}{x}\right)\mathrm{D}u+\left(c^2+ab+a^2+(b+2a)\frac{m}{x}\right)u=\mathrm{P},$$

of which the well-known equation

$$\mathrm{D}^2u+\frac{2m}{x}\mathrm{D}u\pm c^2u=\mathrm{P} \quad \text{. . . . . (8.)}$$

is a particular case; the solution being

$$u=(\mathrm{D}^2\pm c^2)^{m-1}\{x^{-1}(\mathrm{D}^2\pm c^2)^{-m}(x\mathrm{P})\}.$$

If $\mathrm{P}=0$, this is reduced, taking the negative sign only, to

$$u=(\mathrm{D}^2-c^2)^{m-1}\{x^{-1}(k\varepsilon^{cx}+k'\varepsilon^{-cx})\},$$

which will be found to be

$$u=x^{-m}\Big\{k\varepsilon^{cx}\Big(1-(m-1)\frac{m}{2cx}+(m-1)\frac{m-2}{2}\frac{m(m+1)}{(2cx)^2}-\,..\Big)$$
$$+k'\varepsilon^{-cx}\Big(1+(m-1)\frac{m}{2cx}+(m-1)\frac{m-2}{2}\frac{m(m+1)}{(2cx)^2}+\,..\Big)\Big\};$$

and the solution is expressible in finite terms when m is a positive or negative integer. This equation is merely the simplest form of (7.), and is soluble by (5.) without the aid of (1.); for in it ψx is taken to be unity.

Now let $\psi x=x^n$; then the solution of

$$\mathrm{D}^2u+\left(b+\frac{2(m+n)}{x}\right)\mathrm{D}u+\left(c^2+\frac{b(m+n)}{x}+\frac{n(n-1)+2mn}{x^2}\right)u=\mathrm{P},$$

is $$u=x^{-n}(\mathrm{D}^2+b\mathrm{D}+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{-m}(x^{n+1}\mathrm{P})\}.$$

If in this $b=0$ and $n=-2m+1$, we have another form of solution of the equation

$$\mathrm{D}^2u-\frac{2(m-1)}{x}\mathrm{D}u+c^2u=\mathrm{P};$$

namely, $$u=x^{2m-1}(\mathrm{D}^2+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+c^2)^{-m}(x^{-2m+2}\mathrm{P})\}.$$

If $n=-m$, we have for the solution of

$$\mathrm{D}^2u+b\mathrm{D}u+\left(c^2-\frac{m(m-1)}{x^2}\right)u=\mathrm{P}$$

$$u=x^m(\mathrm{D}^2+b\mathrm{D}+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+b\mathrm{D}+c^2)^{-m}(x^{-m+1}\mathrm{P}\};$$

of which the well-known equation

$$D^2u-\left(\frac{m(m-1)}{x^2}\pm c^2\right)u=0$$

is a case; the solution being

$$u=x^m(D^2+c^2)^{m-1}\{x^{-1}(k\sin cx+k'\cos cx)\}$$
$$u=x^m(D^2-c^2)^{m-1}\{x^{-1}(k\varepsilon^{cx}+k'\varepsilon^{-cx})\}..$$

The latter form is simply the series before found in the solution of (8.) without the factor x^{-m}. In fact the solution of (8.) is the type of the solution of (7.), when there is no second term P; for if $u=u_1$ is a solution of (8.), $u=(\psi x)^{-1}u_1$ is the solution of

$$D^2u+\left(\frac{2m}{x}+\frac{\psi'x}{\psi x}\right)Du+\left(c^2+\frac{2m}{x}\frac{\psi'x}{\psi x}+\frac{\psi''x}{\psi x}\right)u=0.$$

The suppression of the terms containing b does not materially impair the generality of the form; for it is well known, and follows immediately from (1.), that

$$\varphi\left(D+\frac{b}{2}\right)u=\varepsilon^{-\frac{b}{2}x}\varphi(D)\{\varepsilon^{\frac{b}{2}x}u\}.$$

I have found the most convenient form of (7.) to be

$$\left.\begin{aligned}&D^2u+2QDu+\left(c^2+Q^2+Q'-\frac{m(m+1)}{x^2}\right)u=P\\&u=x^m\varepsilon^{-Q_1}(D^2+c^2)^{m-1}\{x^{-1}(D^2+c^2)^{-m}(x^{-(m-1)}\varepsilon^{Q_1}P)\}\end{aligned}\right\}\quad.\;.\;.\;.\;.\;.\quad(9.)$$

which are obtained by making $Q=\frac{\psi'x}{\psi x}+\frac{m}{x}$, Q_1 being $\int Qdx$.

Useful applications may be made by eliminating the second term from (9.) by a change of the independent variable from x to t; t and x being connected by the equation $\frac{dt}{dx}=\varepsilon^{-2Q_1}$.

One of these applications leads to an investigation calculated to throw some light upon the limited character of the solution of RICCATI's equation. If in (9.) Q be taken $\frac{n}{x}$, we have as a soluble form,

$$D^2u+\frac{2n}{x}Du+\left(c^2+\frac{n(n-1)-m(m-1)}{x^2}\right)u=P;$$

and the elimination of the second term gives, $\left(\text{making } \frac{dt}{dx}=x^{-2n} \text{ and } z=-(2n-1)t\right)$

$$\frac{d^2u}{dz^2}+(2n-1)^{-2}\left(c^2z^{-\frac{4n}{2n-1}}+(n(n-1)-m(m-1))z^{-2}\right)u=\text{R or } Pz^{-\frac{4n}{2n-1}}.$$

This form, therefore, and the cognate form

$$\frac{dv}{dz}+v^2+(2n-1)^{-2}\left(c^2z^{-\frac{4n}{2n-1}}+(n(n-1)-m(m-1))z^{-2}\right)=0,$$

are soluble, without the restriction that n must be a whole number; but when this

equation is made to assume RICCATI's form by equating n to m, the restriction on the values to be given to n takes effect.

If for n be written $-n$, we obtain the other form of the equation, viz.

$$\frac{dv}{dz}+v^2+(2n+1)^{-2}(c^2z^{-\frac{4n}{2n+1}}+(n(n+1)-m(m-1))z^{-2})=0,$$

which is subject to the same restriction, when assimilated to RICCATI's form.

The solution of

$$\frac{d^2u}{dz^2}+\left(\frac{c}{2n-1}\right)^2 z^{-\frac{4n}{2n-1}}u=0$$

is $$(\mathrm{D}^2+c^2)^{n-1}\{x^{-1}(k\sin cx+k'\cos cx)\},\ x \text{ being } z^{-\frac{1}{2n-1}};$$

and that of

$$\frac{d^2u}{dz^2}+\left(\frac{c}{2n+1}\right)^2 z^{-\frac{4n}{2n+1}}u=0$$

is $$x^{2n+1}(\mathrm{D}^2+c^2)^n\{x^{-1}(k\sin cx+k'\cos cx\};$$

from which, general expressions for the solution of the two corresponding forms of RICCATI's equation may be deduced, subject to proper precautions with reference to the arbitrary constants.

If we now, in a similar manner, apply the equations (6.) in conjunction with the original theorem (1.), we shall find, making $\varphi(\mathrm{D})=\mathrm{D}^2+b\mathrm{D}$, that equations of the form

$$\mathrm{D}^2u+\left(b+\frac{m}{x}+\frac{2\psi'x}{\psi x}\right)\mathrm{D}u+\left(\frac{bm}{x}+\left(b+\frac{m}{x}\right)\frac{\psi'x}{\psi x}+\frac{\psi''x}{\psi x}\right)u=\mathrm{P}$$

are soluble; the solution being

$$u=(\psi x)^{-1}(\mathrm{D}^2+b\mathrm{D})^{-1}\mathrm{D}^m\left\{\frac{1}{x}\mathrm{D}^{-m}(x\psi x.\mathrm{P})\right\};$$

which assumes, m being integer, the form of the finite series,

$$u=(\psi x)^{-1}(\mathrm{D}+b)^{-1}\left\{\frac{1}{x}\mathrm{D}^{-1}(x\psi x.\mathrm{P})-\frac{m-1}{x^2}\mathrm{D}^{-2}(x\psi x.\mathrm{P})+2\frac{(m-1)(m-2)}{x^3}\mathrm{D}^{-3}(x\psi x.\mathrm{P})\right.$$
$$\left.-2.3\frac{(m-1)(m-2)(m-3)}{x^4}\mathrm{D}^{-4}(x\psi x.\mathrm{P})+\ldots\right\}.$$

And again, making $b=0$ and $\mathrm{Q}=\frac{\psi'x}{\psi x}+\frac{m}{2}x$, we have for the integral of

$$\mathrm{D}^2u+2\mathrm{QD}u+\left(\mathrm{Q}^2+\mathrm{Q}'-\frac{\frac{m}{2}\left(\frac{m}{2}-1\right)}{x^2}\right)u=\mathrm{P},$$

$$u=x^{\frac{m}{2}}\varepsilon^{-\mathrm{Q}_1}\mathrm{D}^{m-2}\left\{\frac{1}{x}\mathrm{D}^{-m}\left(x^{-\frac{m}{2}+1}\varepsilon^{\mathrm{Q}_1}\mathrm{P}\right)\right\}.$$

By processes in all respects similar, integrable forms of equations of the third and higher orders may be obtained. For equations of the third order, it will be found

that the expressions (5.) give, suppressing b,

$$D^3u+\left(\frac{3m}{x}+3\frac{\psi'x}{\psi x}\right)D^2u+\left(c+\frac{6m}{x}\frac{\psi'x}{\psi x}+3\frac{\psi''x}{\psi x}\right)Du+\left(f+c\left(\frac{m}{x}+\frac{\psi'x}{\psi x}\right)+\frac{3m}{x}\frac{\psi''x}{\psi x}+\frac{\psi'''x}{\psi x}\right)u=P,$$

$$u=(\psi x)^{-1}(D^3+cD+f)^{m-1}\{x^{-1}(D^3+cD+f)^{-m}(x\psi x.P)\};$$

or

$$D^3u+3QD^2u+\left(c+3(Q^2+Q')-3\frac{m(m-1)}{x^2}\right)Du+\left(f+cQ+Q^3+Q''+3QQ'\right.$$

$$\left.-3\frac{m(m-1)}{x^2}Q+2\frac{(m-1)m(m+1)}{x^3}\right)u=P,$$

$$u=x^m\varepsilon^{-Q_1}(D^3+cD+f)^{m-1}\{x^{-1}(D^3+cD+f)^{-m}(x^{-m+1}\varepsilon^{Q_1}P)\}.$$

And the expressions (6.) give

$$D^3u+\left(b+\frac{m}{x}+3\frac{\psi x}{\psi x}\right)D^2u+\left(c+\frac{bm}{x}+2\left(b+\frac{m}{x}\right)\frac{\psi'x}{\psi x}+\frac{3}{x}\frac{\psi''x}{\psi x}\right)Du$$

$$+\left(c\frac{m}{x}+\left(c+\frac{bm}{x}\right)\frac{\psi'x}{\psi x}+\left(b+\frac{m}{x}\right)\frac{\psi''x}{\psi x}+\frac{\psi'''x}{\psi x}\right)u=P$$

$$u=(\psi x)^{-1}(D^2+bD+c)^{-1}D^{m-1}\left\{\frac{1}{x}D^{-m}(x\psi x.P)\right\}.$$

It is obvious, however, that the generality of the soluble forms becomes less, as the order of the equation rises.

The solutions derived from (5.) and (6.) as particular forms of (4.), have been given in the first instance on account of their peculiar simplicity; but more general forms are derived by the use of (4.).

The expressions (4.) represent the solution of linear equations of any order, in the factors of which no power of x higher than the first appears.

The general form is

$$(a_nx+b_n)D^nu+(a_{n-1}x+b_{n-1})D^{n-1}u+\ldots+(a_1x+b_1)Du+(a_0x+b_0)u=X, \quad . \quad . \quad (10.)$$

in which

$$\psi t=b_nt^n+b_{n-1}t^{n-1}+..+b_1t+b_0$$

$$\varphi t=a_nt^n+a_{n-1}t^{n-1}+\ldots+a_1t+a_0$$

$$\int\frac{\psi t}{\varphi t}dt=\int\left(\frac{b_n}{a_n}+\frac{A}{t-\alpha}+\frac{B}{t-\beta}+\frac{C}{t-\gamma}+\ldots\right)dt=\frac{b_n}{a_n}t+\log\left((t-\alpha)^A(t-\beta)^B(t-\gamma)^C..\right),$$

where α, β, γ, &c. are the roots of $\varphi t=0$; and A, B, C, &c. are found from the rational fraction $\frac{\psi t}{\varphi t}$. Consequently the solution of this equation is

$$u=(a_nD^n+a_{n-1}D^{n-1}+..+a_1D+a_0)^{-1}\varepsilon^{\frac{b_n}{a_n}D}(D-\alpha)^A(D-\beta)^B\ldots.\left(x^{-1}\left\{\varepsilon^{-\frac{b_n}{a_n}D}(D-\alpha)^{-A}(D-\beta)^{-B}\ldots.X\right\}\right).$$

The factor $\varepsilon^{-\frac{b_n}{a_n}D}$ denotes that x is to be changed into $x-\frac{b_n}{a_n}$; and the factor $\varepsilon^{\frac{b_n}{a_n}D}$ denotes that x is to be changed into $x+\frac{b_n}{a_n}$; but these factors may be dispensed with by making $b_n=0$, which does not diminish the generality of the form.

If, for example, we apply this theorem to the solution of

$$D^2u+(a_1x+b_1)Du+(a_0x+b_0)u=X;$$

we have the rational fraction

$$\frac{b_1t+b_0}{t^2+a_1t+a_0}, \quad A=\frac{b_1\alpha+b_0}{\alpha-\beta}, \quad B=\frac{b_1\beta+b_0}{\beta-\alpha}$$

$$u=(D^2+a_1D+a_0)^{-1}(D-\alpha)^A(D-\beta)^B(x^{-1}\{(D-\alpha)^{-A}(D-\beta)^{-B}X\}).$$

The performance of the operations requires that A, B, &c. shall be whole numbers; and these are the conditions under which the equations are soluble in finite terms.

If we combine with the above general form the formula (1.), we obtain the solution of the equation

$$(a_nx+b_n)D^nu+\left\{a_{n-1}x+b_{n-1}+n(a_nx+b_n)\frac{\psi'x}{\psi x}\right\}D^{n-1}u$$

$$+\left\{a_{n-2}x+b_{n-2}+(n-1)(a_{n-1}x+b_{n-1})\frac{\psi'x}{\psi x}+n\frac{n-1}{2}(a_nx+b_n)\frac{\psi''x}{\psi x}\right\}D^{n-2}u$$

$$+\left\{a_{n-3}x+b_{n-3}+(n-2)(a_{n-2}x+b_{n-2})\frac{\psi'x}{\psi x}+(n-1)\frac{n-2}{2}(a_{n-1}x+b_{n-1})\frac{\psi''x}{\psi x}+n\frac{n-1}{2}\,\frac{n-2}{3}(a_nx+b_n)\frac{\psi'''x}{\psi x}\right\}D$$

$$+\ldots=X$$

in the form

$$u=(\psi x)^{-1}(a_nD^n+a_{n-1}D^{n-1}+..)^{-1}\varepsilon^{\frac{b_n}{a_n}D}(D-\alpha)^A(D-\beta)^B\ldots\left(x^{-1}\left\{\varepsilon^{-\frac{b_n}{a_n}D}(D-\alpha)^{-A}(D-\beta)^{-B}\ldots(\psi x.X)\right\}\right).$$

The important limitation, that A, B, &c. must be whole numbers in order that the operations may be practicable, must not be overlooked; and with reference to this point, the attention of the reader is called to the solutions by means of definite integrals given in a subsequent part of this paper.

If two or more of the roots α, β, &c. are equal, we obtain amongst the operations expressions of the form $\varepsilon^{\pm\frac{C}{(D-\alpha)^m}}$, which do not appear to be interpretable in finite terms; but the corresponding solution in the form of a definite integral will apply.

III. *Solution of Equations by interchange of Symbols alone.*

It has been already observed, that the operation or set of operations denoted by any function of D is not of itself intelligible, unless the function is capable of expansion in integer powers of D, so that fractional operations may not be introduced; but if, by means of the transformation above indicated, the function of D becomes in result changed into a function of x, such a result is intelligible, and may be relied on as true, although the expressions introduced during the process may be purely symbolical and incapable of interpretation.

Thus $$u=(D^2-c^2)^{m-1}\{x^{-1}(D^2-c^2)^{-m}X\},$$

regarded as the solution of

$$x(D^2-c^2)u+2mDu=X,$$

is not interpretable in finite terms when m is fractional. Yet

$$u=(x^2-c^2)^{m-1}\mathrm{D}^{-1}\{x^2-c^2)^{-m}\mathrm{X}\},$$

regarded as the solution of

$$\mathrm{D}\{(x^2-c^2)u\}-2mxu=\mathrm{X}, \text{ or } (x^2-c^2)\mathrm{D}u-2(m-1)xu=\mathrm{X},$$

(these forms being derived from the others by changing D into x and x into $-\mathrm{D}$), is interpretable for all values of m, and is correct.

Again, it has already appeared that the solution of

$$\mathrm{D}^2u+\frac{2n}{x}\mathrm{D}u+\left(c^2+\frac{n(n-1)-m(m-1)}{x^2}\right)u=\mathrm{P},$$

is $$u=x^{m-n}(\mathrm{D}^2+c^2)^{m-1}\{x^{-1}(\mathrm{D}^2+c^2)^{-m}(x^{n-m+1}\mathrm{P})\};$$

which is not interpretable in finite terms when m is fractional.

But if this equation be multiplied by x^2 and transformed as before, we get

$$(x^2+c^2)\mathrm{D}^2u-2(n-2)x\mathrm{D}u+((n-1)(n-2)-m(m-1))u=\mathrm{D}^2\mathrm{P}=\mathrm{R} \text{ suppose};$$

and we infer the solution to be

$$u=\mathrm{D}^{m-n}\{(x^2+c^2)^{m-1}\mathrm{D}^{-1}\{(x^2+c^2)^{-m}\mathrm{D}^{n-m-1}\mathrm{R}\}\};$$

which is interpretable though m be fractional, provided $m-n$ be an integer; or the equation

$$(x^2+c^2)\mathrm{D}^2u-2ax\mathrm{D}u+b(2a-b+1)u=\mathrm{R}$$

is soluble when b is a whole number; the solution being

$$u=\mathrm{D}^{-(b+1)}\{(x^2+c^2)^{a-b}\mathrm{D}^{-1}\{(x^2+c^2)^{-(a-b+1)}\mathrm{D}^b\mathrm{R}\}\};$$

of which a remarkable case is, $(a=0)$

$$\mathrm{D}^2u-\frac{b(b-1)}{x^2+c^2}u=\frac{\mathrm{R}}{x^2+c^2}=\mathrm{X} \text{ (suppose)}$$

$$u=\mathrm{D}^{-(b+1)}\{(x^2+c^2)^{-b}\mathrm{D}^{-1}\{(x^2+c^2)^{b-1}\mathrm{D}^b((x^2+c^2)\mathrm{X})\}\}.$$

In applying these forms great caution must be used with reference to the introduction of constants. The processes indicated in the value of u show $b+2$ constants; which renders it necessary to determine b of them in terms of the other two by reference to the original equation.

Thus, suppose we require the integrals of the equations

$$(1+x^2)\mathrm{D}^2u-2u=x,$$

and $$(1+x^2)\mathrm{D}^2u-2u=a.$$

The form of both solutions is the same, viz.

$$u=k\mathrm{D}^{-3}\frac{1}{(x^2+1)^2};$$

whence we have

$$D^2u=k\left(\frac{1}{2}\frac{x}{1+x^2}+\frac{1}{2}\tan^{-1}x\right)+k'$$

$$Du=\frac{k}{2}x\tan^{-1}x+k'x+k''$$

$$u=\frac{k}{2}\left(\frac{1}{2}(1+x^2)\tan^{-1}x-\frac{1}{2}x\right)+\frac{k'x^2}{2}+k''x+k'''.$$

Verifying the first equation by these values, we find

$$k''=\frac{k-1}{2} \text{ and } k'''=\frac{k'}{2};$$

the solution, therefore, is

$$u=\frac{k}{4}((1+x^2)\tan^{-1}x+x)-\frac{x}{2}+\frac{k'}{2}(1+x^2).$$

Verifying the second equation, we find $k''=\frac{k}{2}$ and $k'''=\frac{k'-a}{2}$, and the solution is

$$u=\frac{k}{4}((1+x^2)\tan^{-1}x+x)+\frac{k'}{2}(1+x^2)-\frac{a}{2}\cdot$$

If the general form of the above differential equation be divided by x^2+c^2, and the second term be eliminated, by changing the independent variable from x to t, by means of the assumed equation $\frac{dt}{dx}=(x^2+c^2)^a$, we obtain

$$\frac{d^2u}{dt^2}+b(2a-b+1)(x^2+c^2)^{-(2a+1)}u=R(x^2+c^2)^{-(2a+1)}=R_0 \text{ suppose.}$$

In order that the factor of u may be expressed in terms of the new variable, we must solve $\frac{dt}{dx}=(x^2+c^2)^a$, and then find x^2+c^2 in terms of t.

For most values of a, the equation between x and t is transcendental; but particular soluble cases may be found.

Thus, if $a=-1$, then $x=c\tan ct$, $x^2+c^2=\frac{c^2}{\cos^2 ct}$, and the equation becomes

$$\frac{d^2u}{dt^2}-b(b+1)\frac{c^2}{\cos^2 ct}u=R_0;$$

whose solution, therefore, is

$$u=D^{-(b+1)}\{(x^2+c^2)^{-(b+1)}D^{-1}\{(x^2+c^2)^bD^b((x^2+c^2)^{-1}R_0)\}\};$$

which becomes, when $R_0=0$,

$$u=kD^{-(b+1)}(x^2+c^2)^{-(b+1)}, \ x \text{ being } c\tan ct.$$

The original equation is not altered by writing $-b$ for $b+1$; so that a particular solution may be readily deduced from the simple form

$$u=kD^b(x^2+c^2)^b.$$

This equation will reappear under another form in the sequel.

Again, if $a=-\frac{3}{2}$, then $c^2+x^2=\frac{c^2}{1-c^4t^2}$; and the equation becomes

$$\frac{d^2u}{dt^2}-b(b+2)\frac{c^4}{(1-c^4t^2)^2}u=\mathrm{R}_0\,;$$

whose solution, therefore, is

$$u=\mathrm{D}^{-(b+1)}\{(x^2+c^2)^{-(b+\frac{3}{2})}\mathrm{D}^{-1}\{(x^2+c^2)^{b+\frac{1}{2}}\mathrm{D}^b(x^2+c^2)^{-2}\mathrm{R}_0)\}\}.$$

Thus the solution of

$$\frac{d^2u}{dt^2}=\frac{3}{(1-t^2)^2}u$$

is

$$u=k\left(\frac{1+t^2+k't}{(1-t^2)^{\frac{1}{2}}}\right).$$

The principle illustrated in this section may be further exemplified and usefully applied, by attempting the solution of the general equation of the second order,

$$\varphi x.\mathrm{D}^2u+\psi x.\mathrm{D}u+\chi x.u=\mathrm{P}.$$

By the interchange of symbols, we have

$$\varphi(\mathrm{D})(x^2u)-\psi(\mathrm{D})(xu)+\chi(\mathrm{D})u=\mathrm{P}\,;$$

or

$$\left.\begin{array}{r} x^2\varphi(\mathrm{D})u+2x\varphi'(\mathrm{D})u+\varphi''(\mathrm{D})u \\ -x\psi(\mathrm{D})u-\psi'(\mathrm{D})u \\ +\chi(\mathrm{D})u \end{array}\right\}=\mathrm{P}.$$

This equation is soluble if $\chi(\mathrm{D})u=\{\psi'(\mathrm{D})-\varphi''(\mathrm{D})\}u$; for it then assumes the form

$$x\varphi(\mathrm{D})u+(2\varphi'(\mathrm{D})-\psi(\mathrm{D}))u=x^{-1}\mathrm{P}\,;$$

the solution of which, by (4.), is

$$u=\varphi(\mathrm{D})\varepsilon^{-\int\frac{\psi(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D}}\left\{x^{-1}(\varphi(\mathrm{D}))^{-2}\varepsilon^{\int\frac{\psi(\mathrm{D})}{\varphi(\mathrm{D})}d\mathrm{D}}\{x^{-1}\mathrm{P}\}\right\}\,;$$

and, by restoring the symbols, we get for the solution of

$$\varphi x.\mathrm{D}^2u+\psi x.\mathrm{D}u+(\psi'x-\varphi''x)u=\mathrm{P},$$

$$u=\varphi x.\varepsilon^{-\int\frac{\psi x}{\varphi x}dx}\mathrm{D}^{-1}\left\{(\varphi x)^{-2}\varepsilon^{\int\frac{\psi x}{\varphi x}dx}\mathrm{D}^{-1}(\mathrm{P})\right\},$$

the correctness of which may be ascertained by verification.

If $\psi x=(n+1)\varphi'x$, we have for the solution of

$$\varphi x.\mathrm{D}^2u+(n+1)\varphi'x.\mathrm{D}u+n\varphi''x.u=\mathrm{P},$$

$$u=(\varphi x)^{-n}\int(\varphi x)^{n-1}\int\mathrm{P}dxdx.$$

If ψx be made equal to $\mathrm{Q}\varphi x$, and P to $\mathrm{R}\varphi x$, we get for the solution of

$$\mathrm{D}^2u+\mathrm{QD}u+\left(\mathrm{Q}'+\mathrm{Q}\frac{\varphi'x}{\varphi x}-\frac{\varphi''x}{\varphi x}\right)u=\mathrm{R},$$

$$u=\varphi x.\varepsilon^{-\mathrm{Q}_1}\mathrm{D}^{-1}\{(\varphi x)^{-2}\varepsilon^{\mathrm{Q}_1}\mathrm{D}^{-1}(\mathrm{R}\varphi x)\}.$$

And if ϕx be made equal to $\varepsilon^{\int \mathrm{T}dx}$ or $\varepsilon^{\mathrm{T}_1}$, we have for the solution of

$$\mathrm{D}^2u+\mathrm{QD}u+(\mathrm{Q}'+\mathrm{QT}-\mathrm{T}^2-\mathrm{T}')u=\mathrm{R},$$

$$u=\varepsilon^{\mathrm{T}_1-\mathrm{Q}_1}\mathrm{D}^{-1}\{\varepsilon^{\mathrm{Q}_1-2\mathrm{T}_1}\mathrm{D}^{-1}(\mathrm{R}\varepsilon^{\mathrm{T}_1})\},$$

a form easily arrived at by ordinary processes.

IV. *Application to Partial Differential Equations.*

Most of the soluble forms above deduced are readily convertible into analogous forms of partial differential equations, by substituting for the constants any function of D′, where D′ denotes differentiation with regard to a new independent variable.

Thus if in (9.) for Q we write $f(x, \mathrm{D}')$ and for c we write $\sqrt{-1}k\mathrm{D}'$, we have the symbolical solution of

$$\frac{d^2u}{dx^2}+2f\left(x, \frac{d}{dy}\right)\frac{du}{dx}-k^2\frac{d^2u}{dy^2}+\left(f^2+f'-\frac{m(m-1)}{x^2}\right)u=\mathrm{P},$$

where P may be a function of x and y.

For example, if $f\left(x, \frac{d}{dy}\right)$ be of the form $\frac{n}{x}\frac{d}{dy}$, the equation becomes

$$\frac{d^2u}{dx^2}+\frac{2n}{x}\frac{d^2u}{dxdy}+\left(\frac{n^2}{x^2}-k^2\right)\frac{d^2u}{dy^2}-\frac{n}{x^2}\frac{du}{dy}-\frac{m(m-1)}{x^2}u=\psi(x, y)\,;$$

of which the solution is

$$u=x^m\varepsilon^{-n\log x.\mathrm{D}'}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{m-1}\{x^{-1}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{-m}\{x^{-(m-1)}\varepsilon^{n\log x.\mathrm{D}'}\psi(x, y)\}\}.$$

Now $\varepsilon^{n\log x.\mathrm{D}'}\psi(x, y)=\psi(x, y+n\log x)$, and $\varepsilon^{-n\log x.\mathrm{D}'}\psi(x, y)=\psi(x, y-n\log x)$.

The question, therefore, is reduced to the solution of

$$\frac{d^2u}{dx^2}-k^2\frac{d^2u}{dy^2}=\Psi(x, y),$$

or $\quad \mathrm{D}^2u-a^2u=\Psi(x, y)$, writing a for $k\mathrm{D}'$;

whence $\quad u=\frac{1}{2a}\varepsilon^{ax}\mathrm{D}^{-1}\varepsilon^{-ax}\Psi(x, y)-\frac{1}{2a}\varepsilon^{-ax}\mathrm{D}^{-1}\varepsilon^{ax}\Psi(x, y),$

$$=\frac{1}{2k\mathrm{D}'}\varepsilon^{kx\mathrm{D}'}\int\Psi(x, y-kx)dx-\frac{1}{2k\mathrm{D}'}\varepsilon^{-kx\mathrm{D}'}\int\Psi(x, y+kx)dx,$$

which is $\quad \frac{1}{2k}(\Psi_1(x, y)+\Psi_2(x, y))+\lambda(y+kx)+\mu(y-kx),$

where λ and μ are arbitrary, and Ψ_1 and Ψ_2 are derived from Ψ as follows; for y in Ψ write $y-kx$, integrate to x; change y into $y+kx$, and integrate to y; this gives Ψ_1, from which Ψ_2 is formed by changing the sign of k.

If $f\left(x, \frac{d}{dy}\right)$ be of the form $\lambda x.\frac{d}{dy}+\mu x$, the equation becomes

$$\frac{d^2u}{dx^2}+2\lambda x.\frac{d^2u}{dxdy}+((\lambda x)^2-k^2)\frac{d^2u}{dy^2}+2\mu x.\frac{du}{dx}+(\lambda' x+2\lambda x.\mu x)\frac{du}{dy}+\left((\mu x)^2+\mu' x-\frac{m(m-1)}{x^2}\right)u$$
$$=\psi(x, y)\,;$$

and the solution is

$$u=x^m\varepsilon^{-\mu_1 x}\varepsilon^{-\lambda_1 x.\mathrm{D}'}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{m-1}\{x^{-1}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{-m}\{x^{-(m-1)}\varepsilon^{\mu_1 x}\psi(x, y+\lambda_1 x)\}\},$$

where $\varepsilon^{-\lambda_1 x.\mathrm{D}'}\psi(x, y)$ means $\psi(x, y-\lambda_1 x)$.

Transforming this equation to new independent valuables p and q, by the equations

$$p=y-\lambda_1 x+kx, \qquad -q=y-\lambda_1 x-kx,$$

or

$$x=\frac{p+q}{2k}, \qquad y=\lambda_1\left(\frac{p+q}{2k}\right)+\frac{p-q}{2};$$

we have, as a soluble form,

$$\frac{d^2u}{dpdq}+\frac{1}{2k}\mu\left(\frac{p+q}{2k}\right)\cdot\left(\frac{du}{dp}+\frac{du}{dq}\right)+\left(\frac{1}{4k^2}\left(\mu^2+2k\left(\frac{d\mu}{dp}+\frac{d\mu}{dq}\right)\right)-\frac{m(m-1)}{(p+q)^2}\right)u=\phi(p, q),$$

the function λ disappearing in the process.

If μx be of the form $\frac{a}{x}$, we obtain a well-known equation, solved by Euler in a series when there is no second term,

$$\frac{d^2u}{dpdq}+\frac{a}{p+q}\left(\frac{du}{dp}+\frac{du}{dp}\right)+\frac{a(a-1)-m(m-1)}{(p+q)^2}u=\phi(p, q),$$

which is therefore soluble; the solution being

$$x^{m-a}(\mathrm{D}^2-\mathrm{D}'^2)^{m-1}\{x^{-1}(\mathrm{D}^2-\mathrm{D}'^2)^{-m}\{x^{a-m+1}\phi_1(x, y)\}\},$$

where ϕ_1 is determined from ϕ by the equations

$$p=x+y \text{ and } q=x-y.$$

If $a=0$, we have the solution of

$$\frac{d^2u}{dpdq}-\frac{m(m-1)}{(p+q)^2}u=\phi(p, q);$$

and if $a=m$, we have the solution of

$$\frac{d^2u}{dpdq}+\frac{m}{p+q}\left(\frac{du}{dp}+\frac{du}{dq}\right)=\phi(p, q);$$

from which the solution of

$$\frac{d^2u}{ds^2}=c^2s^{-\frac{4m}{2m-1}}\frac{d^2u}{dt^2}$$

may be obtained by making

$$\phi=0$$
$$s=(p+q)^{-2m+1}$$
$$t=\frac{2m-1}{c}(p-q).$$

Returning to the general form and making $\lambda x=0$, we have for the solution of

$$\frac{d^2u}{dx^2}-k^2\frac{d^2u}{dy^2}+2\mu x\cdot\frac{du}{dx}+\left(\mu^2+\mu'-\frac{m(m-1)}{x^2}\right)u=\psi(x, y)$$

$$u=x^m\varepsilon^{-\mu_1 x}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{m-1}\{x^{-1}(\mathrm{D}^2-k^2\mathrm{D}'^2)^{-m}\{x^{-(m-1)}\varepsilon^{\mu_1 x}\psi(x, y)\}\}.$$

Again, equating the factors of $\frac{du}{dy}$ and u to zero, we have $\mu x=\frac{m}{x}$ and $\lambda x=x^{-2m}$; therefore

$$\frac{d^2u}{dx^2}+2x^{-2m}\frac{d^2u}{dxdy}+(x^{-4m}-k^2)\frac{d^2u}{dy^2}+\frac{2m}{x}\frac{du}{dx}=\psi(x,y)$$

is soluble; and if the variables be changed by the equations

$$p=kx+\frac{1}{2m-1}x^{-(2m-1)}+y$$

$$q=kx-\frac{1}{2m-1}x^{-(2m-1)}-y,$$

the form

$$\frac{d^2u}{dpdq}+\frac{k}{2}m\left(\frac{p+q}{2k}\right)^{4m-1}\left(\frac{du}{dp}+\frac{du}{dq}\right)=\varphi(p,q)$$

becomes soluble.

By a process of a similar nature applied to (10.), it will be found that the form

$$(a_nx+b_n)\frac{d^nu}{dx^n}+(a_{n-1}x+b_{n-1})\frac{d^nu}{dx^{n-1}dy}+\dots+(a_1x+b_1)\frac{d^nu}{dxdy^{n-1}}+(a_0x+b_0)\frac{d^nu}{dy^n}=f(x,y)$$

has for its solution

$$u=(a_n\text{D}^n+a_{n-1}\text{D}^{n-1}.\text{D}'+\dots+a_1\text{D}.\text{D}'^{n-1}+a_0\text{D}'^n)^{-1}\varepsilon^{\frac{b_n}{a_n}\text{D}}(\text{D}-\alpha\text{D}')^{\text{A}}(\text{D}-\beta\text{D}')^{\text{B}}\dots$$

$$x^{-1}\{\{\varepsilon^{-\frac{b_n}{a_n}\text{D}}(\text{D}-\alpha\text{D}')^{-\text{A}}(\text{D}-\beta\text{D}')^{-\text{B}}..f(x,y)\}\}.$$

V. *Connection with Definite Integrals.*

It is well known that many of the differential equations integrated by the above processes, and whose integrals are in some cases capable of an expression merely symbolical by reason of the number of operations to be performed being fractional, may be integrated generally, when there is no second term, by means of definite integrals.

Now with reference to most of the equations of this description here integrated, I have observed that the symbolical form above given is capable of being instantly (and, as it were, mechanically) converted into a definite integral of the form

$$u=\int_a^b \varphi z.\varepsilon^{zx}dz,$$

the function φz being typified in the symbolical solution by the form of the operations preceding the factor x^{-1}.

To explain this, let us take the equation

$$\text{D}^2u+\frac{2m}{x}\text{D}u-c^2u=0;$$

its symbolical solution is

$$u=k(\text{D}^2-c^2)^{m-1}\{x^{-1}(\text{D}^2-c^2)^{-m}0\};$$

and the assertion is that a solution of the equation in the form of a definite integral is obtained by writing for ϕz, $k(z^2-c^2)^{m-1}$ and selecting the limits properly; in fact, it is known that

$$u=k\int_{-c}^{c}(z^2-c^2)^{m-1}\varepsilon^{zx}dz$$

is a partial solution; and as it is a known theorem, that if $u=u_1$ solve

$$D^2u+\frac{n+1}{x}Du-c^2u=0,$$

it is also solved by $u=x^n u_1$ if for n be written $-n$, we have for the complete solution

$$u=k\int_{-c}^{c}(z^2-c^2)^{m-1}\varepsilon^{zx}dz+k'x^{-2m+1}\int_{-c}^{c}(z^2-c^2)^{-m}\varepsilon^{zx}dz$$

$$=k\int_{-1}^{1}(z^2-1)^{m-1}\varepsilon^{czx}dz+k'x^{-2m+1}\int_{-1}^{1}(z^2-1)^{-m}\varepsilon^{czx}dz.$$

If for x be written $((1-2m)t)^{-\frac{1}{2m-1}}$ we obtain the solution of $\frac{d^2u}{dt^2}-a^2t^nu=0$, m and c being properly taken in terms of a and n.

In like manner, if we apply this mode of conversion to the more general form

$$D^2u+2QDu+\left(Q^2+Q'-c^2-\frac{m(m-1)}{x^2}\right)u=0,$$

of which the solution expressed symbolically is

$$u=x^m\varepsilon^{-Q_1}(D^2-c^2)^{m-1}\{x^{-1}(D^2-c^2)^{-1}0\},$$

the definite integral ought to be

$$u=kx^m\varepsilon^{-Q_1}\int_{-1}^{1}(z^2-1)^{m-1}\varepsilon^{czx}dz+k'x^{-m+1}\varepsilon^{-Q_1}\int_{-1}^{1}(z^2-1)^{-m}\varepsilon^{czx}dz,$$

and this is in fact the solution.

The limits must be determined by verifying the equation and assigning them accordingly; the verification at the same time establishing in the particular cases the correctness of the results arrived at by the substitution. Thus, if we take the general form

$$x\phi(D)u+\psi(D)u=0,$$

and its symbolical solution

$$u=\{\phi(D)\}^{-1}\{\varepsilon^{\chi(D)}x^{-1}\varepsilon^{-\chi(D)}0\},$$

where
$$\chi t=\int\frac{\psi t}{\phi t}dt,$$

the conversion here indicated gives

$$u=k\int(\phi z)^{-1}\varepsilon^{\chi z}\varepsilon^{zx}dz.$$

To verify this, we have

$$\phi(\mathrm{D})u=\int\varepsilon^{\chi z}\varepsilon^{zx}dz$$

$$\psi(\mathrm{D})u=\int\frac{\psi z}{\phi z}\varepsilon^{\chi z}\varepsilon^{zx}dz=\int\varepsilon^{zx}d(\varepsilon^{\chi z})=\varepsilon^{zx}\varepsilon^{\chi z}-x\int\varepsilon^{\chi z}\varepsilon^{zx}dz,$$

and the equation is verified, if $\varepsilon^{zx}\varepsilon^{\chi z}$ vanishes between the limits. Whether the limits can be so taken depends upon the form of χz.

Apply this to $$x\mathrm{D}^n u+u=0.$$

Here $$\phi z=z^n \qquad \psi z=1,$$

and $$\chi z=\frac{z^{-n+1}}{-n+1}$$

and $$u=k\int z^{-n}\varepsilon^{\frac{z^{-n+1}}{-n+1}}\varepsilon^{zx}dz.$$

The limits 0 and $-\infty$ cause $\varepsilon^{zx}\varepsilon^{\frac{z^{-n+1}}{-n+1}}$ to vanish and satisfy the equation. So that

$$u=k\int_{-\infty}^{0}z^{-n}\varepsilon^{\frac{z^{-n+1}}{-n+1}}\varepsilon^{zx}dz$$

is a partial solution; which may be completed by writing for z, αz, βz, &c. where 1, α, β, &c. are the roots $t^n=1$.

Again, if we apply the above solution to

$$\mathrm{D}^n u-xu=0,$$

we have $$\phi z=-1,\quad \psi z=z^n,\quad \chi z=-\frac{z^{n+1}}{n+1}$$

and $$u=k\int\varepsilon^{-\frac{z^{n+1}}{n+1}}\varepsilon^{zx}dz.$$

Here, there do not appear to be any limits which will make $\varepsilon^{-\frac{z^{n+1}}{n+1}}\varepsilon^{zx}$ vanish; but if we take for the limits 0 and ∞ we have $\mathrm{D}^n u-xu=k$. This last equation is also solved by $u=k\alpha\int_0^{\infty}\varepsilon^{-\frac{z^{n+1}}{n+1}}\varepsilon^{\alpha zx}dz$, α being a root of $t^{n+1}=1$.

Therefore the original equation is solved by

$$u=k\int_0^{\infty}\varepsilon^{-\frac{z^{n+1}}{n+1}}(\varepsilon^{zx}-\alpha\varepsilon^{\alpha zx})dz,$$

which may be completed by the use of the other roots*.

Let us apply this method to the solution of

$$(a_n x+b_n)\mathrm{D}^n u+(a_{n-1}x+b_{n-1})\mathrm{D}^{n-1}u+\ldots+(a_1 x+b_1)\mathrm{D}u+(a_0 x+b_0)u=0,$$

and we have at once

$$u=k\int(a_n z^n+a_{n-1}z^{n-1}+\ldots+a_1 z+a_0)^{-1}(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}\ldots\varepsilon^{\frac{b_n}{a_n}z}\varepsilon^{zx}dz,$$

where

$$\frac{b_n z^n+b_{n-1}z^{n-1}+\ldots}{a_n z^n+a_{n-1}z^{n-1}+\ldots}=\frac{b_n z^n+b_{n-1}z^{n-1}+\ldots}{a_n(z-\alpha)(z-\beta)\ldots}=\frac{b_n}{a_n}+\frac{\mathrm{A}}{z-\alpha}+\frac{\mathrm{B}}{z-\beta}+\ldots.$$

* See MOIGNO's Calcul. vol. ii. p. 644.

To verify this and find the limits, we have

$$x(a_n\mathrm{D}^n u+a_{n-1}\mathrm{D}^{n-1}u+..+a_1\mathrm{D}u+a_0u)=kx\int(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\varepsilon^{\frac{b_n}{a_n}z}\varepsilon^{zx}dz=k\int(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\varepsilon^{\frac{b_n}{a_n}z}d\varepsilon^{zx}$$

$$=k\varepsilon^{zx}(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\varepsilon^{\frac{b_n}{a_n}z}-k\int(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\left(\frac{b_n}{a_n}+\frac{\mathrm{A}}{z-\alpha}+\frac{\mathrm{B}}{z-\beta}+..\right)\varepsilon^{\frac{b_n}{a_n}z}\varepsilon^{zx}dz$$

$$=k\varepsilon^{zx}(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\varepsilon^{\frac{b_n}{a_n}z}-k\int(b_nz^n+b_{n-1}z^{n-1}+...)(a_nz^n+a_{n-1}z^{n-1}+..)^{-1}(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\varepsilon^{\frac{b_n}{a_n}z}\varepsilon^{zx}dz$$

$$=-(b_n\mathrm{D}^nu+b_{n-1}\mathrm{D}^{n-1}u+..+b_1\mathrm{D}u+b_0),$$

if $k\varepsilon^{zx}(z-\alpha)^{\mathrm{A}}(z-\beta)^{\mathrm{B}}...\varepsilon^{\frac{b_n}{a_n}z}$ can be made to vanish between the limits; and this condition is satisfied if $-\infty$ be taken as the lower limit, and α, β, &c. be taken successively as the upper limit, whence the complete solution.

If the expression $a_nx^n+..$ has m roots equal to α, the form of the solution will be modified. If in such case the rational fraction

$$\frac{b_nz^n+...}{a_nz^n+...}=\frac{b_n}{a_n}+\frac{\mathrm{A}_m}{(z-\alpha)^m}+\frac{\mathrm{A}_{m-1}}{(z-\alpha)^{m-1}}+..+\frac{\mathrm{A}_1}{z-\alpha}+\frac{\mathrm{B}}{z-\beta}+...$$

the solution becomes

$$u=k\int(a_nz^n+.)^{-1}\varepsilon^{-\frac{\mathrm{A}_m}{m-1}\frac{1}{(z-\alpha)}^{m-1}-\frac{\mathrm{A}_{m-1}}{m-z}\frac{1}{(z-\alpha)}^{m-2}-...}(z-\alpha)^{\mathrm{A}_1}(z-\beta)^{\mathrm{B}}....\varepsilon^{\frac{b_n}{a_n}z}\varepsilon^{zx}dz$$

between the limits $-\infty$ and α, β, &c. This solution is incomplete; but it may be completed by using instead of $a_nz^n+...$ its first, second, ... $(m-1)$th differential coefficients.

There can be no doubt that this remarkable connection between the symbolical solution and the solution by definite integrals is not merely accidental, but is founded upon a similarity in the processes by which they would be respectively arrived at in a general system of solution.

The following considerations are offered as in some measure explanatory of the connection above adverted to. The equation to be solved is of the form

$$x\phi(\mathrm{D})u+\psi(\mathrm{D})u=0.$$

Now if $u=k\int_a^b \varpi z.\varepsilon^{zx}dz$, we have

$$\phi(\mathrm{D})u=k\int_a^b\varpi z.\phi z.\varepsilon^{zx}dz=\frac{k}{x}\varepsilon^{zx}\varpi z.\phi z-\frac{k}{x}\int\frac{d(\varpi z.\phi z)}{dz}\varepsilon^{zx}dz$$

between the limits. And

$$\psi(\mathrm{D})u=k\int_a^b\varpi z.\psi z.\varepsilon^{zx}dz.$$

If the limits be $-\infty$ and a root of $\phi z=0$, the equation is verified if $\varpi z.\psi z=\frac{d}{dz}(\varpi z.\phi z)$, which requires

$$\log(\varpi z)=\int\frac{\psi z-\phi' z}{\phi z}dz,$$

or
$$\varpi z=(\varphi z)^{-1}\varepsilon^{\int\frac{\psi z}{\varphi z}dz},$$
being the same process as that by which, in the symbolical solution, the form of the function of D preceding the factor x^{-1} is obtained.

VI. *Linear Equations in Finite Differences.*

If in (3.) we change the form of the functions by writing ε^{D} for D, we have
$$x\varphi(\varepsilon^{\mathrm{D}})u+\psi(\varepsilon^{\mathrm{D}})u=\mathrm{X},$$
or
$$x\varphi(\mathrm{E})u_x+\psi(\mathrm{E})u_x=\mathrm{Q}_x, \quad \dots\dots\dots\dots \quad (11.)$$
where E is equivalent to $1+\Delta$, or denotes the operation which is performed in changing u_x into u_{x+1}.

It follows that the symbolical solution of (11.) is
$$u_x=(\varphi\mathrm{E})^{-1}\varepsilon^{\chi\mathrm{E}}x^{-1}\varepsilon^{-\chi\mathrm{E}}\mathrm{Q}_x,$$
where
$$\chi v=\int\frac{\psi(\varepsilon^t)}{\varphi(\varepsilon^t)}dt=\int\frac{\psi v}{v\varphi v}dv \text{ if } \varepsilon^t=v.$$
A case obviously interpretable is that in which $\psi v=mv\varphi' v$; and in that case the solution of
$$x\varphi(\mathrm{E})u_x+m\mathrm{E}\varphi'(\mathrm{E})u_x=\mathrm{Q}_x$$
is
$$u_x=\{\varphi(\mathrm{E})\}^{m-1}x^{-1}\{\varphi(\mathrm{E})\}^{-m}\mathrm{Q}_x.$$
If we take the equation
$$(a_nx+b_n)u_{x+n}+\dots+(a_1x+b_1)u_{x+1}+(a_0x+b_0)u_x=\mathrm{Q}_x,$$
we have
$$\chi v=\int\frac{b_nv^n+\dots+b_1v+b_0}{v(a_nv^n+\dots+a_1v+a_0)}dv=\int\left(\frac{b_0}{a_0}\frac{1}{v}+\frac{\mathrm{A}_1}{v-\alpha}+\frac{\mathrm{A}_2}{v-\beta}+\dots\right)dv,$$
where α, β, &c. are the roots of $a_nv^n+\dots+a_1v+a_0=0$.
and
$$u_x=(a_n\mathrm{E}^n+\dots+a_1\mathrm{E}+a_0)^{-1}\mathrm{E}^{\frac{b_0}{a_0}}(\mathrm{E}-\alpha)^{\mathrm{A}_1}(\mathrm{E}-\beta)^{\mathrm{A}_2}\dots\{x^{-1}(\mathrm{E}^{-\frac{b_0}{a_0}}(\mathrm{E}-\alpha)^{-\mathrm{A}_1}(\mathrm{E}-\beta)^{-\mathrm{A}_2}\dots\mathrm{Q}_x)\}.$$
In like manner, if
$$(a_nx+b_n)\Delta^nu_x+\dots+(a_1x+b_1)\Delta u_x+(a_0x+b_0)u_x=\mathrm{Q}_x,$$
we have
$$u_x=(a_n\Delta^n+\dots+a_1\Delta+a_0)^{-1}(1+\Delta)^{\mathrm{B}}(\Delta-\alpha)^{\mathrm{A}_1}(\Delta-\beta)^{\mathrm{A}_2}\dots\{x^{-1}((1+\Delta)^{-\mathrm{B}}(\Delta-\alpha)^{-\mathrm{A}_1}(\Delta-\beta)^{-\mathrm{A}_2}\dots\mathrm{Q}_x)\},$$
where
$$\frac{b_nv^n+\dots+b_1v+b_0}{(1+v)(a_nv^n+\dots+a_1v+a_0)}=\frac{\mathrm{B}}{1+v}+\frac{\mathrm{A}_1}{v-\alpha}+\frac{\mathrm{A}_2}{v-\beta}+\dots$$
Thus the solution of
$$x(\mathrm{E}^2-c^2)u_x-2m\mathrm{E}u_x=\mathrm{Q}_x,$$
or
$$u_{x+2}-\frac{2m}{x}u_{x+1}-c^2u_x=\frac{\mathrm{Q}_x}{x},$$
is
$$u_x=(\mathrm{E}+c)^{\frac{m}{c}-1}(\mathrm{E}-c)^{-\frac{m}{c}-1}\left\{x^{-1}(\mathrm{E}+c)^{-\frac{m}{c}}(\mathrm{E}-c)^{\frac{m}{c}}\mathrm{Q}_x)\right\};$$

and that of

$$x(\Delta^2-c^2)u_x-2m\Delta u_x=Q_x,$$

or

$$\Delta^2 u_x-\frac{2m}{x}\Delta u_x-c^2u_x=\frac{Q_x}{x},$$

is $\quad u_x=(\Delta^2-c^2)^{-1}E^{\frac{2m}{1-c^2}}(\Delta-c)^{-\frac{m}{c+1}}(\Delta+c)^{\frac{m}{c-1}}\left\{x^{-1}\left(E^{-\frac{2m}{1-c^2}}(\Delta-c)^{\frac{m}{c-1}}(\Delta+c)^{-\frac{m}{c+1}}Q_x\right)\right\}.$

The results thus obtained are not interpretable unless B, $\frac{b_0}{a_0}$, A_1, A_2, &c. are integer; in the event of any of them being fractional, however, the solution of the equation, (the second side being suppressed,) may be found as before in the form of a definite integral, by introducing the factor ε^{zx} and changing D into z, and writing for $\frac{1}{x}$ the integration with regard to z between the proper limits.

Thus it will be found that the solution of

$$(a_nx+b_n)u_{x+n}+..+(a_1x+b_1)u_{x+1}+(a_0x+b_0)u_x=0$$

is

$$u_x=\int(a_n\varepsilon^{nz}+..+a_1\varepsilon^z+a_0)^{-1}\varepsilon^{\frac{b_0z}{a_0}}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}....\varepsilon^{zx}dz$$

between proper limits.

For taking this value of u_x, we have

$$x(a_nu_{x+n}+..+a_1u_{x+1}+a_0u_x)=x\int\varepsilon^{\frac{b_0z}{a_0}}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...\varepsilon^{zx}dz$$

$$=\int\varepsilon^{\frac{b_0z}{a_0}}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...\,d\varepsilon^{zx}=\varepsilon^{zx}\varepsilon^{\frac{b_0z}{a_0}}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...$$

$$-\int\varepsilon^{zx}\left(\varepsilon^{\frac{b_0z}{a_0}}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...\right)\left(\frac{b_0}{a_0}+\varepsilon^z\left(\frac{A_1}{\varepsilon^z-\alpha}+\frac{A_2}{\varepsilon^z-\beta}+..\right)\right)dz.$$

Of these two terms, the first vanishes if one limit be $z=-\infty$, and the other have the successive values $\log\alpha$, $\log\beta$, &c., and the second term is equal to

$$-\int\varepsilon^{zx}\left(\varepsilon^{\frac{b_0z}{a_0}}\left((\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...\right)\frac{b_n\varepsilon^{nz}+..+b_1\varepsilon^z+b_0}{a_n\varepsilon^{nz}+..+a_1\varepsilon^z+a_0}dz=-(b_nu_{x+n}+..+b_1u_{x+1}+b_0u_x),\right.$$

so that the equation is verified.

The complete solution therefore is

$$u_x=c_1\int_{-\infty}^{\log\alpha}(a_n\varepsilon^{nz}+..+a_1\varepsilon^z+a_0)^{-1}\varepsilon^{\frac{b_0}{a_0}z}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...\varepsilon^{zx}dz$$

$$+c_2\int_{-\infty}^{\log\beta}(a_n\varepsilon^{nz}+..+a_1\varepsilon^z+a_0)^{-1}\varepsilon^{\frac{b_0}{a_0}z}(\varepsilon^z-\alpha)^{A_1}(\varepsilon^z-\beta)^{A_2}...\varepsilon^{zx}dz$$

$$+\&c.$$

or more simply,

$$u_x=c_1\int_0^{\alpha}(a_nv^n+..+a_1v+a_0)^{-1}v^{\frac{b_0}{a_0}}(v-\alpha)^{A_1}(v-\beta)^{A_2}....v^{x-1}dv$$

$$+c_2\int_0^{\beta}(a_nv^n+..+a_1v+a_0)^{-1}v^{\frac{b_0}{a_0}}(v-\alpha)^{A_1}(v-\beta)^{A_2}....v^{x-1}dv$$

$$+\&c. \qquad \&c.$$

In a manner in all respects analogous, it may be shown that the integral of

$$(a_nx+b_n)\Delta^n u_x+..+(a_1x+b_1)\Delta u_x+(a_0x+b_0)u_x=0,$$

is

$$u_x=c_1\int_{-1}^{\alpha}(a_nv^n+..+a_1v+a_0)^{-1}v^{\frac{b_0}{a_0}}(v-\alpha)^{A_1}(v-\beta)^{A_2}...(1+v)^{x-1}dv$$

$$+c_2\int_{-1}^{\beta}(a_nv^n+..+a_1v+a_0)^{-1}v^{\frac{b_0}{a_0}}(v-\alpha)^{A_1}(v-\beta)^{A_2}...(1+v)^{x-1}dv$$

$$+\&c. \qquad \&c.$$

If the expressions $v(a_nv^n+..+a_1v+a_0)$, $(1+v)(a_nv^n+..+a_1v+a_0)$ should have two or more equal roots, we shall obtain factors of the form $\varepsilon^{\frac{B}{(v-\alpha)^n}}$, as in the case of linear differential equations of analogous forms.

The process of changing the symbols may be used to obtain solutions of differential equations from known solutions of equations in finite differences.

The solution of the general equation of the first order

$$(1+\Delta)u_x-P_xu_x=Q_x$$

is

$$u_x=...P_n....P_{x-2}P_{x-1}\Sigma\left(\frac{Q_x}{..P_n...P_{x-2}P_x}\right).$$

A similarity existing between this form and the solution of linear differential equations of the first order will be seen, by writing the above equations in the following form,—

$$\varepsilon^D u-\phi x.u=X,$$

$$u=\varepsilon^{\Sigma\log\phi x}\Sigma(\varepsilon^{-\Sigma\log\phi(x+1)}X)\ ;$$

and the conversion of symbols would give

$$\varepsilon^{-x}u-\phi(D)u=X,$$

$$u=\varepsilon^{\frac{1}{\varepsilon^{-x}-1}\log\phi D}\left\{\frac{1}{\varepsilon^{-x}-1}\varepsilon^{-\frac{1}{\varepsilon^{-x}-1}\log\phi(D+1)}.X\right\}\ ;$$

a symbolical solution apparently incapable of rational interpretation, at least in finite terms.

If however we suppress X, and by a conversion similar to the one before proposed, ϕD be changed into $\phi(-z)$; and x be changed into $-D'$, D' denoting differentiation with regard to z and a factor ε^{-zx} be introduced, we shall find that the substitution for $\frac{1}{\varepsilon^{-x}-1}$ is $\frac{1}{\varepsilon^{D'}-1}$ or $\frac{1}{\Delta'}$; and the general result is

$$u=\Sigma\varepsilon^{\Sigma\log\phi(-z)}\varepsilon^{-zx},$$

where Σ denotes summation with reference to z, and the first summation must be taken between proper limits.

The form of this is

$$u=\Sigma(...\phi(n)...\phi(-z-2).\phi(-z-1).\varepsilon^{-zx})$$

between proper limits; which, changing the sign of z, may be more conveniently written

$$u=\Sigma(....\phi(z-2).\phi(z-1).\varepsilon^{zx}).$$

To verify this, we have

$$\varepsilon^{-x}u=\Sigma(\ldots.\phi(z-2).\phi(z-1).\varepsilon^{(z-1)x})=\mathrm{P}_{z-1} \text{ suppose,}$$

$$\phi(\mathrm{D})u=\Sigma(\ldots.\phi(z-2).\phi(z-1).\phi z.\varepsilon^{zx})=\mathrm{P}_z;$$

consequently

$$\varepsilon^{-x}u-\phi(\mathrm{D})u=\Sigma(\mathrm{P}_{z-1}-\mathrm{P}_z)=-\Sigma\Delta\mathrm{P}^{z-1}=\chi x-\mathrm{P}_{z-1}=\chi x-(\ldots\phi(z-2).\phi(z-1).\varepsilon^{(z-1)x}),$$

the last term being taken between proper limits.

Let α, β, &c. be the roots of $\phi z=0$; then if at one limit z have the values $1+\alpha$, $1+\beta$, &c. successively, P_{z-1} will vanish; and $z=-\infty$ will give a vanishing value at the other limit. The form of the function χx must be ascertained by verification.

Let $$\phi z=z^p+az^{p-1}+..=(z-\alpha)(z-\beta)(z-\gamma)\ldots.$$

$$\phi(n)\ldots.\phi(z-2).\phi(z-1)=\{(n-\alpha)\ldots.(z-1-\alpha)\}\{(n-\beta)\ldots(z-1-\beta)\}\ldots.$$

$$=\frac{\Gamma(z-\alpha)}{\Gamma(n-\alpha)}\cdot\frac{\Gamma(z-\beta)}{\Gamma(n-\beta)}\cdots$$

$$u=\Sigma_{-\infty}^{1+\alpha}(\phi(n)\ldots\phi(z-2).\phi(z-1).\varepsilon^{zx})=\frac{1}{\Gamma(n-\alpha)\Gamma(n-\beta)..}\Sigma_{-\infty}^{\alpha+1}(\Gamma(z-\alpha)\Gamma(z-\beta)\ldots\varepsilon^{zx})$$

$$=\frac{1}{\Gamma(n-\alpha)\Gamma(n-\beta)\ldots}\Sigma\int_0^\infty dv_1\varepsilon^{-v_1}v_1^{\,z-\alpha-1}\int_0^\infty dv_2\varepsilon^{-v_2}v_2^{\,z-\beta-1}\int_0^\infty dv_3\ldots\varepsilon^{zx}.$$

Now $$\Sigma v_1^{\,z-\alpha-1}v_2^{\,z-\beta-1}\ldots\varepsilon^{zx}=v_1^{\,-\alpha-1}v_2^{\,-\beta-1}\Sigma\varepsilon^{z(x+\log(v_1v_2\ldots))}=v_1^{\,-\alpha-1}v_2^{\,-\beta-1}\ldots\frac{\varepsilon^{zx}v_1^{\,z}v_2^{\,z}\ldots}{\varepsilon^xv_1v_2..-1},$$

which taken between the limits $-\infty$ and $1+\alpha$ is

$$\frac{\varepsilon^{(\alpha+1)x}v_2^{\,\alpha-\beta}v_3^{\,\alpha-\gamma}\ldots}{\varepsilon^xv_1v_2\ldots-1};$$

and, if this be integrated successively with respect to $v_1\,v_2\ldots$ and taken between the proper limits, we get

$$-[n-\alpha-1][n-\beta-1]\ldots u=\varepsilon^{(\alpha+1)x}\{[\alpha-\beta][\alpha-\gamma]..+[\alpha-\beta+1][\alpha-\gamma+1]..\varepsilon^x$$
$$+[2][\alpha-\beta+2][\alpha-\gamma+2]\ldots\varepsilon^{2x}+\ldots\}.$$

If $n=\alpha+1$, we have

$$u=-\varepsilon^{(\alpha+1)x}\{1+(\alpha-\beta+1)(\alpha-\gamma+1)..\varepsilon^x+1.2(\alpha-\beta+1)(\alpha-\beta+2)(\alpha-\gamma+1)$$
$$(\alpha-\gamma+2)\ldots\varepsilon^{2x}+\ldots\}.$$

To verify this, we have

$$\varepsilon^{-x}u=-\varepsilon^{\alpha x}\{1+(\alpha-\beta+1)(\alpha-\gamma+1)..\varepsilon^x+..\}$$

$$\phi(\mathrm{D})u=-\varepsilon^{\alpha x}\quad\{(\alpha-\beta+1)(\alpha-\gamma+1)..\varepsilon^x+..\}$$

$$\varepsilon^{-x}u-\phi(\mathrm{D})u=-\varepsilon^{\alpha x}.$$

If all the roots of ϕz are zero, we have for a partial solution of

$$\varepsilon^{-x}u-\mathrm{D}^nu=c,$$

$$u=c\varepsilon^x\int_0^\infty dv_1\int_0^\infty dv_2\ldots\int_0^\infty dv_n\left\{\varepsilon^{-(v_1+v_2+..+v_n)}\frac{1}{1-v_1v_2\ldots v_n\varepsilon^x}\right\}$$

$$=c\varepsilon^x(1+\varepsilon^x+(1.2)^n\varepsilon^{2x}+(1.2.3)^n\varepsilon^{3x}+\ldots.).$$

VII. *Miscellaneous Forms.*

In the course of the preceding investigations, I was led to attempt the solution of some forms of equations by means of successive operations not consisting exclusively of D combined with constants, but involving also functions of x. The only important result which I obtained is the following, being a slight generalization of the method originally employed by me in effecting the solution of the equation of LAPLACE's coefficients.

The equation

$$D^2u+bDu+c^2u-n(n+1)\frac{u}{\cos^2 x}=X$$

is solved as follows.

Let α and β be the roots of $z^2+bz+c^2=0$, and assume

$$Du-\alpha u=u_1.$$

Then

$$Du_1-\beta u_1-n(n+1)\frac{u}{\cos^2 x}=X,$$

$$u=\frac{\cos^2 x}{n(n+1)}(Du_1-\beta u_1-X),$$

$$Du-\alpha u=\frac{\cos^2 x}{n(n+1)}(D^2u_1+bDu_1+c^2u_1)-\frac{2\cos x.\sin x}{n(n+1)}(Du_1-\beta u_1)-(D-\alpha)\left(\frac{\cos^2 x}{n(n+1)}X\right)=u_1,$$

or $D^2u_1+bDu_1+c^2u_1-2\tan x.(Du_1-\beta u_1)-\frac{n(n+1)}{\cos^2 x}u_1=X'-\alpha X-2\tan x.X=X_{\prime}$ suppose.

Assume

$$Du_1-\alpha u_1-2\tan x.u_1=u_2,$$

then by a similar process we obtain

$$D^2u_2+bDu_2+c^2u_2-4\tan x.(Du_2-\beta u_2)-\frac{n(n+1)-2}{\cos^2 x}u_2=X'_{\prime}-\alpha X_{\prime}-4\tan x.X_{\prime}=X_{\prime\prime} \text{ suppose.}$$

Similarly, if we assume

$$Du_2-\alpha u_2-4\tan x.u_2=u_3,$$

we obtain

$$D^2u_3+bDu_3+c^2u_3-6\tan x.(Du_3-\beta u_3)-\frac{n(n+1)-6}{\cos^2 x}u_3=X'_{\prime\prime}-\alpha X_{\prime\prime}-6\tan x.X_{\prime\prime}=X_{\prime\prime\prime}$$

suppose.

Proceed in like manner until we arrive at the assumption

$$Du_n-\alpha u_n-2n\tan x.u_n=u_{n+1}.$$

Then

$$D^2u_{n+1}+bDu_{n+1}+c^2u_{n+1}-2(n+1)\tan x.(Du_{n+1}-\beta u_{n+1})=X_{(n+1)}.$$

Let

$$Du_{n+1}-\beta u_{n+1}=Q,$$

then

$$DQ-\alpha Q-2(n+1)\tan x.Q=X_{(n+1)},$$

and

$$Q=\varepsilon^{\alpha x}(\cos x)^{-2(n+1)}\int\varepsilon^{-\alpha x}(\cos x)^{2(n+1)}X_{(n+1)}dx,$$

$$u_{n+1}=\varepsilon^{\beta x}\int\varepsilon^{(\alpha-\beta)x}\int(\cos x)^{-2(n+1)}\int\varepsilon^{-\alpha x}(\cos x)^{2(n+1)}X_{(n+1)}dxdx,$$

$$u_n=\varepsilon^{\alpha x}(\cos x)^{-2n}\int\varepsilon^{-\alpha x}(\cos x)^{2n}u_{n+1}dx;$$

and so on down to u, which will be found to be,

$$\varepsilon^{\alpha x}\left(\frac{d}{d(\tan x)}\right)^{-n}\{\textstyle\int\varepsilon^{(\beta-\alpha)}x(\cos x)^{2n}\int\varepsilon^{(\alpha-\beta)x}(\cos x)^{-2(n+1)}\int\varepsilon^{-\alpha x}(\cos x)^{2(n+1)}\mathrm{X}_{(n+1)}dxdxdx\};$$

and if $\mathrm{X}=0$, this becomes

$$u=k\varepsilon^{\alpha x}\left(\frac{d}{d(\tan x)}\right)^{-n}\{\textstyle\int\varepsilon^{(\beta-\alpha)x}(\cos x)^{2n}\int\varepsilon^{(\alpha-\beta)x}(\cos x)^{-2(n+1)}dxdx\},$$

proper precautions being taken with regard to the introduction of constants.

Perhaps the difficulties relating to the constants may be evaded by writing the solution in the form

$$u=k\varepsilon^{\alpha x}\left(\frac{d}{d(\tan x)}\right)^{-(n+1)}\{\varepsilon^{(\beta-\alpha)x}(\cos x)^{2(n+1)}\textstyle\int\varepsilon^{(\alpha-\beta)x}(\cos x)^{-2(n+1)}dx\},$$

and then substituting $-(n+1)$ for n, which does not alter the original equation, we have

$$u=k\varepsilon^{\alpha x}\left(\frac{d}{d(\tan x)}\right)^{n}\{\varepsilon^{(\beta-\alpha)x}(\cos x)^{-2n}\textstyle\int\varepsilon^{(\alpha-\beta)x}(\cos x)^{2n}dx\}.$$

If α and β are both zero, we have for the solution of

$$\mathrm{D}^2u-n(n+1)\frac{u}{\cos^2 x}=0,$$

$$u=k\left(\frac{d}{d(\tan x)}\right)^{n}\{(\cos x)^{-2n}\textstyle\int(\cos x)^{2n}dx\}$$

$$=k\left(\frac{d}{dy}\right)^{n}\left\{(1+y^2)^n\int\frac{dy}{(1+y^2)^{n+1}}\right\}+k^1\left(\frac{d}{dy}\right)^{n}(1+y^2)^n,$$

where $y=\tan x$.

Let $\alpha=c$ and $\beta=-c$, so that the original equation becomes

$$\frac{d^2u}{dx^2}-c^2u-n(n+1)\frac{u}{\cos^2 x}=0,$$

then the solution is

$$u=k\varepsilon^{cx}\left(\frac{d}{d(\tan x)}\right)^{-n}\{\textstyle\int\varepsilon^{-2cx}(\cos x)^{2n}\int\varepsilon^{2cx}(\cos x)^{-2(n+1)}dxdx\}$$

$$=k\varepsilon^{cx}\left(\frac{d}{d(\tan x)}\right)^{n+1}\{\textstyle\int\varepsilon^{-2cx}(\cos x)^{-2(n+1)}\int\varepsilon^{2cx}(\cos x)^{2n}dxdx\},$$

which contains the proper number of constants; as the constant which enters by reason of the first integration disappears by the subsequent differentiations.

This solution will apply to LAPLACE's equation, if for c be written $c\frac{d}{dy}$.

This gives for the solution of

$$\frac{d^2u}{dx^2}-c^2\frac{d^2u}{dy^2}=n(n+1)\frac{u}{\cos^2 x},$$

$$u=\varepsilon^{cx\frac{d}{dy}}\left(\frac{d}{d(\tan x)}\right)^{n}\{\varepsilon^{-2cx\frac{d}{dy}}(\cos x)^{-2n}\textstyle\int(\cos x)^{2n}\phi(y+2cx)dx\}$$

$$+\varepsilon^{cx\frac{d}{dy}}\left(\frac{d}{d(\tan x)}\right)^{n}\{(\cos x)^{-2n}\chi(y-2cx)\},$$

the symbol $\varepsilon^{-2cx\frac{d}{dy}}$ denoting that, after the operations are performed, $y-2cx$ must be written in lieu of y; and the symbol $\varepsilon^{cx\frac{d}{dy}}$ denoting that, after the further operations are performed, $y+cx$ must be written in lieu of y.

The solution is simplified by considering the latter function alone as a partial solution, and completing the solution by changing the sign of c with a new arbitrary function.

Now if in LAPLACE's equation

$$\frac{d}{d\mu}\left((1-\mu_2)\frac{du}{d\mu}\right)+\frac{1}{1-\mu^2}\frac{d^2u}{dy^2}+n(n+1)u=0,$$

we make

$$x=\tan^{-1}(\mu\sqrt{-1}),$$

we obtain

$$\frac{d^2u}{dx^2}-\frac{d^2u}{dy^2}-n(n+1)\frac{u}{\cos^2 x}=0.$$

The solution of LAPLACE's equation, therefore, by this process assumes the form

$$u=\varepsilon^{\tan^{-1}(\mu\sqrt{-1})\frac{d}{dy}}\left(\frac{d}{d\mu}\right)^n\{(1-\mu^2)^n\varphi(y-2\tan^{-1}(\mu\sqrt{-1}))\}$$
$$+\varepsilon^{-\tan^{-1}(\mu\sqrt{-1})\frac{d}{dy}}\left(\frac{d}{d\mu}\right)^n\{(1-\mu^2)^n\chi(y+2\tan^{-1}(\mu\sqrt{-1}))\}.$$

POSTSCRIPT.—Received March 16, 1848.

The following brief investigation is more general in its results than that developed in pages 50 and 51.

By applying the fundamental theorem to the linear equation

$$u_{x+n}-\varphi x.u_x=\psi x,$$

and its solution

$$u_x=\varphi(x-n).\varphi(x-2n)\ldots\Sigma\{(\varphi x.\varphi(x-n).\varphi(x-2n)\ldots)^{-1}\psi x\},$$

(where the sign of summation has reference to x, Δx being n,) we obtain the equation

$$\varepsilon^{-nx}u-\varphi(\mathrm{D})u=\psi x, \quad \ldots\ldots\ldots \quad (12.)$$

and its symbolical solution

$$u=\varphi(\mathrm{D}-n).\varphi(\mathrm{D}-2n)\ldots\left\{\frac{1}{\varepsilon^{-nx}-1}((\varphi(\mathrm{D}).\varphi(\mathrm{D}-n).\varphi(\mathrm{D}-2n)\ldots)^{-1}\psi x)\right\};$$

and by expanding the factor $\frac{1}{\varepsilon^{-nx}-1}$, and reducing, we obtain for the solution of (12.) the series (Σ referring to p),

$$u=\Sigma\varepsilon^{-pnx}(\varphi(\mathrm{D}).\varphi(\mathrm{D}-n)\ldots.\varphi(\mathrm{D}-pn))^{-1}\psi x,$$
$$=\Sigma(\varphi(\mathrm{D}+pn).\varphi(\mathrm{D}+(p-1)n)\ldots.\varphi(\mathrm{D}))^{-1}\{\varepsilon^{-pnx}\psi x\}.$$

If $\varepsilon^x=y$, and χy consist of powers of y, the above formula gives the solution in series of powers of y of the equation

$$\varphi\left(y\frac{d}{dy}\right).u+qy^n=\chi y.$$

Several equations of this form solved by Mr. BOOLE's general method, are given in the Philosophical Transactions for 1844, pp. 236–240.

III. *On a new substance occurring in the Urine of a patient with Mollities Ossium.*

By Henry Bence Jones, *M.A., F.R.S., Physician to St. George's Hospital.*

Received February 25,—Read April 22, 1847.

On the 1st of November 1845 I received from Dr. Watson the following note, with a test tube containing a thick, yellow, semi-solid substance:—"The tube contains urine of very high specific gravity; when boiled it becomes highly opake; on the addition of nitric acid it effervesces, assumes a reddish hue, becomes quite clear, but, as it cools, assumes the consistence and appearance which you see: heat reliquifies it. What is it?"

A few hours afterwards a specimen of the same urine, passed by a grocer forty-seven years of age, who had been out of health for thirteen months, was sent to me by Dr. MacIntyre. He being in attendance on the case with Dr. Watson, had two days previously first observed the peculiar reactions of the urine.

The specimen of urine was slightly acid; specific gravity 1034·2; it contained a sediment consisting of crystalline phosphate of lime, oxalate of lime, and cylinders of fibrin. The urine became thick with heat from a deposit of phosphates, but cleared with a drop of acid. It gave no precipitate with an excess of nitric acid, unless left to stand, or unless heated and left to cool, when it became solid. This solid redissolved by heat, and again formed on cooling. Continued boiling with strong nitric acid evolved but little gas, and did not quickly hinder this reaction. Hydrochloric acid gave the same solid precipitate, soluble by heat. Strong acetic acid gave only a slight precipitate, which redissolved by heat. Caustic potash and sulphate of copper gave a splendid bright blue, clear liquid, passing over when heated to claret colour.

516·84 grains evaporated to dryness *in vacuo* over sulphuric acid, gave 48·37 grains solid residue =93·58 per 1000 urine.

November 3rd.—I received more urine from the same patient. It gave a greater sediment, consisting of urate of ammonia, and some amorphous phosphate of lime, and some coarse cylinders. Filtered, specific gravity =1043·2. The reactions were the same as before.

521·39 grains gave 65·85 grains solid residue =126·30 per 1000 urine.

All the phosphoric acid was precipitated in combination with lime by adding chloride of calcium and ammonia. Phosphate of lime =5·68 per 1000 urine.

November 7th.—Water from the same patient contained some sediment of crystalline phosphate, some granular and laminar phosphate of lime: some coarse cylinders

of fibrin, very coarse. The urine was slightly alkaline from ammonia; it gave a plentiful precipitate with alcohol; coagulated firmly with heat, very perfectly with a drop or two of acetic acid. Ferro-prussiate of potash gave no immediate precipitate; in less than half an hour a considerable precipitate had formed, which was soluble in liq. potassæ.

November 8th.—I saw multitudes of phosphate of lime crystals: no cylinders: few octohedral crystals. The urine coagulated with heat even when rendered feebly alkaline by liq. potassæ or liq. ammoniæ: if these were in excess coagulation by heat did not take place.

November 9th.—The urine was loaded with urates. Not nearly so readily coagulable by heat as it had been the two previous days; it bore today brisk and prolonged boiling: specific gravity =1037·2.

November 15th.—I saw the patient. He stated that he usually passed about 35℥ of urine daily.

November 18th.—The urine had a specific gravity = 1039·6 and was acid. It contained much urate of ammonia, phosphate of lime, and oxalate of lime; and bore long-continued boiling without coagulating. The urine was filtered and the following analysis was made:—

519·54 grains evaporated to dryness *in vacuo* over sulphuric acid, gave,—

56·78 grains of solid residue =109·28 per 1000 urine.
Ash =6·02 grains =11·58 per 1000 urine.

This ash dissolved in water, filtered, precipitated by nitrate of silver and nitric acid, gave—

Fused chloride of silver =48·84 grains=1·99 grain ch. sod. =3·83 per 1000 urine.
Sulphate of baryta =1·47 grain =1·10 grain sulphate of potash =2·11 per 1000 urine.

519·62 grains of urine precipitated by alcohol—

Precipitate =37·16 grains =71·51 per 1000 urine.

29·26 grains of this precipitate gave,—

1·47 grain ash (=5·01 per cent.) =3·58 per 1000 urine.

519·62 grains treated with alcohol—

Dissolved =19·72 grains=37·93 per 1000.
Ash . . = 4·17 grains = 8·03 per 1000.

1024·48 grains precipitated by strong acetic acid—

Uric acid =·99 grain =·96 per 1000 urine.

Filtered fluid precipitated by ammonia,—

Earthy phosphate =1·23 grain =1·20 per 1000.

Hence,

Water	=890·72 per 1000 urine.
Total solid residue	=109·28
Total ash	= 11·58

Substance soluble in alcohol	=37·93 per 1000 urine.
Ash	= 8·03
Whence urea and alcoholic extract .	=29·90

Substance insoluble in alcohol	=71·51 per 1000 urine.
Ash	= 3·58
Uric acid	= ·96
Whence new substance	=66·97

Earthy phosphate	= 1·20 per 1000 urine.
Chloride of sodium	= 3·83
Sulphate of potash	= 2·10
Thence phosphate of soda	= 4·45, because total ash=11·58.

Hence,

Water	=890·72
New substance . .	= 66·97
Urea	= 29·90
Uric acid	= ·96
Earthy phosphate .	= 1·20
Chloride of sodium	= 3·83
Sulphate of potash	= 2·10
Alkaline phosphate	= 4·45
	1000·13

January 6th.—I was given by Dr. MacIntyre the water passed on the 27th, 29th and 30th of December, all strongly acid, containing much urate of ammonia, oxalate of lime and phosphate of lime.

In the water of December the 27th, I saw some dead spermatozoa. The urine when filtered had a specific gravity of 1031·3.

515·37 grains precipitated by chloride of calcium and ammonia. Phosphate of lime =4·15 grains =8·05 per 1000 urine.

1018·85 grains, precipitated by ammonia. Earthy phosphates =1·47 grain =1·44 per 1000.

In that of December 29th, the specific gravity when filtered was =1037·9.

518·69 grains, precipitated as above. Phosphate of lime=4·27 grains=8·24 per 1000.

1027·71 grains, precipitated as above. Earthy phosphate =1·62 grain =1·57 per 1000.

In that of December 30th, specific gravity =1042·7, filtered. No fibrinous cylinders. The urine remained acid for a month in an open vessel.

521·05 grains, precipitated as above. Phosphate of lime =6·17 grains =11·85 per 1000.

717·04 grains, precipitated by ammonia. Earthy phosphate =1·23 grain =1·72 per 1000.

January 2nd.—The patient died. The following day I saw that the bony structure of the ribs was cut with the greatest ease, and that the bodies of the vertebræ were capable of being sliced off with the knife*. For an account of the structure of the bone, see a paper by Mr. Dalrymple in the third number of the Dublin Journal, August 1846.

The variation in the coagulability by heat observed in these different examinations of the urine might possibly have arisen from the variation in the acidity of the secretion. Free acid might have hindered the coagulation. The only other supposition is that the composition of this new substance varied slightly at different times, and, on the whole, this seems not unlikely to have been the case.

The variations in the amount of earthy and alkaline phosphates in this case may be arranged in the following Table:—

		Earthy phosphates.	Total phosphates.
1st analysis.	Urine, specific gravity =1043·2		5·68 per 1000 urine.
2nd analysis.	Urine, specific gravity =1039·6	1·20	5·65 per 1000 urine.
3rd analysis.	Urine, specific gravity =1031·3	1·43	8·05 per 1000 urine.
4th analysis.	Urine, specific gravity =1037·9	1·57	8·24 per 1000 urine.
5th analysis.	Urine, specific gravity =1042·7	1·72	11·85 per 1000 urine.

Reactions of this new substance precipitated from the Urine by Alcohol, well-washed, dried and powdered.

The substance slowly but entirely dissolved when thrown into cold water.

It was much more readily dissolved by boiling water; if the boiling was continued, at first no appearance of coagulation was evident, but, after ten minutes, in repeated experiments a gelatinous coagulation took place. By continuing the boiling, adding small quantities of water from time to time, the coagulum in an hour entirely redissolved.

If water was poured on the powdered substance it sometimes caked together, and on boiling became horny, and resisted the action even of long-continued heat.

The substance was soluble in caustic potash, and, after long standing, at the ordinary temperature, or at 140°. When the alkaline solution was neutralized by acetic acid, no precipitate formed, but if an excess of acid was added, a plentiful precipitate fell, which was insoluble in more acetic acid, but was readily soluble by heat. If a great excess of acetic acid was added the precipitate was dissolved.

The solution of this substance in water gave an immediate precipitate with nitric acid, which entirely and readily dissolved when heated. Boiling caused no precipitation. On cooling, the precipitate again was formed. If the watery solution was acidulated with acetic acid, an immediate white precipitate fell on the addition of ferro-prussiate of potash. The precipitate was soluble in caustic potash.

* The pericardium contained an ounce or two of fluid, which gave a precipitate immediately by heat and acid. The coagulum did not redissolve when heated. The fluid was feebly alkaline; specific gravity 1015·3. Heat and acid coagulated the blood, and on heating the acid coagulum it did not redissolve. The kidneys were healthy to the naked eye and to the microscope.

The watery solution gave a precipitate with sulphate of copper and with bichloride of mercury; each precipitate readily dissolved when acetic acid was added. The precipitate with sulphate of copper was soluble in caustic potash, forming a splendid deep blue solution, which became more claret-coloured when heated.

Strong hydrochloric acid dissolved the substance, giving a splendid purple blue solution.

Dissolved in caustic potash and boiled, a deep inky blackness was produced by dropping acetate of lead into the solution.

On the Analysis of this Substance.

The substance was prepared in the following way. The urine was mixed with a large excess of alcohol. The precipitate which formed was thrown on a filter, and well-washed with cold alcohol until it passed through, leaving no stain when evaporated on platinum.

It was then dried *in vacuo*, finely powdered, and treated with æther at the boiling temperature, until no traces of fatty matter were left on evaporation of the æther. The substance was long dried *in vacuo* over sulphuric acid.

6·72 grains of the substance so prepared gave 0·19 ash =2·8 per cent.

Of second preparation 4·59 grains of the substance gave 0·13 ash =2·9 per cent.

4·10 grains, minus ash, burnt with chromate of lead, gave 7·77 carbonic acid and 2·64 water.

C =51·68
H= 7·15

4·12 grains, minus ash, burnt as before, gave 7·78 carbonic acid and 2·63 water.

C =51·50
H= 7·09

Of another preparation,

3·57 grains, minus ash, gave 6·84 carbonic acid and 2·24 water.

C =52·25
H= 6·97

4·12 grains, minus ash, dried at 280° Fahr., gave 7·90 carbonic acid and 2·66 water.

C =52·29
H= 7·17

Another preparation,

3·83 grains of the substance, minus ash, gave 7·33 carbonic acid and 2·47 water.

C =52·16
H= 7·16

4·42 grains of the substance, minus ash, gave 8·46 carbonic acid.

C =52·19

3·57 grains, minus ash, gave 6·84 carbonic acid and 2·24 water.

C = 52·25
H = 6·97

2·85 grains of the substance of the 1st preparation, minus ash, gave 6·89 ammonio-chloride of platinum.

N = 15·24

4·37 grains of the 2nd preparation, minus ash, gave 10·27 grains ammonio-chloride of platinum,

N = 14·81

Hence,

C . .	51·68	51·50	52·25	52·29	52·16	52·19	52·25
H . .	7·15	7·09	6·97	7·17	7·16		6·97
N . .	15·24			14·81			
O.							

5 grains of the substance, mixed with 50 grains of pure nitre, free from chloride and sulphate, heated gently in a platinum crucible, inflamed (some was lost); the residue, dissolved in water acidulated with pure nitric acid, gave no precipitate with nitrate of silver; but gave a very plentiful precipitate with chloride of barium, insoluble in nitric acid, when boiled. After being filtered it gave a cloud with ammonia, soluble in acid, reprecipitated by ammonia.

Previously to determining the sulphur in this substance, I determined the quantity present in some dried albumen from an egg. Treated with hydrochloric acid, it gave scarcely a trace of precipitate with chloride of barium.

9 grains of dry albumen, fused with 45 grains of nitre, and the same quantity of pure carbonate of potash, gave ·83 grain of sulphate of baryta.

= ·11 grain of sulphur.
= 1·27 per cent. sulphur.

9·56 grains of the new substance, fused with 45 grains of nitre, and the same quantity of carbonate of potash, gave 1·01 grain of sulphate of baryta.

= ·139 grain of sulphur.
= 1·45 per cent. sulphur.

Some of the substance, prepared at a different time, was next examined.

20·67 grains, when treated with boiling dilute hydrochloric acid for about half an hour, gave . . . . ·17 grain of sulphate of baryta.

= ·023 grain of sulphur.
= ·09 per cent. sulphur.

The clear liquid, after separation of the sulphate of baryta, precipitated by ammonia, filtered, washed with boiling water, burnt, redissolved in hydrochloric acid, and reprecipitated by ammonia, gave ·16 grain of phosphate of baryta; redissolved in

hydrochloric acid, precipitated with sulphate of soda = ·059 grain of baryta.
= ·022 grain of phosphorus.
=0·1 per cent. phosphorus.

13·53 grains of the substance, fused with 69 grains of nitre, and the same quantity of carbonate of potash, gave 1·17 grain of sulphate of baryta.

=0·1614 grain of sulphur.
=1·19 per cent. sulphur.

The clear liquid precipitated by ammonia—

Phosphate of baryta	=·29
Phosphorus	=·041
Sulphate of baryta	=·16=·10 baryta
Hence phosphorus	=·30 per cent.

11·88 grains of the substance, fused with 55 grains of nitre and the same quantity of carbonate of potash, gave ·97 grain of sulphate of baryta.

= ·1338 grain sulphur.
=1·12 per cent. sulphur.

The clear liquid precipitated by ammonia—

Phosphate of baryta	=·25.
Phosphorus	=·0349.
Sulphate of baryta	=·14=·9 baryta.
Hence phosphorus	=·29 per cent.

There is, then, deducting the sulphur and phosphorus dissolved out by the boiling hydrochloric acid,—

	1st.	2nd.	3rd.
Sulphur, per cent.	1·36	1·09	1·03
Phosphorus, per cent. . . .		·20	·19

The presence of sulphur and phosphorus in this new substance proves that it is not an oxide of protein. The solubility of this body in water might lead to the comparison of it with the hydrated tritoxide of protein of Mulder, which is also soluble in water; but according to Mulder, this hydrated tritoxide of protein gives no precipitate on the addition of ferro-prussiate of potash, and it contains no sulphur or phosphorus, while this new substance gives both sulphur and phosphorus, and also a precipitate with ferro-prussiate of potash. In these respects there is a most marked and essential difference between these substances. The peculiar reaction of this new body with nitric acid, the solubility of the nitric acid precipitate when heated, I find to be also common to the so-called hydrated tritoxide of protein prepared from the inflammatory crust of the blood; from albumen of the blood by chlorine; and from albumen of the egg. Lastly, this peculiar reaction with nitric acid hinders all possibility of confusing this new substance with albumen. Indeed ordinary albumen may be separated from this new substance by adding nitric acid,

boiling and filtering whilst hot; on cooling, the hydrated oxide will be precipitated from the filtered liquid, and it will again be redissolved by heat, whilst the albumen will remain on the filter.

Conclusions.

1st. The ultimate analysis of this substance may be represented by $C_{48}, H_{38}, N_6, O_{18}$, or by $C_{40}, H_{31}, N_5, O_{15}$; according as protein is $=C_{48}, H_{37}, N_6, O_{15}$, or $C_{40}, H_{30}, N_5, O_{12}$.

	1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	$C_{48}, H_{38}, N_6, O_{18}$; Reckoned.	or $C_{40}, H_{31}, N_5, O_{15}$. Reckoned.
C	51·68	51·50	52·25	52·29	52·16	52·19	52·25	52·10	52·00
H	7·15	7·09	6·97	7·17	7·16		6·97	6·70	6·85
N	15·24			14·81				15·17	15·15
O								26·00	25·99

The sulphur and phosphorus in this substance, per cent., gave

Sulphur	=1·36	1·09	1·03 per cent.
Phosphorus	=	·20	·19 per cent.

Hence it is an oxide of albumen, and from the ultimate analysis, it is the hydrated deutoxide of albumen.

2nd. In the above case of mollities ossium 66·97 parts of this hydrated deutoxide of albumen were passing out of the body in every 1000 parts of the urine. Hence, therefore, there was as much of this peculiar albuminous substance in the urine as there is ordinary albumen in healthy blood. So far, then, as the albumen alone is concerned, each ounce of urine passed was equivalent to an ounce of blood lost.

3rd. The peculiar characteristic of this hydrated deutoxide of albumen was its solubility in boiling water, and the precipitate with nitric acid being dissolved by heat and reformed when cold. By this reaction a similar substance in small quantity may be detected in pus and in the secretion from the vesiculæ seminales.

4th. This substance must again be looked for in acute cases of mollities ossium. The reddening of the urine on the addition of nitric acid might perhaps lead to the rediscovery of it; when found, the presence of chlorine in the urine, of which there was a suspicion in the above case, should be a special subject of investigation, as it may lead not only to the explanation of the formation of this substance, but to the comprehension of the nature of the disease which affects the bones.

Lastly, I am much indebted to Dr. Watson and Dr. MacIntyre for enabling me to follow out this case, and also to Professor Fownes for the use of his laboratory at University College.

IV. *Examination of the proximate Principles of some of the Lichens.*

By John Stenhouse, *Esq., Ph.D., Glasgow.*
Communicated by Thomas Graham, *Esq., F.R.S., &c.*

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THE examination of the proximate principles of the lichens, especially of those which yield red colouring matters with ammonia, attracted the attention of some very eminent chemists at a comparatively early period in the history of organic chemistry, and by the labours of Messrs. Robiquet, Heeren, Dumas and Kane, very considerable progress was made in their investigation. Within the last four or five years, Messrs. Schunck, Rochleder, Heldt and Knop have resumed the subject and greatly extended our acquaintance with this interesting but rather difficult department of organic research. It is but justice to Mr. Schunck to state that he has been by far the most successful cultivator of this field, and that he has done more to elucidate it than any of his predecessors.

Nearly two years ago my attention was directed by Dr. Pereira to a kind of Orcella weed which had been recently imported into London from the Cape of Good Hope, but which had been rejected by the London archil manufacturers as unfit for their use, from the small quantity of colouring matter it yielded when subjected to the usual process. The lichen was of considerable size, from eight to ten inches long, and was pronounced by an eminent botanist, Dr. Scouler of Dublin, to whom I submitted it, to be merely a large variety of the *Roccella tinctoria*. I soon ascertained on a very cursory examination, why the lichen had been rejected by the archil makers, for it only contained a small portion of a crystalline principle which yields a red colour with ammonia; I found in its stead, however, a considerable quantity of another crystalline body on which ammonia had no action, and which appeared to have been hitherto undescribed. As it seemed important to ascertain whether or not the red dyes obtained from the various lichens resulted from the action of ammonia on the same crystalline principle, described by Mr. Schunck under the name of Lecanorin, I procured quantities of the several lichens usually employed by the archil makers, and subjected them to investigation. These lichens consisted,—1st, of a large species of *Roccella tinctoria* from the west coast of South America; 2nd, of the *R. tinctoria* from the Cape of Good Hope; 3rd, of the *R. Montagnei* from Angola; and 4th, of the *Lecanora tartarea*. I had made considerable progress with the investigation of these lichens, and also with that of the *Evernia prunastri*, when Mr. Schunck's elaborate paper on the Angola lichen appeared, from which it was evident that at least two varieties of the red colouring principle existed in these lichens. I now therefore proceed to give a detail of the results of these examinations.

I. *The South American variety of* Roccella tinctoria.

This lichen, which is imported in considerable quantities from the west coast of South America, is found, I have reason to believe, in the neighbourhood both of Lima and of Valparaiso. It is a large and handsome lichen from six to eight inches long, and its stems are in some instances as thick as those of a goose-quill. It was pronounced by Dr. Scouler to be a large variety of the *R. tinctoria*. The lichen, after being cut into small pieces, was macerated with a considerable quantity of water for some hours, so as to moisten it completely; a quantity of quicklime was then put into the liquid; the whole was well-stirred and then suffered to repose. The clear liquid, which had a slightly yellow colour, was then drawn off and filtered. The mixture of the lime and lichen had about half as much water poured on it as before, and after standing for a quarter of an hour this was also drawn off, filtered and mixed with the first quantity. In order to exhaust the lichen completely, it should be treated with a considerable excess of lime, but care should be taken not to permit the lichen to remain very long in contact with the lime, otherwise the colouring principle will be oxidized and rendered brown. An excess of muriatic acid was then added to the lime solution, when the whole of the colouring principle of the lichen was precipitated as a white gelatinous bulky mass. When this gelatinous precipitate had been repeatedly washed by decantation to remove adhering muriatic acid, it was collected on a cloth filter, and dried upon a plate of gypsum. When pretty well freed from moisture, it was dissolved in hot spirits of wine, but at a temperature much under boiling, as otherwise an ether compound would have been formed. On the cooling of the solution, the colouring principle was deposited in small white prismatic needles arranged in stars. Should these crystals not be quite colourless, they may be easily rendered perfectly so by redissolving them in alcohol, and digesting them with a little purified animal charcoal. The above is the best process for procuring the colouring principle of this lichen, to which I shall give the name of Orsellic acid, in a state of purity. Orsellic acid may also be obtained, but much less economically, as the greater portion of it is then destroyed, by boiling the lichen in large quantities of water, and then purifying the precipitate obtained by repeatedly crystallizing it out of weak spirit. This is the process by which Mr. Schunck has extracted erythric acid from the Angola lichen. Besides the great loss of the acid which this method occasions, I have always found that the orsellic acid extracted by hot water, even after repeated crystallizations out of alcohol, is never free from traces of a resinous matter and of a fatty acid, which I could only remove by dissolving it in cold lime or baryta water, and purifying it in the way already described.

(*Alpha*) *Orsellic Acid.*

Orsellic acid is nearly insoluble in cold water, but it dissolves, though sparingly, in boiling water, from which it is deposited in small prisms arranged in stars. It is

pretty soluble in cold alcohol and ether, and very readily so in boiling alcohol. Its solutions redden litmus paper very distinctly, and it neutralizes the alkalies and alkaline earths, forming soluble and crystallizable salts. Its most characteristic reaction, by which its presence can be very readily detected, is the deep blood-red colour it instantly strikes when it is brought in contact with a solution of hypochlorite of lime. The red colour only remains for a minute or two, and then changes to a deep yellow, which also gradually disappears. This is a property which orsellic acid possesses in common with most of the red colouring principles of the lichens with which we are acquainted. The orsellic acid is rapidly oxidized by contact with the hypochlorite of lime, and is converted into a dark green uncrystallizable resin. This reaction of hypochlorite of lime on these colouring principles forms a very beautiful class experiment. It may be readily shown by pouring a weak solution of bleaching powder into a basin containing either an alkaline or an alcoholic solution of the lichen, or perhaps best of all, simply on the gelatinous precipitate itself. The only precaution to be observed is that no free acid be present. A solution of orsellic acid in ammonia on exposure to the air soon assumes a bright red colour, which gradually becomes darker and more purple-coloured on standing. When heated on platinum foil, it burns with a yellowish flame, leaving no residue ; when it is distilled, it yields a little empyreumatic oil and orcin.

Orsellic acid gives a copious white precipitate with ammonio-nitrate of silver, which however is soon decomposed. It is also precipitated by basic, but not by neutral acetate of lead.

I. 0·216 grm. acid dried at 212° Fahr. and burned with chromate of lead, gave 0·475 carbonic acid and 0·098 water.

II. 0·233 grm. acid gave 0·521 carbonic acid and 0·105 water.

III. 0·3033 grm. acid gave 0·676 carbonic acid and 0·1361 water.

	Calculated numbers.		I.	II.	III.
32 C	2445·920	60·46	60·00	60·98	60·78
16 H	199·672	4·93	5·03	5·00	4·98
14 O	1400·000	34·61	34·97	34·02	34·24
	4045·592	100·00	100·00	100·00	100·00

These analyses give $C_{32} H_{15} O_{13}+HO$ for the rational formula of hydrated alpha-orsellic acid.

The Baryta Salt.

In order to form the baryta salt, pure orsellic acid was dissolved in a slight excess of cold baryta water, and a current of carbonic acid gas was passed through the solution till all excess of baryta was removed. The precipitate, which consisted of carbonate of baryta and the organic acid, was collected on a filter and dried. It was then treated with hot alcohol, which dissolved out the salt and deposited it on cooling in small shining crystals arranged in stars.

(*Alpha*) *Orsellate of Baryta.*

I. 0·593 grm. salt gave 0·149 carb. of barytes=0·1156 BaO=19·49 per cent. BaO.
II. 0·350 grm. salt gave 0·088 carb. of barytes=0·0682 BaO=19·49 per cent. BaO.
III. 0·505 grm. salt gave 0·126 carb. of barytes=0·0977 BaO=19·34 per cent. BaO.

I. 0·1505 grm. salt gave 0·264 carbonic acid and 0·052 water.
II. 0·5773 grm. salt gave 1·041 carbonic acid and 0·197 water.

	Calculated numbers.		Found numbers. I.	II.	III.
32 C	2445·9200	50·01	49·36	49·18	
15 H	187·1925	3·82	3·83	3·79	
1 BaO	956·8800	19·57	19·49	19·49	19·34
13 O	1300·0000	26·60	27·32	27·54	
	4889·9925	100·00	100·00	100·00	

These analyses give $C_{32}\,H_{15}\,O_{13}+BaO$ for the formula of orsellate of baryta.

(*Alpha*) *Orsellesic Acid.*

When the gelatinous precipitate thrown down from the lime solution of the lichen by muriatic acid, and which constitutes crude orsellic acid, is mixed with a little water, and is again cautiously neutralized by the addition of small quantities of lime or baryta, and boiled till the whole is dissolved, the orsellic acid quickly undergoes a rather singular decomposition. Great care must be taken that no excess of base is present, and the boiling continued till the solution becomes quite clear, which shows that the decomposition of the orsellic acid is complete. If the solution has been quite neutral, no precipitation of carbonate of lime takes place. The liquid is allowed to cool and is then filtered. It is next neutralized with muriatic acid, which throws down an abundant gelatinous precipitate, which is a new acid, to which I shall give the name of orsellesic acid. This precipitate is to be collected on a filter and washed with cold water to remove adhering muriatic acid. It is then to be dried and dissolved in weak spirits, out of which it readily crystallizes. It is to be still further purified by being digested with animal charcoal in water, which, for a reason to be presently mentioned, must not be allowed to boil. The acid crystallizes out of alcohol and water in perfectly white needle-shaped crystals arranged in stars. It reddens litmus paper distinctly, and has a feebly acid and somewhat bitter taste. It is very soluble in alcohol, both hot and cold; is much more soluble in hot water than orsellic acid, and crystallizes out of its aqueous solutions in much larger crystals. When orsellesic acid is boiled in water it gives off carbonic acid, and is pretty rapidly converted into orcin, which, when the solution is concentrated, is deposited in perfectly colourless crystals. About half an hour's boiling is sufficient to convert any quantity of orsellesic acid into perfectly colourless orcin. When orsellesic acid, on the other hand, is boiled with an excess of lime or baryta, it is also converted into

orcin with the deposition of a carbonate of the base, and the formation of a reddish colouring matter, from which the orcin can never be effectually freed. This subject will be again referred to in a subsequent part of the paper. Orsellesic acid gives a fugitive bluish red, or rather violet colour with hypochlorite of lime, thus differing from the reaction of orsellic acid, but not easily distinguishable from that of orcin with the same reagent.

When orsellesic acid is dissolved in ammonia and exposed to the air, it yields a red colouring matter similar to orcein. The acid was dried at 212° Fahr.

I. 0·3313 grm. acid gave with chromate of lead 0·7045 carbonic acid and 0·1567 water.

II. 0·3301 grm. acid gave 0·7008 carbonic acid and 0·151 water.

	Calculated numbers.		I.	II.
16 C	1222·960	57·27	57·99	57·90
9 H	112·315	5·26	5·25	5·08
8 O	800·000	37·47	36·76	37·02
	2135·275	100·00	100·00	100·00

These analyses give $C_{16} H_8 O_7 + HO$ as the rational formula of hydrated orsellesic acid.

(*Alpha*) *Orsellesiate of Baryta.*

The baryta salt was prepared from crude orsellesic acid which had been carefully freed from adhering muriatic acid. The crude orsellesic acid was dissolved in hot spirits, to which small quantities of dry crystals of baryta were cautiously added. The solution of the salt may be concentrated by evaporation so long as it is acid, but any excess of base must be avoided, as otherwise the acid would be decomposed and carbonate of baryta deposited. When the hot solution has been sufficiently concentrated, it should be carefully neutralized while hot, and set aside to crystallize. The salt is exceedingly soluble both in water and in spirits. When evaporated to the consistence of a syrup, it crystallizes in small prisms; but when the spirits have been much diluted, the salt is deposited in long four-sided prisms. Its aqueous solutions must be evaporated *in vacuo*. As the baryta salt is decomposed when kept at 212° F., it was also dried *in vacuo*. Six or seven days were required for this purpose, and it lost from seven to eleven per cent. of water, according to the strength of the spirits out of which it had crystallized.

			Per cent.
I.	0·342 grm. salt gave 0·1407 carb. of baryta	=0·1091 BaO	=31·90 BaO.
II.	0·221 grm. salt gave 0·091 carb. of baryta	=0·0706 BaO	=31·94 BaO.
III.	0·293 grm. salt gave 0·120 carb. of baryta	=0·0931 BaO	=31·77 BaO.

0·4915 grm. salt gave 0·7442 carbonic acid and 0·161 water.

	Calculated numbers.		I.	II.	III.
16 C	1222·960	41·04	41·29		
8 H	99·836	3·35	3·63		
1 BaO	956·880	32·11	31·90	31·94	31·77
7 O	700·000	23·50	23·18		
	2979·676	100·00	100·00		

These numbers give $C_{16} H_8 O_7 + BaO$ for the rational formula of orsellesiate of baryta.

Orsellesic Ether.

When (alpha) orsellic acid is boiled for some hours in strong alcohol an ether compound is formed. This ether is most advantageously prepared from the gelatinous precipitate obtained from the lime solution by muriatic acid. This precipitate, when it has been cautiously dried, is to be boiled for six or seven hours in strong alcohol. The solution is then to be cautiously evaporated on the water-bath till nearly the whole of the alcohol has been driven off; care, however, being taken not to evaporate it to dryness, otherwise much of the ether would be converted into a resinous matter. The residue is then to be boiled with a considerable quantity of water and filtered. On the cooling of the liquid the ether precipitates in long flat needles, which have at first a yellowish colour from adhering resin, but when they are treated with animal charcoal and recrystallized they are quite colourless. This ether cannot be distinguished in its external properties from the lecanoric and the erythric ethers. When it is distilled with dry potash it gives off alcoholic vapours, and orcin remains in the retort. Its reactions with ammonia and hypochloride of lime are similar to those of orsellesic acid. It might naturally have been expected that, as orsellic acid was the acid employed in its formation, this ether would have been the orsellic ether. From the results of the subjoined analysis I have been unable to deduce the formula which the orsellic ether ought theoretically to have, while the analyses agree pretty well with the formula of the orsellesic ether.

I am therefore disposed to regard this compound as the orsellesic ether, the more so as I have ascertained that by merely boiling orsellic acid in water without the presence of a base, it is resolved into orsellesic acid. In a subsequent part of this paper, in the case of evernic acid, an instance of the formation of an ether of a second acid different from that of the acid originally put into the alcohol appears to occur.

I. 0·327 grm. ether dried at 212° Fahr., gave 0·734 carbonic acid and 0·1842 water.

II. 0·3195 grm. ether gave 0·716 carbonic acid and 0·177 water.

	Calculated numbers.		Found numbers. I.	II.
20 C	1528·700	61·38	61·24	61·13
13 H	162·233	6·51	6·26	6·15
8 O	800·000	32·11	32·50	32·72
	2490·933	100·00	100·00	100·00

The rational formula of orsellesic ether, therefore, appears to be $C_{16} H_8 O_7 + C_4 H_5O$.

Roccella tinctoria *from the Cape of Good Hope.*

This lichen, the history and botanical characters of which were partially described in a previous part of this paper, was pronounced by Dr. SCOULER to be also a large variety of the *Roccella tinctoria*. It contains two crystalline principles, one very similar to orsellic acid; and another with much less determinately acid characters, to which I have given the provisional name of roccellinin. These two principles may be both extracted by treating the lichen with milk of lime, and precipitating with muriatic acid in the way already described. The gelatinous mass which precipitates consists of a mixture of both principles; but they may be easily separated, as the roccellinin is quite insoluble both in cold and in hot water, while the acid analogous to orsellic acid, and which I purpose to call (beta) orsellic acid, is pretty readily soluble in boiling water. By repeatedly treating therefore the mixture of the two substances with boiling water and filtering, the (beta) orsellic acid is dissolved and deposited on the cooling of the liquid in small silky crystals. These crystals may be obtained quite white and of a much larger size by repeatedly crystallizing them out of weak spirits, care being taken not to boil the solutions. The (beta) orsellic acid may also be prepared by boiling the lichen repeatedly with water, in the way already described for (alpha) orsellic acid. The orsellic acid prepared by this process crystallizes in small silky needles, which are quite free from roccellinin, but which still retain traces of a fatty acid and a little resin, from which I could only purify them by dissolving them in lime or baryta water, precipitating by muriatic acid, and treating them in the way already described.

(Beta) Orsellic Acid.

The following are the characters of (beta) orsellic acid when purified by repeated crystallizations out of weak spirits. Its aqueous and alcoholic solutions redden litmus paper. It is soluble in hot and cold alcohol, and in ether. A solution of hypochlorite of lime yields the same fugitive blood-red colour with it as with (alpha) orsellic acid, with erythric acid, and with the colouring principle in *Lecanora tartarea*. The action of ammonia on all these bodies appears perfectly similar. (Beta) orsellic acid yields but a trifling precipitate with acetate, but a bulky white precipitate with subacetate of lead. In short, it is intermediate in its properties between (alpha) orsellic acid and erythric acid, but approaches the former more closely.

I. 0·2535 grm. acid dried at 212° FAHR., gave with chromate of lead 0·558 carbonic acid and 0·1155 water.

II. 0·2555 grm. substance gave 0·564 carbonic acid and 0·121 water.

	Calculated numbers.		Found numbers. I.	II.
34 C	2598·790	60·10	60·07	60·20
18 H	224·631	5·19	5·06	5·26
15 O	1500·000	34·71	34·87	34·54
	4323·421	100·00	100·00	100·00

The rational formula of the hydrated (beta) orsellic acid is C_{34}, H_{17}, $O_{14}+HO$.

Baryta Salt.

The baryta salt of (beta) orsellic acid is prepared in exactly the same way as the (alpha) orsellate of baryta, by dissolving the acid in an excess of baryta water in the cold, removing the excess of baryta by a stream of carbonic acid gas, collecting the precipitates on a filter and drying them.

The organic salt was then separated from the carbonate of baryta by means of hot alcohol, out of which it crystallized in small white prismatic needles.

Per cent.

I. 0·237 grm. salt gave 0·067 $BaOSo_3$=0·0439 BaO=18·52 BaO.

II. 0·357 grm. salt gave 0·101 $BaOSo_3$=0·0662 BaO=18·54 BaO.

0·346 grm. salt gave with chromate of lead 0·6275 carbonic acid and 0·1285 HO.

	Calculated numbers.		Found. I.	II.
34 C	2598·7900	50·29	49·46	
17 H	212·1515	4·15	4·12	
1 BaO	956·8800	18·51	18·52	18·54
14 O	1400·0000	27·05	27·90	
	5167·8215	100·00	100·00	

The rational formula of the salt is $C_{34}\,H_{17}\,O_{14}$+BaO.

When (beta) orsellic acid is exactly neutralized with lime or baryta and boiled for a short time, it is decomposed just as (alpha) orsellic acid is when similarly treated, yielding a crystallizable acid, which I shall call (beta) orsellesic acid, whose properties very closely resemble those of (alpha) orsellesic acid. (Beta) orsellesic acid also forms a very soluble baryta salt, which crystallizes in long four-sided prisms. When an aqueous solution of (beta) orsellesic acid is boiled, it gives off carbonic acid and is wholly resolved into colourless orcin, exactly in the same way as (alpha) orsellesic acid. A more precise examination of this and another corresponding acid derived from erythric acid will form the subject of a future communication.

An ether compound may be readily procured by boiling pure (beta) orsellic acid in strong alcohol, evaporating the solution, and treating the residue with boiling water in the way already described for orsellesic ether. It may also be prepared by boiling the crude precipitate already mentioned, consisting of orsellic acid and roccellinin, when dried, in strong spirits. In this case the ether is apt to contain a little roccellinin, but from this it can be easily separated by crystallizing it out of boiling water, in which the roccellinin is nearly insoluble. The ether compound crystallizes on the cooling of the liquid in long flat prisms, which cannot be distinguished in appearance from the orsellesic, lecanoric or erythric ethers, with which bodies in its properties and reactions it very closely corresponds.

I. 0·365 grm. ether dried at 212° Fahr., gave with chromate of lead 0·814 carbonic acid and 0·206 water.

II. 0·396 grm. ether gave 0·882 carbonic acid and 0·219 water.

III. 0·2515 grm. ether gave 0·561 carbonic acid and 0·142 water.

	I.	II.	III.
C	60·82	60·75	60·83
H	6·27	6·15	6·27
O	32·91	33·10	33·00
	100·00	100·00	100·00

This substance when distilled with caustic potash gave off alcoholic vapours, leaving orcin in the retort. It is certainly an ether, but of what acid I am unable to determine. These analyses do not agree with the formula of (beta) orsellic ether. It is not improbable therefore that it is the ether of (beta) orsellesic acid, but this is a point which can only be determined when that acid has been subjected to analysis.

Roccellinin.

The Cape of Good Hope lichen contains a much larger quantity of roccellinin than of (beta) orsellic acid. The most convenient mode of extracting the roccellinin is by drying the gelatinous mass precipitated from the lime solution of the lichen by muriatic acid, and then boiling it for a considerable time in strong spirits. The orsellic acid is generally converted into the ether compound, while the roccellinin remains unchanged. On evaporating the solution nearly to dryness, and then treating the residue with boiling water, the ether compound is readily removed, while the roccellinin remains undissolved. By boiling the roccellinin in a large quantity of strong spirits it also dissolves, and on the cooling of the liquid it is deposited in long white hair-like crystals. By repeated crystallizations out of strong spirits, aided by a little animal charcoal, the roccellinin is rendered perfectly pure. It consists of soft hair-like crystals of a silky lustre, and about half an inch long, usually arranged in stars. If these crystals, when treated with hypochlorite of lime, acquire a reddish tinge, they are impure, from containing a little adhering orsellic acid. When quite pure, hypochlorite of lime gives them a greenish-yellow colour, which is permanent. Roccellinin requires a considerable quantity of boiling alcohol to dissolve it, and it is but very moderately soluble either in cold alcohol or in ether. It dissolves readily in the fixed alkalies and in ammonia. Its solutions remain quite colourless. When roccellinin is boiled in baryta water no carbonate of baryta is deposited, and a hot solution of caustic potash is equally inoperative upon it. When it is boiled in alcohol, saturated with muriatic acid gas, no ether is produced.

I. 0·406 grm. substance dried at 212° Fahr., gave with chromate of lead 0·9328 CO_2 and 0·1785 water.

II. 0·351 grm. gave 0·807 carbonic acid and 0·156 water.

III. 0·4705 grm. gave 1·079 carbonic acid and 0·205 water.

IV. 0·4695 grm. gave 1·075 carbonic acid and 0·1965 water.

	Calculated numbers.		I.	II.	III.	IV.
38 C	2904·5300	62·91	62·66	62·67	62·54	62·44
17 H	212·1515	4·59	4·88	4·90	4·84	4·65
15 O	1500·0000	32·50	32·46	32·43	32·62	33·01
	4616·6815	100·00	100·00	100·00	100·00	100·00

The results of these analyses agree pretty closely with the formula $C_{38} H_{17} O_{15}$, which however is merely empirical. I have made many attempts to determine the atomic weight of roccellinin by endeavouring to combine it with the alkalies and earths so as to form definite salts, but hitherto without success. It produces no precipitate with acetate or subacetate of lead, or with nitrate or ammonio-nitrate of silver. When a current of chlorine gas was passed for four days through a quantity of roccellinin diffused through water, the roccellinin assumed a slightly yellow colour. It was collected on a filter and washed to free it from adhering muriatic acid; when crystallized out of alcohol its properties were unchanged, and when subjected to analysis it contained no chlorine. Nitric acid in the cold had no action upon it, with the assistance of heat it converted it into oxalic acid. An attempt was made to prepare a baryta salt by boiling the roccellinin with freshly-precipitated carbonate of baryta. A quantity of prismatic crystals were obtained, but the quantity of base they contained varied according to the concentration of the solution. With caustic baryta, magnesia and lime, the results were equally unsatisfactory. When dissolved in an excess of ammonia and dried *in vacuo*, it was found to be unchanged, and to contain no ammonia. Roccellinin appears therefore to be a very indifferent body, which, like santonin, enters into no stable combinations with either alkalies or acids, though it appears to have a slight affinity for bases, and may therefore be regarded as a feeble acid. I am quite unable to determine what relation roccellinin bears to the orsellic series of acids, with which, however, it is not improbably connected, as the (beta) orsellic acid appears to be partially replaced by it in the Cape of Good Hope lichen.

Roccella Montagnei.

This lichen, which is imported in large quantity from the Portuguese settlement of Angola, and also from Madagascar, where it grows upon trees, was examined by Mr. SCHUNCK under the name of *Roccella tinctoria* var. *fuciformis*. Dr. SCOULER pronounces it to be the *R. Montagnei* of BETENGER, who* found it growing on mango trees at Madras. The branches of the true *R. fuciformis* are much rounder than those of the *R. Montagnei*, which are nearly quite flat. This lichen is by much the richest in colouring matter of any of those employed by the archil manufacturers. Mr. SCHUNCK extracted its colouring principle by treating the lichen with boiling water, and purifying the crystalline precipitate by repeatedly crystallizing it out of weak spirits. This is a very wasteful process; Mr. SCHUNCK states that he only got 60 grs. of erythric acid from a pound weight of lichen. The erythric acid employed in my experiments was prepared partly in this way, but chiefly by the more economical method of macerating the lichen in milk of lime, as already described. By the lime process, the average product of crude erythric acid amounted to twelve per cent. of the weight of the lichen employed. The erythric acid prepared by either method

* See Voyage aux Ind. Orient.

had similar properties, and when subjected to analysis gave identical results. I regret to add that in all my analyses, amounting to about a dozen, I always obtained about one and a half per cent. less carbon than Mr. Schunck, though the combustions were made with all possible care, and a stream of oxygen gas was sent through the apparatus towards the close of the operation. The alcoholic and aqueous solutions of erythric acid do not redden litmus paper, its acid properties are therefore rather feeble. Erythric acid agrees with (alpha) and (beta) orsellic acids in yielding red-coloured compounds with ammonia, and also in its reaction with hypochlorite of lime. There is, generally speaking, a great similarity among these bodies in their behaviour towards reagents; still their atomic weights and the products of their decompositions are very different.

I. 0·5951 grm. acid dried at 212° Fahr., gave with chromate of lead 1·2404 Co_2 and 0·298 HO.

II. 0·3846 grm. gave 0 8031 carbonic acid and 0·1846 water.

III. 0·6737 grm. gave 1·4115 carbonic acid and 0·3415 water.

	Calculated numbers.		I.	II.	III.
C 20	1528·7000	57·34	56·85	56·94	57·14
H 11	137·2745	5·15	5·56	5·33	5·63
O 10	1000·0000	37·51	37·59	37·73	37·23
	2665·9745	100·00	100·00	100·00	100·00

These analyses give $C_{20} H_{10} O_9 + HO$ for the formula of hydrated erythric acid.

I made repeated attempts to form the lead salt, but I could not obtain it of a constant composition. I had no better success with an attempt to form a baryta salt. When erythric acid was dissolved in an excess of baryta in the cold, and a stream of carbonic acid gas sent through the solution, the whole of the baryta was thrown down in the state of carbonate. This plainly shows that erythric acid is a much feebler acid than either of the orsellic acids.

Erythric Ether.

Erythric ether may be readily obtained by boiling the crude acid in strong spirits. It is crystallized out of water and purified in the same way as orsellesic and lecanoric ethers, which substances it very closely resembles in appearance and properties.

I. 0·4646 grm. ether dried at 212° Fahr., gave 1·032 carbonic acid and 0·265 water.

II. 0·372 grm. substance gave 0·8285 Co_2 and 0·2115 water.

	Calculated numbers.		I.	II.
24 C	1834·440	60·70	60·65	60·74
15 H	187·192	6·19	6·33	6·31
10 O	1000·000	33·11	33·02	33·05
	3021·632	100·00	100·00	100·00

The rational formula for erythric ether therefore is $C_{20} H_{10} O_9 + C_4 H_5 O$.

Erythric ether combines with basic acetate of lead. This compound is readily obtained by adding a hot solution of basic acetate of lead to a solution of the ether in boiling water. It falls as a bulky white precipitate. It was washed and dried at 212° Fahr.

Per cent.

I. 1·5145 grm. salt gave 0·5545 PbO and 0·4805 metallic lead=70·78 oxide.

II. 1·0825 grm. salt gave 0·3000 PbO and 0·4295 Pb =71·01 oxide.

I. 0·7188 grm. salt gave with chromate of lead 0·491 Co_2 and 0·1075 water.

II. 1·504 grm. salt gave with chromate of lead 1·043 carbonic acid and 0·232 water.

	I.	II.
C	18·63	18·91
H	1·66	1·71
PbO	71·01	71·01
O	8·70	8·37
	100·00	100·00

I have been quite unable to deduce any rational formula from these analyses of the lead compound of this ether, though the combination appears to be a very stable one, as the salt prepared at different times had always the same composition.

Erythric Methylic Ether.

The methyle compound is readily obtained by boiling erythric acid in strong wood spirit. It crystallizes in longer and narrower prisms than erythric ether. The reactions of both bodies with ammonia and hypochlorite of lime are perfectly similar.

I. 0·4205 grm. ether gave 0·9144 carbonic acid, and 0·209 water.

II. 0·3285 grm. gave 0·715 carbonic acid and 0·1685 water.

Calculated numbers.			I.	II.
22 C	1681·570	59·14	59·30	59·36
13 H	162·233	5·70	5·52	5·69
10 O	1000·000	35·16	35·18	35·04
	2843·803	100·00	100·00	100·00

The rational formula of erythrate of methyle is $C_{20}H_{10}O_9+C_2H_3O$.

When the ethyl compound of erythric acid is boiled with caustic potash or baryta, alcoholic vapours are given off, and orcin and another crystalline body, pseudo-orcin, remain in the solution.

When erythric acid is exactly neutralized with lime or baryta and boiled for a short time it is decomposed, two new compounds being formed, one an acid, which is very similar in appearance and properties to (alpha) and (beta) orsellesic acids, and the other is picro-erythrin. The quantity of the new acid, which I shall call erythrelesic acid, which erythric acid yields, is much less than what is obtained when either of the orsellic acids is decomposed in a similar manner. The reason of this appears to be that a large portion of the erythric acid goes to the formation of the

picro-erythrin, while in the case of the orsellic acids, orsellesic acid and some carbonic acid are the only products. Erythrelesic acid is not quite so soluble in water as either of the orsellesic acids; it crystallizes in small flat scales. When erythrelesic acid is boiled for a short time in water, it is, like the two preceding acids, converted wholly into colourless orcin with evolution of carbonic acid gas. When, on the other hand, it is boiled with an excess of lime or baryta, it is also converted into orcin, which, by this process, has always a deep red colour. Erythrelesic acid forms a very soluble salt with baryta, which crystallizes in long four-sided prisms, which are flatter than those of orsellate of baryta. I intend subjecting this and its kindred acids to a more minute examination. The taste of erythrelesic acid is sourer and more bitter than the orsellesic acid, and it exhibits the same violet colour with hypochlorite of lime.

Picro-erythrin.

When erythric acid has been neutralized by lime or baryta and boiled, as above described, and the erythrelesic acid thrown down by muriatic acid, and removed, the mother-liquor contains a considerable amount of picro-erythrin. When the mother-liquor is concentrated considerably and set aside for a few days in a cold place, the picro-erythrin is deposited in yellowish-coloured crystals. These may be purified by washing them with cold water, and repeatedly crystallizing them out of boiling water, aided by a little animal charcoal. When pure, picro-erythrin crystallizes in long colourless needles arranged in stars. It has a very bitter taste, and is exceedingly soluble in hot water, but very slightly soluble in cold water. Picro-erythrin strikes the same blood-red colour with hypochlorite of lime as erythric acid does. Its solutions in ammonia, on standing for some time exposed to the air, yield a red compound similar to orcein.

I. 0·5225 grm. picro-erythrin dried at 212° Fahr. and burned with chromate of lead, gave 1·0169 carbonic acid and 0·286 water.

II. 0·2825 grm. picro-erythrin gave 0·5527 Co_2 and 0·1513 water.

	Calculated numbers.		Found. I.	Found. II.
34 C	2598·790	53·19	53·07	53·23
23 H	287·028	5·87	6·08	5·95
20 O	2000·000	40·94	40·85	40·82
		100·00	100·00	100·00

The formula which I have found for picro-erythrin differs from Mr. Schunck's by one atom of hydrogen, his being $C_{34} H_{24} O_{20}$.

I was unable to procure the lead salt, mentioned by Mr. Schunck, of uniform composition, though I made repeated attempts. I was not more successful in my endeavours to form a baryta salt; the picro-erythrin crystallized out of its solution, retaining scarce a trace of baryta. Mr. Schunck's description of the other properties

of picro-erythrin are generally speaking very correct, but he has fallen into one or two errors. Mr. Schunck supposes that picro-erythrin is formed from erythric acid by that acid merely taking up five equivalents of water. Now, I have fully ascertained that erythric acid is never converted into picro-erythrin without the evolution of a considerable amount of carbonic acid gas, so that in this process carbon is always eliminated, and the change is not produced merely by the absorption of the elements of water. Picro-erythrin is a very stable body; a quantity of it after being boiled in water for thirty-six hours had undergone very little change, a very small portion of it only having been converted into orcin, and probably into pseudo-orcin. Mr. Schunck asserts that when picro-erythrin is boiled with an excess of lime or baryta it is wholly converted into orcin. A small quantity of orcin is always produced by this process, but the great bulk of the compound then formed is a new sweet crystallizable body to be described in the next section, and which I shall call pseudo-orcin.

Pseudo-orcin.

I have as yet only succeeded in obtaining this very curious substance from erythric acid and its compounds, by acting on them with alkalies. As already mentioned, it is only from picro-erythrin itself, or from the portion of erythric acid which yields picro-erythrin, that pseudo-orcin is obtained. Erythrelesic acid does not yield a trace of it. The best way of preparing pseudo-orcin in quantity is the following. The lime solution of the *Roccella Montagnei* should be boiled in an open pan for two or more hours, and the liquid concentrated to a third or a fourth of its bulk. A stream of carbonic acid gas should be sent through the liquid so long as carbonate of lime is precipitated, and the liquid filtered and evaporated on the water-bath to a thick syrup. This syrup consists chiefly of orcin and pseudo-orcin, with much reddish colouring matter containing a good deal of resin. The syrup should then be introduced into a flask, and treated with a large quantity of ether, which dissolves the orcin and most of the colouring matter, leaving the pseudo-orcin undissolved. A more economical proceeding is to mix the syrup with three times its bulk of alcohol. After standing for a couple of days the pseudo-orcin crystallizes out in small shining crystals. These crystals should be collected on a cloth filter, pressed, and washed with cold strong spirits, which removes most of the colouring matter and any adhering orcin. On being crystallized twice or thrice out of strong boiling spirits, the pseudo-orcin is obtained in large shining colourless crystals. Pseudo-orcin may be obtained in still finer crystals by crystallizing it out of water, in which liquid it is exceedingly soluble, much more so than in spirits. I have obtained it from its aqueous solutions in curiously truncated pyramids, nearly an inch broad, having a brilliant diamond lustre. In fact, there are very few organic substances which form more splendid crystals than pseudo-orcin. I sent a quantity of these crystals to Professor Miller of Cambridge, who was so kind as to determine their measurements and optical properties. The following are the results of his examination:—

Pyramidal . . . . . 111, $\bar{1}\bar{1}1 = 38° 58'$
Simple forms . . . . . . a 100, r 111
s 311.

The form to which the faces s belong is usually hemihedral with parallel faces, 11 $3\bar{1}\bar{3}$. In two crystals only, out of ten which were examined, faint indications of the faces of the other half form were observed.

Fig. 1.

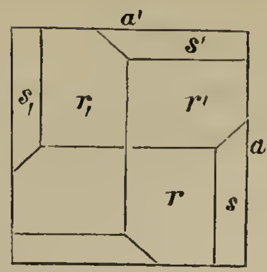
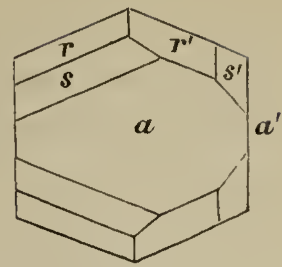


Fig. 2.



The angles between normals to the faces are,—

	°	′
aa'	90	0
ra	70	31
sa	43	18
rr'	38	58
$rr_{,}$	56	17
$ss_{,}$	100	12

No cleavage observable.

The index of refraction, for the brightest part of the spectrum, of the ordinary ray =1·545 ; of a ray in a plane perpendicular to the faces a, a', polarized in that plane =1·523.

Pseudo-orcin is quite neutral to test paper. It has a very sweet taste, though scarcely so sweet as that of orcin. When heated on platinum foil it burns with a blue flame, and emits a smell similar to caramel. When distilled it yields a thin yellowish liquid, emitting at the same time the smell of burnt sugar. The liquid is soluble both in water and in alcohol; it does not crystallize on standing.

Ammonia and hypochlorite of lime have no action upon pseudo-orcin; neither is it affected by any of the alkalies. It is not acted on by bromine. It is not affected by cold nitric acid, but hot nitric acid converts it into oxalic acid. It is not blackened by cold sulphuric acid, but when heated with that acid it becomes brown. Pseudo-orcin gives no precipitate with neutral or basic acetate of lead, with nitrate or ammonio-nitrate of silver, or with salts of copper. It is incapable of fermentation. Dried *in vacuo*, and then at 212° Fahr., it lost no water.

I. 0·507 grm. dried at 212° Fahr., gave with chromate of lead 0·734 Co_2 and 0·3813 water.

II. 0·6275 grm. dried at 212° Fahr., gave 0·907 Co_2 and 0·467 water.

III. 0·4702 grm. dried at 212° Fahr., gave 0·679 Co_2 and 0·355 water.

	Calculated numbers.		I.	II.	III.
10 C	764·3500	39·67	39·46	39·42	39·36
13 H	162·2335	8·42	8·55	8·30	8·60
10 O	1000·0000	51·91	51·99	52·28	52·04
	1926·5835	100·00	100·00	100·00	100·00

These numbers agree pretty well with the formula $C_{10} H_{13} O_{10}$, which however is merely empirical, as I have been quite unable to form any compound with pseudo-orcin so as to determine its atomic weight. Pseudo-orcin appears to be an exceedingly indifferent substance with properties intermediate between those of orcin and mannite.

Mode of extracting the Colouring Principles of the Lichens employed by the Archil makers, so as to render them more portable for commercial purposes.

The amount of colouring principle contained in even the richest of the lichens employed by the archil manufacturers, bears but a small proportion to the weight of the lichen itself. In the case of the *Roccella Montagnei* from Angola, it amounts to about twelve per cent.; in the South American lichen to about seven; and in the Cape variety and in the *Lecanora tartarea* it varies from about two to one and a half per cent. From the great distances from which most of these lichens must be brought, the cost of their transport is so considerable as materially to diminish their commercial value. This is especially the case with their poorer varieties. I think it would in many cases therefore be found advantageous to extract the colouring matter in the countries where the lichens grow, and as this can be effected very readily, a large proportion of the expense of their transport might be easily saved. All that would be necessary for this purpose would be to cut the lichens into small pieces; to macerate them in wooden vats with milk of lime, and to saturate the solution either with muriatic or acetic acid. The gelatinous precipitate could then be collected on cloths and dried by a gentle heat. Almost the whole of the colouring matter in a lichen could thus be easily extracted at a comparatively small expense, and the value of the dried extract, amounting to more than a thousand pounds per ton, would abundantly defray the expense from even the most distant inland localities, such as the Andes or the Himalayas.

Mode of estimating the quantity of Colouring Matter in the Lichens.

A solution of hypochlorite of lime affords a very ready method of approximating pretty closely to the amount of red colouring matter contained in a lichen. Any convenient quantity of the lichen, say one hundred grains, may be cut into very small pieces and then macerated with milk of lime till all the colouring principle is extracted. Three or four macerations are quite sufficient for this purpose if the lichen has been sufficiently comminuted. The clear liquors should be filtered and mixed together. A solution of bleaching powder of known strength should then be poured into the lime solution from a graduated alkalimeter. The moment the bleaching liquor comes in contact with the lime solution of the lichen, a blood-red colour is produced which disappears in a minute or two, and the liquid has only a deep yellow colour. A new quantity of the bleaching liquid should then be poured into the lime solution and the mixture carefully stirred. This operation should be

repeated so long as the addition of the hypochloride of lime causes the production of the red colour; for this shows that the lime solution still contains unoxidized colouring principle. Towards the end of the process, the bleaching solution should be added by only a few drops at a time, the mixture being carefully stirred between each addition. We have only to note how many measures of the bleaching liquor have been required to destroy the colouring matter in the solution, to determine the amount of the colouring principle it contained. The following are the results of trials with the same test liquor upon four varieties of lichen.

	Measures.
Angola lichen required	200=1·00
American lichen	120=0·60
Cape lichen	035=0·17
Lecanora tartarea from Germany, near Giessen . .	025=0·12

The amount of colouring principle in a lichen may also be directly determined by extracting the lichen with milk of lime, by precipitating by means of acetic acid, collecting the precipitate on a weighed filter, drying it at the ordinary temperature, and then weighing it.

Evernia Prunastri.

This lichen was examined in 1843 by Messrs. Rochleder and Heldt, who found in it a substance which they regarded as identical with lecanoric acid. The results of my examination are so different that I am under the necessity of supposing, either that the proximate principles of the *Evernia Prunastri* which grows in Germany are quite different from those of the same plant in Scotland, or, what is much more probable, that these gentlemen had not examined the true *Evernia Prunastri*, but some other lichen in its stead. The lichen on which I operated was pronounced to be the true *Evernia Prunastri* by Dr. Scouler and by two other botanists to whom I submitted it. The lichen was extracted by milk of lime in the way already so often described. Its solution had a bright yellow colour, and on being neutralized by muriatic acid, it yielded an abundant pale yellow precipitate. This was washed, collected on a filter and cautiously dried. It was then digested, at a temperature much under that of boiling, in a quantity of very weak spirits, and this digestion was repeated several times till about two-thirds of the precipitate had dissolved. On the cooling of the solution a mass of small yellowish crystals was deposited. These crystals were rendered perfectly colourless by being treated with animal charcoal, and by being repeatedly crystallized out of weak spirits. They constitute a new acid, which I purpose to call Evernesic acid. The portion of the precipitate which did not dissolve in the weak spirits is usnic acid, which requires pretty strong boiling alcohol for its solution. The usnic acid will be again referred to at a subsequent part of this paper.

Evernic Acid.

Evernic acid is insoluble in cold water ; boiling water dissolves a little of it, which, on the cooling of the liquid, is deposited in white flocks. It is pretty soluble in cold, and exceedingly soluble in hot alcohol; on cooling the liquid is filled with a mass of small adhering needles. Evernic acid is also pretty soluble in ether. This acid has neither taste nor smell. Its alcoholic solution reddens litmus paper. When heated on platinum foil, evernic acid burns readily without leaving any residue; and when it is heated in a test tube, it yields a little empyreumatic oil and a sublimate which has all the characters of orcin. When evernic acid is dissolved in an excess of ammonia and exposed to the air, on standing a few days the liquid slowly assumes a dark red colour. Evernic acid yields only a slightly yellowish colour with hypochlorite of lime. Evernic acid loses no water at 212° Fahr.

I. 0·304 grm. dried at 212° gave with chromate of lead 0·687 Co_2 and 0·137 water.

II. 0·3325 grm. gave 0·7285 carbonic acid and 0·1525 water.

	Calculated numbers.		I.	II.
34 C	2598·790	61·89	61·63	61·61
16 H	199·672	4·75	5·00	5·16
14 O	1400·000	33·36	33·37	33·23
	4198·462	100·00	100·00	100·00

These analyses give $C_{34}\,H_{15}\,O_{13}+HO$ for the rational formula of hydrated evernic acid.

Potash Salt.

A quantity of evernic acid was dissolved in a considerable excess of potash in the cold ; the solution had a yellowish colour and frothed like soap. A current of carbonic acid was then sent through the solution till all the caustic potash present was neutralized, when the liquid became filled with a mass of small crystals. These are the potash salt, which is very soluble in caustic potash, but very slightly soluble either in carbonate of potash or in cold water. The crystals were collected on a filter, dried, purified by digestion with animal charcoal, and finally crystallized out of spirits *in vacuo*. The crystals were then perfectly colourless and had a fine silky lustre.

I. 0·0803 grm. salt gave 0·0183 sulphate of potash =0·098 potash =12·31 Ko per cent.

II. 0·235 grm. salt gave 0·054 sulphate of potash =0·0291 Ko =12·30 Ko per cent.

I. 0·192 grm. salt burned with chromate of lead, gave 0·393 carbonic acid and 0·076 water.

II. 0·319 grm. salt gave 0·6467 carbonic acid and 0·1235 water.

		Calculated numbers.	I.	II.
34 C	2598·7900	55·57	55·82	55·28
15 H	187·1925	4·00	4·39	4·30
13 O	1300·0000	27·82	27·49	28·12
KO	589·9200	12·61	12·30	12·30
	4675·9025	100·00	100·00	100·00

These numbers give $C_{34} H_{15} O_{13}+KO$ as the formula of evernate of potash. This salt loses no water at 212° F., and appears to be anhydrous.

The Baryta Salt.

The baryta salt was prepared in a way similar to the potash salt, by dissolving the acid in an excess of cold baryta water, and neutralizing the solution by a stream of carbonic acid gas. This salt is but little soluble in water, but very soluble in weak spirits. It crystallizes in small prisms. It was first dried *in vacuo*, and then at 212° F., when it lost no water.

I. 0·207 grm. salt gave 0·049 carb. of baryta =0·038 BaO =18·35 per cent. BaO.

II. 0·1683 grm. salt gave 0·0470 sulphate of BaO =0·0308 BaO =18·30 per cent. BaO.

I. 0·2435 grm. salt gave 0·449 carbonic acid and 0·09 water.

II. 0·2534 grm. salt gave with chromate of lead 0·4675 Co_2 and 0·0·94 water.

		Calculated numbers.	I.	II.
34 C	2598·790	50·40	50·30	50·31
16 H	199·672	3·87	4·10	3·96
14 O	1400·000	27·17	27·08	27·38
BaO	956·880	18·56	18·52	18·35
	5155·342	100·00	100·00	100·00

The rational formula of evernate of baryta appears therefore to be $C_{34} H_{15} O_{13} + BaO + Aq$. It contains one atom of water.

Evernesic Acid.

A quantity of evernic acid was dissolved in a slight excess of caustic potash, and the solution boiled for a few minutes. A stream of carbonic acid gas was then sent through the dark-coloured liquid till all the caustic alkali was neutralized. The solution was then concentrated, and on its being set aside for some time, a quantity of scaly crystals was deposited. These crystals are the potash salt of a new acid which I shall call evernesic acid. They are very soluble in water and in hot spirits, but they may be washed with cold spirits without much loss. They are rendered quite colourless by digestion with animal charcoal. If the aqueous solution of the potash salt is decomposed by muriatic acid, a white flocculent precipitate is obtained. This precipitate readily dissolves in boiling water, and is deposited on the cooling of the liquid in long hair-like crystals of a silky lustre. These crystals are evernesic acid.

Evernesic acid may be still more advantageously prepared by employing baryta water instead of potash. When evernic acid is boiled for a short time in a slight excess of baryta water, much carbonate of baryta falls. This is to be removed and the solution saturated with muriatic acid, when the evernesic acid precipitates in silky needles which have a slightly yellowish tint. By being redissolved and treated with animal charcoal, they are rendered colourless. The mother-liquor from which the evernesic acid has been precipitated, when evaporated to dryness, yields a considerable quantity of orcin. Evernesic acid has neither taste nor smell. It is little soluble in cold, but very soluble in boiling water. It dissolves readily in hot spirits, and to some extent also in ether. Its aqueous and alcoholic solutions redden litmus paper. When heated in a test tube, it emits an agreeable smell and yields a white crystalline sublimate. Though orcin is always found in the mother-liquors from which the evernesic acid has been obtained, yet when evernesic acid itself is boiled a second time with either potash or baryta, no orcin is produced, showing that the orcin has been derived solely from the decomposition of *evernic* acid. Hypochlorite of lime merely gives a yellow colour with evernesic acid; and when it is dissolved in ammonia and exposed to the air, no red colour is produced. When evernesic acid was dried *in vacuo* and then at 212° F., it lost no water.

I. 0·191 grm. substance gave with chromate of lead 0·415 Co_2 and 0·0995 water.

II. 0·243 grm. substance gave 0·53 carbonic acid and 0·124 water.

	Calculated numbers.		I.	II.
18 C	1375·830	59·80	59·25	59·48
10 H	124·795	5·42	5·78	5·66
8 O	800·000	34·78	34·97	34·86
	2300·625	100·00	100·00	100·00

These analyses give $C_{18}\,H_9\,O_7+HO$ for the rational formula of hydrated evernesic acid.

Baryta Salt.

The baryta salt of evernesic acid is easily formed as follows. A quantity of evernic acid is boiled for a short time with a slight excess of baryta, and the solution neutralized by a stream of carbonic acid gas. The filtered liquid which contains the baryta salt and some orcin is to be evaporated to dryness on the water-bath. The orcin and the colouring matter which accompany the salt may be removed by treating the dry residue with either cold alcohol or ether, in both of which liquids the salt is nearly insoluble. If the salt is then digested in weak spirits, it readily dissolves, and is deposited on the cooling of the solution in large hard four-sided prisms arranged in a fan-like shape. If not quite colourless at first, the crystals may be easily rendered so by repeated crystallizations.

I. 0·228 grm. salt dried *two* days *in vacuo*, gave 0·083 BaO Co_2=0·0643 BaO=28·20 per cent. BaO.

II. 0·355 grm. salt gave 0·5331 carbonic acid and 0·137 water.

		Calculated numbers.	I.
18 C	1375·8300	40·82	41·11
11 H	137·2745	4·07	4·28
BaO	956·8800	28·36	28·20
9 O	900·0000	26·75	26·41
	3369·9845	100·00	100·00

These numbers give $C_{18}\ H_9\ O_7+BaO+2$ aq=*evern.* BaO+2 Aq.

I. 0·168 grm. salt dried *four* days *in vacuo*, gave 0·0635 BaO Co_2 =0·0492 BaO =29·28 per cent. BaO.

II. 0·1525 grm. salt dried *four* days *in vacuo*, gave 0·0577 BaO Co_2 =0·0447 BaO =29·31 per cent. BaO.

0·2091 grm. salt with chromate of lead gave 0·323 carbonic acid and 0·080 water.

		Calculated numbers.	Found. I.	II.
C 18	1375·8300	42·23	42·12	
H 10	124·7900	3·83	4·24	
BaO	956·8800	29·37	29·28	29·31
8 O	800·0000	24·57	24·36	
	3257·5000	100·00	100·00	

These numbers give $C_{18}\ H_9\ O_7+BaO+Aq$ for the rational formula of this salt of baryta = *evern.* BaO+Aq.

0·298 grm. same salt dried at 212°, gave 0·137 BaO So_3 =0·0899 BaO =30·16 per cent. BaO.

0·2611 grm. salt, also dried at 212° Fahr., gave 0·413 carbonic acid and 0·092 water.

		Calculated numbers.	Found.
18 C	1375·8300	43·74	43·13
9 H	112·3155	3·57	3·91
1 BaO	956·8800	30·42	30·16
7 O	100·0000	22·26	22·80
	3145·0255	100·00	100·00

The formula of this salt, which appears to be anhydrous, is $C_{18}\ H_9\ O_7+BaO$=*evern.* +BaO.

It is evident from these results that evernesate of baryta exists in three states, viz. as an anhydrous salt, and with one and two atoms of water.

The Silver Salt.

The silver salt was formed by precipitating a neutral solution of evernesiate of ammonia with nitrate of silver; it is a white precipitate.

I. 0·247 grm. salt gave 0·092 silver =0·0988 AgO=40·00 per cent. oxide.

II. 0·141 grm. salt gave 0·053 silver =0·0569 AgO=40·35 per cent. oxide.

0·3835 grm. salt gave 0·522 carbonic acid and 0·1113 water.

	Calculated numbers.		Found.
18 C	1375·8300	37·80	37·12
9 H	112·3155	3·09	3·22
AgO	1451·6100	39·86	40·00
7 O	700·0000	19·23	19·66
	3639·7555	100·00	100·00

These numbers give $C_{18}\,H_9\,O_7+AgO$ as the formula of the silver salt, which, as might have been expected, is anhydrous, and agrees in this respect with the baryta salt dried at 212° Fahr.

Evernesic Ether.

A quantity of evernic acid was boiled for a short time in strong alcohol to which a few pieces of fused potash had been added, till the whole was dissolved. A current of carbonic acid gas was then sent through the dark-coloured solution till the caustic potash was saturated. On the cooling of the liquid a quantity of large brown-coloured prismatic crystals was deposited. These crystals were washed with cold water, in which they were nearly insoluble, and after being dried between folds of blotting-paper, and being repeatedly crystallized out of spirits, they formed beautiful white prisms about half an inch long. A further small quantity of these crystals was obtained by diluting the mother-liquor with water. The mother-liquor contains a considerable amount of orcin. A substance perfectly identical in its composition and properties with that just described, may also be obtained by boiling evernic acid in absolute alcohol for eight or ten hours. The alcoholic solution is then to be evaporated nearly to dryness by a gentle heat. The residue contains the ether mixed with a good deal of orcin, and some yellowish resinous matter. The orcin is removed by treating the mass with a little cold water, and the ether is freed from the resin which adheres to it very pertinaciously by digesting it with a little animal charcoal in dilute spirits. From the difficulty of separating the resinous matter from the ether, the first process with potash is by far the best.

The properties of the ether prepared by either of the above processes are as follows:— It has neither taste nor smell. It is very soluble in alcohol, whether hot or cold, and in ether. It is quite insoluble in cold water; boiling water dissolves little more than a trace of it. When heated to 133° Fahr. it melts, but crystallizes again when the temperature is slightly lowered. It is insoluble in ammonia, and gives no red colour with that alkali. Hypochlorite of lime has also no action upon it; its solutions give no precipitates with acetate or subacetate of lead. Cold and boiling muriatic acids produce no effect upon it. It is insoluble in carbonate of potash, but caustic potash dissolves it readily. The solution is colourless, and when it is neutralized by muriatic acid, the ether precipitates unchanged. Its alcoholic solution, when boiled with potash, yields no orcin and remains apparently unchanged. When however the vapour of the ether is passed over fused potash in a narrow glass tube with a capillary opening,

the ether is decomposed, and a gas is emitted which burns with the flame characteristic of alcohol. The potash when examined contained no orcin. There is every reason therefore to regard this substance as an ether.

I. 0·2482 grm. ether prepared with absolute alcohol and dried *in vacuo*, gave with chromate of lead 0·5740 carbonic acid and 0·1600 water.

II. 0·2774 grm. ether prepared in the same way, gave 0·640 carbonic acid and 0·171 water.

III. 0·373 grm. ether prepared with potash and alcohol, gave 0·859 carbonic acid and 0·228 water.

IV. 0·2617 grm. ether prepared with potash and alcohol, gave 0·604 carbonic acid and 0·1635 water.

	Calculated numbers.		I.	II.	III.	IV.
22 C	1681·570	63·30	63·07	62·92	62·80	62·94
14 H	174·713	6·58	7·15	6·88	6·79	6·93
8 O	800·000	30·12	29·78	30·20	30·41	30·13
	2656·283	100·00	100·00	100·00	100·00	100·00

$C_{18} H_9 O_7 + C_4 H_5 O$, the formula derived from these analyses, is exactly that which the evernesic ether ought theoretically to have, and which I am strongly inclined to think this substance most certainly is. It is no doubt somewhat singular that when an attempt was made to form the evernesic ether directly, by boiling evernesic acid in absolute alcohol saturated with muriatic acid gas, it totally failed. Still I am inclined to regard the substance above described as evernesic ether, as it certainly cannot be evernic ether, as might naturally have been expected, seeing that evernic acid was the acid employed in its formation ; for its mother-liquors always contained orcin, thus showing that the evernic acid had been decomposed ; and when the evernesic ether was fused with dry potash, it yielded no orcin, which evernic acid always does when similarly treated. It is clear therefore that the ether could not contain any evernic acid. The only probable explanation of this enigma appears to be that evernic acid does not form an ether, and that evernesic ether is only procurable when evernesic acid is generated in contact with alcohol.

On Orcin and its Preparation.

It is a remarkable circumstance that, so far as we know, orcin is always one of the products when any of the colouring principles of the lichens which yield red dyes with ammonia are subjected to particular operations. We have seen that this is the case with the principles of the different varieties of *Roccella tinctoria*, with those of the various kinds of *Lecanora*, and with that of the *Evernia Prunastri*. When the colouring principles of these lichens are destructively distilled, or are boiled with alkalies, or even with pure water or alcohol, orcin is always one, though by no means the only product. This circumstance is characteristic of this whole class of bodies.

and shows that though they differ considerably from each other in some respects, still a very intimate relation exists among them. Mr. SCHUNCK first observed that when lecanoric or erythric acids are boiled with an excess of any of the alkalies, orcin and a carbonate of the alkali are invariably produced. Mr. SCHUNCK was of opinion, and in this he has been followed by Messrs. ROCHLEDER and HELDT, that orcin and carbonic acids are the only products of this decomposition. We have already seen that this decomposition is by no means so simple as these gentlemen supposed; that, first of all, intermediate acids are always formed, and that in the case of the erythric and evernic acids, orcin is not the only ultimate product, but that pseudo-orcin and evernesic acid are also invariably obtained. Though orcin may be procured from (alpha) and (beta) orsellic acids, from erythric acid, and from lecanoric and evernic acids, they are not all of them equally suitable for this purpose. By far the best way, in fact the only way, of procuring colourless orcin, is to boil the (alpha) and (beta) orsellesic acids or erythrelesic acid in water from half an hour to an hour, when a great deal of carbonic acid is given off, and on the solution being concentrated and set aside to cool, abundance of colourless crystals of orcin are deposited. If a little animal charcoal is kept in the solution while it is being concentrated, it will be found useful in preventing any tendency to oxidation. I think it will also be found, though I have not had the opportunity of trying the experiment, that when lecanoric acid is neutralized with lime or baryta and boiled for a short time, it will be decomposed like orsellic and erythric acids, and yield an acid similar to the orsellesic and erythrelesic acids, which I have no doubt, by simply boiling in water, will also be converted into colourless orcin. By long-continued boiling with water, (alpha) and (beta) orsellic and erythric acids may no doubt be slowly converted into orcin without the intervention of an alkali. An immense amount of boiling is however required for this purpose. A quantity of orsellic acid which had been kept continuously boiling for three days, was only partially converted into orcin, and the product was far from colourless, owing to the effects of oxidation.

I. 0·332 grm. colourless orcin, prepared as above described, dried at the ordinary temperature, gave with chromate of lead 0·717 carbonic acid and 0·204 water.

		Calculated.	Mr. SCHUNCK's.	I.
16 C	1222·960	59·36	58·98	58·90
11 H	137·274	6·66	7·68	6·82
7 O	700·000	33·98	33·96	33·96
	2060·234	100·00	100·00	100·00

It will be seen from this that the faint red colour which orcin has when prepared by boiling with an excess of alkali, has no influence upon the analysis.

When it is wished to prepare orcin on a large scale without its being required to be perfectly colourless, the following process will be found very convenient. The clear solution obtained by macerating any of the varieties of the *Roccella tinctoria*, or

any of the different kinds of *Lecanora* in milk of lime, should be boiled in an open pan for some hours, and the liquid concentrated to about a fourth of its bulk. A stream of carbonic acid gas should then be sent through the liquid so long as carbonate of lime precipitates. The clear liquid should again be drawn off, and cautiously evaporated to dryness on the water-bath. The residue should then be boiled with three or four times its bulk of strong spirits, filtered and set aside to crystallize. In two or three days abundance of large dark-coloured crystals are deposited in the liquor. These should be separated and dried between folds of blotting-paper. The dried crystals should then be dissolved in three or four times their weight of anhydrous ether, in which they are exceedingly soluble, and the solution filtered. By evaporation *in vacuo* the orcin crystallizes out of the etherial solution in large six-sided prisms, which have only a light red colour, and which by a subsequent crystallization, also out of ether, may be rendered still fainter. A quantity of orcin prepared in this way was dried under the air-pump for upwards of a week. It is not easily dried, as it is a very hygroscopic substance. When subjected to analysis, it gave numbers which agreed exactly with those which Mr. Schunck found for anhydrous orcin. Orcin repeatedly crystallized out of anhydrous ether appears therefore to be anhydrous. Orcin, however, is a substance which varies so much in the quantity of water it contains, according to the temperature at which it has been dried, that I defer saying more at present than that I am engaged in examining it more minutely.

When orcin is treated with a few drops of hypochlorite of lime, it assumes a dark purple red colour, which quickly changes to a deep yellow. It is quite different from the blood-red colour which orsellic or erythric acids yield when similarly treated, but it cannot be distinguished by the eye from the reaction which orsellesic and the other intermediate acids yield with hypochlorite of lime.

Brom-Orceid.

When bromine is poured into a concentrated aqueous solution of orcin an energetic action immediately ensues ; much heat is evolved, and a brownish red crystalline mass falls to the bottom of the liquid. Bromine was added so long as this action continued, and the crystallized mass was separated from the supernatant liquor, which contained a great deal of hydrobromic acid, and was repeatedly washed with cold water. This bromine compound is but little soluble in either cold or hot water. In hot water it melts like a heavy oil, but crystallizes on the cooling of the liquid. It is very soluble in alcohol, both hot and cold, and also in ether. The brown colour of the crystals, when first precipitated, is owing to their being contaminated with a brown-coloured uncrystallizable resin. The quantity of this brownish resin is, comparatively speaking, small. It also contains bromine, and has a very pungent smell, which affects the eyes and nose very strongly, occasioning considerable pain. The crystalline bromine compound, which I shall call Brom-orceid, is readily purified by digesting it in dilute spirits, with the aid of a little animal charcoal, which absorbs

the brownish resinous matter. When pure, brom-orceid forms long white adhering needles. It has neither taste nor smell. The mother-liquors, from which it crystallizes, assume a pale reddish colour on standing, and contain some hydrobromic acid. The brom-orceid was dried *in vacuo* and subjected to analysis.

I. 0·355 grm. substance gave 0·5810 AgBr=0·2439 Br=68·70 per cent. bromine.

II. 0·407 grm. substance gave 0·665 AgBr=0·2792 Br=68·59 per cent. bromine.

I. 0·5442 grm. substance gave 0·471 carbonic acid and 0·076 water.

II. 0·451 grm. gave with chromate of lead 0·391 carbonic acid and 0·065 water.

	Calculated numbers.		Found numbers. I.	II.
66 C	5044·710	23·63	23·60	23·64
24 H	324·467	1·52	1·55	1·60
3 Br	14674·500	68·75	68·70	68·59
13 O	1300·000	6·10	6·15	6·27
	21343·677	100·00	100·00	100·00

The formula given above, $C_{66} H_{24} Br_3 O_{13}$, is purely empirical. I regret that I have been unable to determine its atomic weight by the analysis of any of its compounds, and to establish a simple relation between it and orcin, if any such exist. Brom-orceid has a feeble acid reaction. It dissolves readily in alkaline solutions. These solutions soon grow dark-coloured, and when they are neutralized by muriatic acid, a yellowish resin precipitates.

Chlor-Orceid.

A similar compound is formed when chlorine is made to act upon orcin. In the course of several trials with dry orcin and with its aqueous and alkaline solutions, I obtained little more than traces of a crystalline chlorine compound. The crystals were always accompanied with a large quantity of a dark-coloured resin, which adhered to them so pertinaciously that I have not as yet been able to procure a sufficient quantity of the crystals to be able to subject them to analysis. Mr. Schunck endeavoured to prepare a similar compound, but, as he has given no analysis of it, I suspect that in this instance his success has been pretty similar with my own.

Usnic Acid.

This acid, which occurs in considerable quantity in several of the lichens, was discovered by Mr. Knop in 1843. It was also examined nearly at the same time by Messrs. Rochleder and Heldt. Knop found it in several species of Usnea, such as *Usnea florida*, *U. hirta* and *U. plicata*. Messrs. Rochleder and Heldt extracted it from the lichen *Rangiferinus*, *Usnea barbata* and *Ramalinea calicaris*. In addition to these sources, I may mention that I have found it in *Evernia Prunastri*, where it occurs along with evernic acid, and in *Ramalinea Fraxinea*. Knop's process for procuring usnic acid is by treating the lichens in a displacement apparatus with ether. This is

both a tedious and a costly method. Messrs. ROCHLEDER and HELDT extracted it by macerating the lichens with a mixture of ammonia and spirits of wine. This latter method succeeds very well, but is also costly, especially in England, where alcohol is so dear. I have found that the method already so often described, viz. macerating the lichens in milk of lime and precipitating with muriatic acid, answers perfectly well, and is much to be preferred to either of the preceding methods. It is not advisable to employ either *Evernia Prunastri* or any of the species of Ramalinea for the preparation of usnic acid, as the acid obtained from these lichens is always accompanied with a good deal of resinous brownish colouring matter, which adheres to the acid so pertinaciously that it is very difficult to purify it completely. The lichens I prefer for the preparation of usnic acid are the *Cladonia Rangiferina*, but especially the *Usnea florida*. The lime solution of these lichens is deep yellow, and the precipitate thrown down either by muriatic or acetic acid has a bright yellow colour. By crystallizing this precipitate repeatedly out of alcohol, aided by a little animal charcoal, the usnic acid is obtained in large flat crystals of a pale-yellow colour. Usnic acid is not readily combustible.

0·2955 grm. acid dried at 212° FAHR. and burned with oxide of copper and a stream of oxygen gas, gave 0·688 carbonic acid gas and 0·134 water.

	Calculated.		Found numbers.
38 C	2854·56	63·90	63·62
17 H	212·16	4·75	5·03
14 O	1400·00	31·35	31·35
	4467·00	100·00	100·00

Potash Salt.

The potash salt was prepared in the way indicated by Mr. KNOP, by boiling the acid with carbonate of potash. The salt crystallizes readily in large plates.

I. 0·407 grm. salt dried at 212° FAHR., gave 0·087 sulphate of potash =0·047 Ko =11·55 per cent.

II. 0·212 grm. salt gave 0·0453 $KoSo_3$=0·0244 Ko=11·50 per cent.

The calculated quantity of potash in the salt is 11·66 per cent.

Usnic acid is oxidized and converted into a dark brown uncrystallizable resin when it is boiled for a considerable time with an excess of either potash or baryta. The action of chlorine upon usnic acid produces a somewhat similar result.

Glasgow, January 25, 1848.

V. *On the Heat disengaged during Metallic Substitutions.*

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Communicated by Michael Faraday, *Esq., F.R.S., &c. &c.*

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In the present communication I propose to give an account of some new investigations on the heat disengaged in chemical actions, which may be considered a continuation of my former inquiries on the same subject*. The greater number of the experiments to be detailed in this paper were made some years ago, and the conclusion at which I arrived was briefly announced in the Philosophical Magazine for August 1844. More recently, I have taken an opportunity to repeat many of my former experiments and to add new ones on the same subject, all of which confirm the general results formerly obtained.

Having originally observed that although a very limited number of bases (potash, soda, barytes and strontia) develope nearly the same quantity of heat, when a chemical equivalent of each enters into combination with an acid, yet that the greater number of bases differ most widely from one another, when so treated, while on the other hand, that different acids (taken in the state of dilute solution) produce with the same base nearly the same amount of heat, I ventured to draw the general inference that the thermal effects produced are more intimately connected with the basic, or electro-positive, than with the acid, or electro-negative element. In conformity with this view, it appeared probable that in the decomposition of solutions of neutral salts by the addition of bases or metallic bodies, the nature of the acid or electro-negative element of the compound would exercise no special influence on the result. I have already endeavoured to establish by experiment the truth of this principle in the case of basic substitutions, and, in the present memoir, I propose to extend the same general law to the other case, in which one metallic element replaces, or is substituted for another.

Few chemical actions are more simple in their final results, or admit more easily of being varied without changing the general type of the reaction, than those which form the subject of the present inquiry. When a neutral solution of any salt of the black oxide of copper, as, for example, the sulphate, the chloride, or the acetate, is precipitated by metallic zinc, the final result is the substitution of an atom of zinc for an atom of copper in the solution, and the precipitation of an atom of copper. If

* Transactions of the Royal Irish Academy, vol. xix. pp. 228, 293. Also Philosophical Transactions for 1844, p. 21.

the physical and chemical properties of equivalent solutions of different salts of copper be compared, they will be found to present almost a complete identity, and the same remark applies to the solutions of the salts of zinc which remain after the reactions are finished. We have, therefore, every condition favourable to the production of simple thermal results. For the present object, it is not necessary to inquire in what state the metallic element exists in an aqueous solution of its salts, or what changes actually occur between the first addition of the zinc and the final precipitation of the copper; it is enough to know that the final result is the same, whether we employ a solution of an oxy-salt, or of a haloid salt.

The general result of the whole investigation may be stated in the following terms:—

When an equivalent of one and the same metal replaces another in a solution of any of its salts of the same order, the heat developed is always the same; but a change in either of the metals produces a different development of heat.

By the expression "solution of a salt of the same order" is understood, a solution in which the same precipitate is produced by the addition of an alkali, or, on one view of the composition of such salts, in which the metal exists in the same state of oxidation.

Salts of Copper with Zinc.

Two distinct series of experiments were made with the salts of the black oxide of copper and metallic zinc. In the first series, concentrated solutions were taken and introduced into a small glass vessel, in which was also placed a glass tube, open above, and containing pure zinc in a state of fine subdivision. The glass vessel, carefully closed, was introduced into a larger vessel of copper furnished with a lid. The latter was filled with water adjusted to the proper temperature and suspended in an outer vessel of tin plate, and the whole introduced into a cylinder closed with a lid and capable of being rotated*. After all parts of the apparatus had acquired the same temperature, a very sensible thermometer was introduced into the water contained in the copper vessel through a small orifice in the lid, and the position of the mercury in the tube observed. The thermometers having been removed and the orifice closed with a cork, the lid of the outer vessel was shut down, and the rotating wheel moved through half a revolution, by which means the metallic zinc was brought into contact with the copper solution. The rotation was afterwards continued for five minutes and a half, which was found to be sufficient not only to complete the precipitation of the copper, but also to diffuse the heat arising from the reaction uniformly through the apparatus. The temperature of the water was so adjusted as to render the corrections required for the heating and cooling influence of the air very inconsiderable; their amount was, however, ascertained in each experiment and the results altered accordingly.

* For a description and representation of a similar apparatus, see Transactions of the Royal Irish Academy, vol. xix.

To remove all uncertainty as to the strength of the solutions, a considerable quantity of each salt was dissolved in water, and a portion of the solution carefully analysed by precipitating the oxide of copper. The solutions were all employed in a perfectly neutral state.

In the second series of experiments, more dilute solutions were taken, and the increment of temperature observed directly in the solution in which the precipitation occurred. The zinc, cooled to the same degree as the liquid, was introduced after the temperature of the former had been observed, and the whole rotated for a period of one minute and a half. After the final temperature was taken, a few drops of the liquid were quickly withdrawn for future examination, and the apparatus was again rotated for a period of one minute and a half. On again introducing the thermometer, the temperature of the liquid was always found to be a few hundredths of a degree higher than at the preceding observation, although the whole of the copper had been previously precipitated, and on repeating the same operation several times, nearly the same development of heat occurred on each occasion. This secondary evolution of heat arose from two distinct causes, the oxidation of the precipitated metal by the air contained in the upper part of the glass vessel, and the voltaic circle formed by the precipitated copper with the zinc in excess. The influence of the former circumstance was clearly proved by repeating the experiment with the vessel as nearly filled as possible with the solution, which considerably diminished the amount of the secondary development of heat. But without entering into a minute discussion of the efficient causes of this rise of temperature, it is sufficient for the present object to observe that the same causes must have been in operation, even in a more intense degree, during the greater part of the first period of agitation, and would render the increment then observed too high. The application of the required correction is very difficult, and the uncertainty on this point prevents absolute accuracy being attained in the following numerical results. As the most probable estimate, I assumed the correction to be equal to the increment observed during the second period of rotation, without applying any correction to this increment for the cooling influence of the air. The amount of this correction was usually about 0°·1 C. It should be carefully remembered that the precipitation was in every experiment proved to be complete at the end of the first agitation, by removing a few drops of the solution and afterwards carefully testing it. In the first series of experiments with the salts of copper, no correction was applied for this secondary development of heat, because it was impossible to ascertain its amount, which however was probably less than in the experiments of the second series.

First Series.—Sulphate of Copper and Zinc.

The solution of sulphate of copper weighed 43·3 grms. and contained 1·100 grm. oxide of copper. The specific heat of the solution of sulphate of zinc which was formed, was found by direct experiment to be 0·935, and consequently its thermal

equivalent 40·5 grms. water. The apparatus in contact with the fluids contained 92 grms. copper, 20 grms. brass, and 43 grms. glass, besides the cork, &c. Its thermal equivalent, the excess of zinc included, was 17·4 grms. water. The degrees are those of the centigrade scale.

I. Air 13°·4. Increment found 2°·54, corrected 2°·53.
Water 242·6 grms. Solution and vessels (equivalent to) 57·9 grms.
II. Air 11°·8. Increment found 2°·53, corrected 2°·53.
Water 243·5 grms. Solution and vessels 57·9 grms.
III. Air 12°·0. Increment found 2°·51, corrected 2°·52.
Water 243·5 grms. Solution and vessels 57·9 grms.
IV. Air 15°·4. Increment found 2°·51, corrected 2°·50.
Water 243·6 grms. Solution and vessels 57·9 grms.

Hence we have for the heat of combination referred to 1 grm. of metallic copper as unit,

I.	II.	III.	IV.	Mean.
866°	868°	865°	858°	864°.

Chloride of Copper and Zinc.—The solution of chloride of copper weighed 43·3 grms. and contained 1·100 grm. oxide of copper. The specific heat of the solution of chloride of zinc was found to be 0·946.

I. Air 13°·5. Increment found 2°·50, corrected 2°·51.
Water 243·3 grms. Solution and vessels (equivalent to) 58·3 grms.
II. Air 14°·0. Increment found 2°·52, corrected 2°·52.
Water 238·3 grms. Solution and vessels 58·3 grms.
III. Air 14°·9. Increment found 2°·49, corrected 2°·50.
Water 244·8 grms. Solution and vessels 58·3 grms.
IV. Air 13°·6. Increment found 2°·49, corrected 2°·50.
Water 241·1 grms. Solution and vessels 58·3 grms.

We have, therefore, for the heat of combination referred to the same unit as before,

I.	II.	III.	IV.	Mean.
862°	851°	863°	852°	857°.

Acetate of Copper and Zinc.—The solution of acetate of copper weighed 43·3 grms. and contained 1·092 grm. oxide of copper. The solution of acetate of zinc formed during the reaction had a specific heat of 0·930.

I. Air 16°·7. Increment found 2°·43, corrected 2°·44.
Water 242·8 grms. Solution and vessels (equivalent to) 57·6 grms
II. Air 16°·7. Increment found 2°·43, corrected 2°·44.
Water 243·6 grms. Solution and vessels 57·6 grms.
III. Air 15°·5. Increment found 2°·40, corrected 2°·41.
Water 242·8 grms. Solution and vessels 57·6 grms.

I.	II.	III.	Mean.
841°	843°	830°	838°.

Second Series.—Sulphate of Copper and Zinc.

The solution of sulphate of copper weighed 100 grms., and contained 0·360 grm. oxide of copper. The specific heat of the solution of sulphate of zinc was ascertained by experiment to be 0·989. A large excess of zinc (4·5 grms.) in the state of fine filings was taken, in order to complete the action in the shortest possible time. The zinc had been carefully distilled, and contained not more than 0·0005 of impurity, which was chiefly lead. The glass vessel in which the experiment was performed, weighed 50 grms., and the thermal value of the entire apparatus with its contents was 106·3 grms. The whole of the copper was precipitated in the course of one minute and a half of agitation.

I. Air 17°·9. Increment found 2°·48, corrected 2°·35.
II. Air 17°·8. Increment found 2°·50, corrected 2°·37.
III. Air 17°·5. Increment found 2°·48, corrected 2°·35.

In these experiments the glass vessel was not entirely filled with the solution, and the correction for the secondary development of heat amounts, it will be observed, to 0°·13. In the two following experiments, a much smaller quantity of air was left in the vessel, and the correction was thereby reduced to 0°·05. The solution now weighed 130·0 grms., and contained 0·468 grm. oxide of copper. The thermal equivalent of the whole apparatus was 136·3 grms.

IV. Air 15°·5. Increment found 2°·40, corrected 2°·35.
V. Air 16°·0. Increment found 2°·42, corrected 2°·37.

I.	II.	III.	IV.	V.	Mean.
869°	876°	869°	857°	867°	868°.

Chloride of Copper and Zinc.—The solution contained the same weight of oxide of copper, and the zinc solution had the same specific heat as the preceding. The last experiment was adjusted in the same manner as the fourth and fifth of the foregoing series.

I. Air 17°·2. Increment found 2°·43, corrected 2°·31.
II. Air 17°·0. Increment found 2°·46, corrected 2°·34.
III. Air 17°·6. Increment found 2°·46, corrected 2°·34.
IV. Air 17°·8. Increment found 2°·44, corrected 2°·32.
V. Air 16°·1. Increment found 2°·40, corrected 2°·35.

I.	II.	III.	IV.	V.	Mean.
854°	865°	865°	858°	857°	860°.

Acetate of Copper and Zinc.—100 grms. of a solution of this salt containing 0·360 grm. oxide of copper were taken. Specific heat of zinc solution 0·987. Thermal value of the whole 106·1 grms.

I. Air 17°·3. Increment found 2°·49, corrected 2°·37.
II. Air 17°·3. Increment found 2°·50, corrected 2°·38.
III. Air 18°·0. Increment found 2°·50, corrected 2°·38.

I.	II.	III.	Mean.
875°	878°	878°	877°

Formiate of Copper and Zinc.—The solution of this salt corresponded in all respects with that of the sulphate of copper.

I. Air 17°·8. Increment found 2°·43, corrected 2°·36.

II. Air 17°·6. Increment found 2°·41, corrected 2°·34.

I.	II.	Mean.
873°	865°	869°.

Collecting the foregoing results, we find for the heat of combination from the first series,

	°
Sulphate of copper	864
Chloride of copper	857
Acetate of copper	838

And from the second series,

Sulphate of copper	868
Chloride of copper	860
Acetate of copper	877
Formiate of copper	869

The agreement among these numbers is as close as can be expected in experiments of this kind, in which other disturbing sources of heat are present, whose precise influence it is difficult to estimate. It is very plain that the heat developed is wholly independent of the acid with which the metal is combined. The results obtained in the two series, in which solutions of very different strengths were employed, differ little from one another, but an accurate comparison cannot be made, as no correction for the disturbing thermal effects was applied to the numbers of the first series. We may, however, conclude that, within the limits of these experiments, the heat developed by the same amount of metallic substitution is nearly the same in solutions of different strengths. It is probable that this observation will not be found strictly to apply to very concentrated solutions.

It is almost unnecessary to remark that if, in the course of the reaction, any chemical change occurs besides the displacement of one metal by another, the heat evolved will no longer be the same. On this account, solutions of the metallic nitrates, especially if concentrated, are not adapted for this investigation.

If we take the mean of the numbers in the second series, and adopt 3·96 as the atomic weight of copper, we shall have for the heat extricated during the displacement of

	C.		F.
1 grm. copper by zinc	868°	or	1562°
1 equiv. copper by zinc	3435°	or	6183°.

Salts of Copper with Iron.

Two distinct series of experiments similar to the preceding were made on the precipitation of the salts of copper by iron. In the first series, the apparatus and solutions were in all respects the same as in the experiments with zinc. A large quantity (from 12 to 13 grms.) of the precipitating metal was required.

First Series.—Sulphate of Copper and Iron.

I. Air 14°·7. Increment found 1°·68, corrected 1°·68.

Water 246·4 grms. Solution and vessels (equivalent to) 59·1 grms.

II. Air 14°·9. Increment found 1°·68, corrected 1°·68.

Water 244·3 grms. Solution and vessels 59·1 grms.

III. Air 14°·4. Increment found 1°·65, corrected 1°·68.

Water 243·5 grms. Solution and vessels 59·1 grms.

Hence we have for the heat of combination referred to 1 grm. copper as unit,

I.	II.	III.	Mean.
584°	580°	579°	581°.

Chloride of Copper and Iron.

I. Air 15°·5. Increment found 1°·77, corrected 1°·77.

Water 243·1 grms. Solution and vessels 59·5 grms.

II. Air 14°·2. Increment found 1°·77, corrected 1°·77.

Water 242·5 grms. Solution and vessels 59·5 grms.

I.	II.	Mean.
609°	610°	609°·5.

The greater amount of heat in the latter experiments arose from the protochloride of iron absorbing oxygen more rapidly than the protosulphate. In the next series this source of error was avoided, and the results agree better with each other.

Second Series.—In these experiments the salts were dissolved in recently boiled water, and a small bubble of air only was left in the containing vessel. Each solution weighed 126·7 grms. and contained 0·456 grm. oxide of copper.

Sulphate of Copper and Iron.

I. Air 15°·5. Increment found 1°·64, corrected 1°·62.

Heat of combination 593°.

Chloride of Copper and Iron.

I. Air 15°·5. Increment found 1°·64, corrected 1°·61.

Heat of combination 590°.

Taking the mean of the last experiments, we obtain for the heat extricated during the precipitation of

	C.	F.
1 grm. copper by iron	592° or	1066°
1 equiv. copper by iron	2342° or	4216°.

Salts of Copper with Lead.

Acetate of Copper and Lead.—As before, 100 grms. of a solution of acetate of copper

containing 0·360 grm. oxide were taken. A large excess of lead (15 grms.) was required for complete precipitation. The thermal value of the entire was 106·0 grms.

I. Air 21°·5. Increment found 0°·89, corrected 0°·77.

II. Air 21°·5. Increment found 0°·89, corrected 0°·77.

I.	II.	Mean.
284°	284°	284°.

Formiate of Copper and Lead.—The solution was adjusted as before.

I. Air 17°·2. Increment found 0°·82, corrected 0°·72.

II. Air 17°·1. Increment found 0°·83, corrected 0°·71.

III. Air 16°·0. Increment found 0°·80, corrected 0°·70.

I.	II.	III.	Mean.
265°	269°	259°	268°.

Hence we have for the heat evolved during the precipitation of

	C.		F.
1 grm. copper by lead	268°	or	482°
1 equiv. copper by lead	1061°	or	1909°.

Salts of Silver with Zinc.

The salts of silver are easily reduced by agitating their solutions with finely divided zinc. The sulphate and acetate were selected for experiment, and the results will be found to afford a further illustration of the general principle laid down in the commencement of this paper. The secondary development of heat was also clearly manifested, and continued for a considerable period of time, but gradually diminished in intensity. The two following observations will exhibit the amount of this heat, which, in the case of the salts of silver, must be chiefly ascribed to voltaic action. The annexed numbers give the increments of heat observed at intervals of two minutes, during each of which the agitation was continued in precisely the same manner. A few drops of the liquid having been removed after the first period of agitation, gave afterwards not the slightest opalescence with chloride of sodium, showing that the metallic precipitation was then finished.

	Sulphate of silver.	Acetate of silver.
First increment . . .	1°·96	1°·94
Second increment. . .	0·14	0·12
Third increment . . .	0·12	0·12
Fourth increment. . .	0·10	0·08
Fifth increment . . .	0·09	0·06
Sixth increment . . .	0·04	0·06

The final temperature of the liquid was about 0°·7 above that of the surrounding air. The existence of a considerable amount of voltaic action was clearly shown by the evolution of hydrogen gas from the surface of the precipitated silver.

Sulphate of Silver and Zinc.—The weight of the solution taken was 100 grms. An

equal portion of the same solution yielded, on analysis, 0·600 grm. chloride of silver, corresponding to 0·452 grm. metallic silver. The thermal equivalent of the solution of sulphate of zinc obtained after precipitation was 99·6 grms. water, and of the vessels, &c. 6·8 grms.

I. Air 16°·6. Increment found 1°·96, corrected 1°·82.
II. Air 15°·2. Increment found 1°·93, corrected 1°·80.
III. Air 15°·2. Increment found 1°·92, corrected 1°·79.

I.	II.	III.	Mean.
428°	424°	421°	424°.

In another set of experiments a weaker solution was employed, which gave, on analysis, 0·571 grm. chloride of silver. The thermal value of the whole was now 106·7 grms.

I. Air 18°·2. Increment found 1°·80, corrected 1°·72.
II. Air 18°·0. Increment found 1°·80, corrected 1°·72.
III. Air 18°·1. Increment found 1°·81, corrected 1°·73.

I.	II.	III.	Mean.
427°	427°	429°	428°.

Acetate of Silver and Zinc.—100 grms. of the solution gave 0·600 grm. chloride of silver.

I. Air 16°·4. Increment found 1°·94, corrected 1°·82.
II. Air 15°·2. Increment found 1°·93, corrected 1°·81.
III. Air 15°·0. Increment found 1°·93, corrected 1°·80.

I.	II.	III.	Mean.
428°	426°	424°	426°.

This result is identical with the mean number deduced from the experiments with the sulphate. We have, therefore, for the heat evolved during the precipitation of

	C.		F.
1 grm. silver by zinc	426°	or	767°
1 equiv. silver by zinc	5747°	or	10345°.

It has been already remarked that the nitrates do not in general yield the same thermal results as other salts, in consequence of the tendency of the nitric acid to decompose, which introduces other chemical actions in addition to the metallic precipitation. Approximate results were, however, obtained with the nitrate of silver. 100 grms. of a solution of nitrate of silver, containing 0·711 grm. of the dry salt, gave in three trials 2°·12, 2°·13 and 2°·11, as increments, without any correction being applied. On continuing the agitation for periods of two minutes, and observing the temperature at the end of each period, the increments followed a singular law, being at first very small, and afterwards suddenly increasing. The march of the thermometer will be readily understood by inspecting the following numbers, which give the temperatures observed at the end of every two minutes of agitation.

First increment . . .	2°·12	2°·13	2°·11
Second increment . .	0·03	0·06	0·07
Third increment. . .	0·03	0·02	0·03
Fourth increment . .	0·14	0·11	0·17
Fifth increment . . .	0·23	0·17	0·25
Sixth increment. . .	0·18	0·16	0·20
Seventh increment. .	0·13	0·15	0·23
Eighth increment . .	0·11	0·11	0·16

The sudden increase in the amount of the increment which took place after the agitation had continued for six minutes is very remarkable, and it occurred uniformly in all the experiments. It plainly shows that some new voltaic or chemical action occurred at that time, with the nature of which I am not precisely acquainted. No analogous irregularity occurred with any of the other salts of silver which were examined.

Salts of Silver with Copper.

All the solutions of silver in these experiments contained the same quantity of silver. 100 grms. of each, precipitated by hydrochloric acid, gave 0·600 grm. chloride of silver. The copper had been reduced from the oxide by hydrogen, and about 2 grms. were taken in each experiment.

Sulphate of Silver and Copper.

I. Air 13°·9. Increment found 0°·73, corrected 0°·68.
II. Air 13°·7. Increment found 0°·71, corrected 0°·69.
III. Air 12°·7. Increment found 0°·76, corrected 0°·71.

I.	II.	III.	Mean.
159°	161°	166°	162°.

Acetate of Silver and Copper.

I. Air 13°·4. Increment found 0°·68, corrected 0°·67.
II. Air 12°·8. Increment found 0°·71, corrected 0°·66.
III. Air 12°·8. Increment found 0°·71, corrected 0°·66.

I.	II.	III.	Mean.
157°	155°	155°	156°.

Nitrate of Silver and Copper.

I. Air 13°·8. Increment found 0°·74, corrected 0°·70.
II. Air 13°·0. Increment found 0°·75, corrected 0°·70.
III. Air 12°·8. Increment found 0°·77, corrected 0°·72.

I.	II.	III.	Mean.
164°	169°	169°	166°.

In this case the numbers obtained with the nitrate agree with the others. Hence we have for the heat developed during the precipitation of

	C.	F.
1 grm. silver by copper	161° or	290°
1 equiv. silver by copper . . .	2176° or	3917°.

Salts of Lead with Zinc.

Acetate of Lead and Zinc.—The precipitation of lead from its solutions by metallic zinc is difficult to complete in a short space of time. A large excess of zinc (8 grms.) was required and four minutes of agitation. The results on this account are only approximations. The solution weighed 130 grms. and contained 1·305 grm. oxide of lead. The thermal equivalent of the whole was 136·0 grms. water.

I. Air 17°·8. Increment found 1°·63, corrected 1°·60.
II. Air 16°·5. Increment found 1°·68, corrected 1°·65.

I.	II.	Mean.
180°	185°	182·5°.

Formiate of Lead and Zinc.—0·433 grm. of the salt employed gave 0·324 grm. oxide of lead. 100 grms. of the solution contained 1·35 grm. of the formiate.

I. Air 12°·7. Increment found 1°·74, corrected 1°·61.

I.
181°·5

Hence we have for the heat evolved during the precipitation of

	C.	F.
1 grm. lead by zinc	182° or	327°
1 equiv. lead by zinc	2357° or	4243°.

Salts of Mercury with Zinc.

Chloride of Mercury and Zinc.—This is the only salt of mercury which was examined. The result however is sufficient to determine the thermal position of mercury among the metals. 100 grms. of a solution containing 1·240 grm. chloride of mercury were taken. The thermal value of the whole was 106·4 grms. water. In this case no further development of heat occurred after the precipitation was completed, nor was there any disengagement of hydrogen gas. The excess of zinc, in fact, became amalgamated, which effectually prevented both oxidation and voltaic action.

I. Air 16°·6. Increment found 2°·86, corrected 2°·86.
II. Air 16°·5. Increment found 2°·85, corrected 2°·88.
III. Air 16°·4. Increment found 2°·88, corrected 2°·88.

I.	II.	III.	Mean.
332°	334°	334°	333°.

We have, therefore, for the heat disengaged during the displacement of

	C.		F.
1 grm. mercury by zinc	333°	or	600°
1 equiv. mercury by zinc . . .	4166°	or	7499°.

Salts of Platinum with Zinc.

Soda-Chloride of Platinum and Zinc.—Of the salts of platinum, the double chloride of sodium and platinum is best adapted for this investigation. The complete precipitation of platinum by zinc is more difficult, and requires a longer time than that of any of the metals hitherto examined. This renders the corrections larger and the final results less exact. To ascertain the composition of the salt employed, 0·692 grm. carefully dried were precipitated by muriate of ammonia and the precipitate ignited; 0·298 grm. metallic platinum were obtained. The solution employed in each experiment weighed 100 grms., and contained 0·721 grm. of the dry salt.

I. Air 15°·4. Increment found 2°·94, corrected 2°·64.

II. Air 16°·2. Increment found 2°·93, corrected 2°·62.

I.	II.	Mean.
902°	896°	899°

Hence we have for the heat disengaged during the precipitation of

	C.		F.
1 grm. platinum by zinc . . .	899°	or	1618°
1 equiv. platinum by zinc . . .	11085°	or	19953°.

It would have been very interesting to have extended this investigation to other cases of metallic substitution, so as to have been able to present in one complete view the quantities of heat developed in all such cases; but the facility with which some metals are oxidized, and the difficulty of precipitating others in a short space of time from their solutions, prevented me from further extending the foregoing results. For convenience, I have collected in the following table the numerical quantities already obtained:—

	Of precipitated metal. 1 grm.	1 equiv.
Salts of copper and zinc	868°	3435°
Salts of copper and iron	592	2342
Salts of copper and lead	268	1061
Salts of silver and zinc	426	5747
Salts of silver and copper	161	2176
Salts of lead and zinc	182	2357
Salts of mercury and zinc	333	4166
Salts of platinum and zinc.	899	11085

To prevent mistake, it may be right here to state that the numbers in the first column express the degrees centigrade through which one gramme of water would

be heated by the precipitation of one gramme of the metal from a solution of any of its salts; and that those in the second column express the degrees through which the same weight of water would be raised by the precipitation of an equivalent (oxygen = 1) of the same metal.

If three metals, A, B, C, be so related that A is capable of displacing B and C from their combinations, and also B capable of displacing C; then the heat developed in the substitution of A for C will be equal to that developed in the substitution of A for B, added to that developed in the substitution of B for C; and a similar rule may be applied to any number of metals similarly related. Several illustrations of this principle are afforded by the preceding table. Thus

1 equiv. lead displaced by zinc	2357°
1 equiv. copper by lead.	1061
1 equiv. copper by zinc	3418

The experimental result for the last case is 3435°, which in such inquiries may be considered to be identical with the theoretical number. Again,

1 equiv. copper by zinc	3435°
1 equiv. silver by copper	2176
1 equiv. silver by zinc	5611

The experimental result is 5747°, which differs $\frac{1}{43}$rd part from theory. This difference only corresponds to an error of about 0°·04 among the three experiments, and the agreement may therefore be considered satisfactory. By applying the same principle, we can easily deduce the amount of heat developed in other cases of metallic substitution. Thus an equivalent of mercury displaced by zinc should give 731 units of heat, of platinum displaced by copper 7650 units, by mercury 6919 units, &c.

Engraved by J. & C. Walker

VI. *Report of Observations made upon the Tides in the Irish Sea, and upon the great similarity of Tidal Phenomena of the Irish and English Channels, and the importance of extending the Experiments round the Land's End and up the English Channel. Embodied in a letter to the Hydrographer. By Captain* F. W. Beechey, *R.N., F.R.S. Communicated by* G. B. Airy, *Esq., F.R.S. &c., Astronomer Royal.*

Received March 22,—Read March 29, 1848.

Sir, London, February 19th, 1848.

It is with much pleasure I lay before you the result of observations which have been made upon the tides in the Irish Sea, during the execution of the survey which has been entrusted to my charge.

Up to the period of these observations the set of the tides in the Irish Sea had been greatly misunderstood, owing to the association of the turn of the stream with the rise and fall of the water on the shore; and it was generally understood that as Holyhead was three hours later in its tides than places at the entrance of the channel, a vessel starting with the first of the flood would carry nine hours' tide in her favour in her run up channel and *vice versâ*. The stream in the Irish Channel hitherto misunderstood.

This was an error sufficiently great in itself, but it was liable to be increased by an entire ignorance as *to the time* when a vessel would take any particular tide; for the times of high water by the shore were very various on different sides of the channel.

The present inquiry, I am happy to inform you, has set these errors and doubts at rest. The observations have shown that, notwithstanding the variety of times of high water throughout the channel, the turn of the stream is *simultaneous*; that the northern and southern streams in both channels commence and end in all parts (practically speaking) *at the same time,* and that time happens to correspond with the time of *high and low water* on the shore at *Morecombe Bay*; an estuary rendered remarkable as being the point where the opposite tides coming round the extremities of Ireland, finally meet. So that it is necessary only to know the times of high and low water at Morecombe Bay to determine the hour when the stream of either tide will commence or terminate; a fact which will, I am sure, be fully appreciated by every person navigating the Irish Channel at night, or in thick weather. Turn of the stream simultaneous throughout north and south channels. Meeting of tides.

The chart of curves or lines of *direction of the stream,* Plate II., which accompanies this report, will show at once the effect of the tide upon a vessel wherever she may be placed in the channel, and especially direct her where, with a beating wind,

she will be benefited by standing in shore or otherwise; and will warn her of the danger of drawing near the shores of Cardigan or Caernarvon Bays, with particular tides and scant winds, and so likewise of the danger of standing close to the banks skirting the Irish coast in light winds.

But it is not to the navigator alone that these observations will, I hope, be found useful: they will, I think, be interesting to men of science. Taken in connection with the very valuable series of observations which were carried round Ireland by the Ordnance at the suggestion of Professor AIRY, we are made acquainted with several curious facts: first, that whilst it is high water at one end of the channel, it is low water at the other; that the same stream makes both high and low water at the same time; that there are two spots in the channel, in one of which the stream runs with considerable velocity without the water either rising or falling, and in the other, that the water rises and falls from sixteen to twenty feet without having any visible horizontal motion of its surface; and that during the first half of the flowing, and last half of the ebbing, tide-wave, the stream in the south channel runs in a contrary direction to the wave, and goes up an ascent of about one foot in $4\frac{1}{2}$ miles. (See Plate IV.)

Peculiar feature of the tides.

To the lines of direction of the stream I have added the rate of the tide at its greatest velocity on the day of syzygy, and have reduced all to the same standard.

I shall now proceed to describe the general course of the streams throughout the channel; point out the situations in which the meeting of the tides occurs; and offer such remarks on the course of the stream and upon the tidal phenomena of this sea, as will, I conceive, benefit the navigator, and be interesting to science.

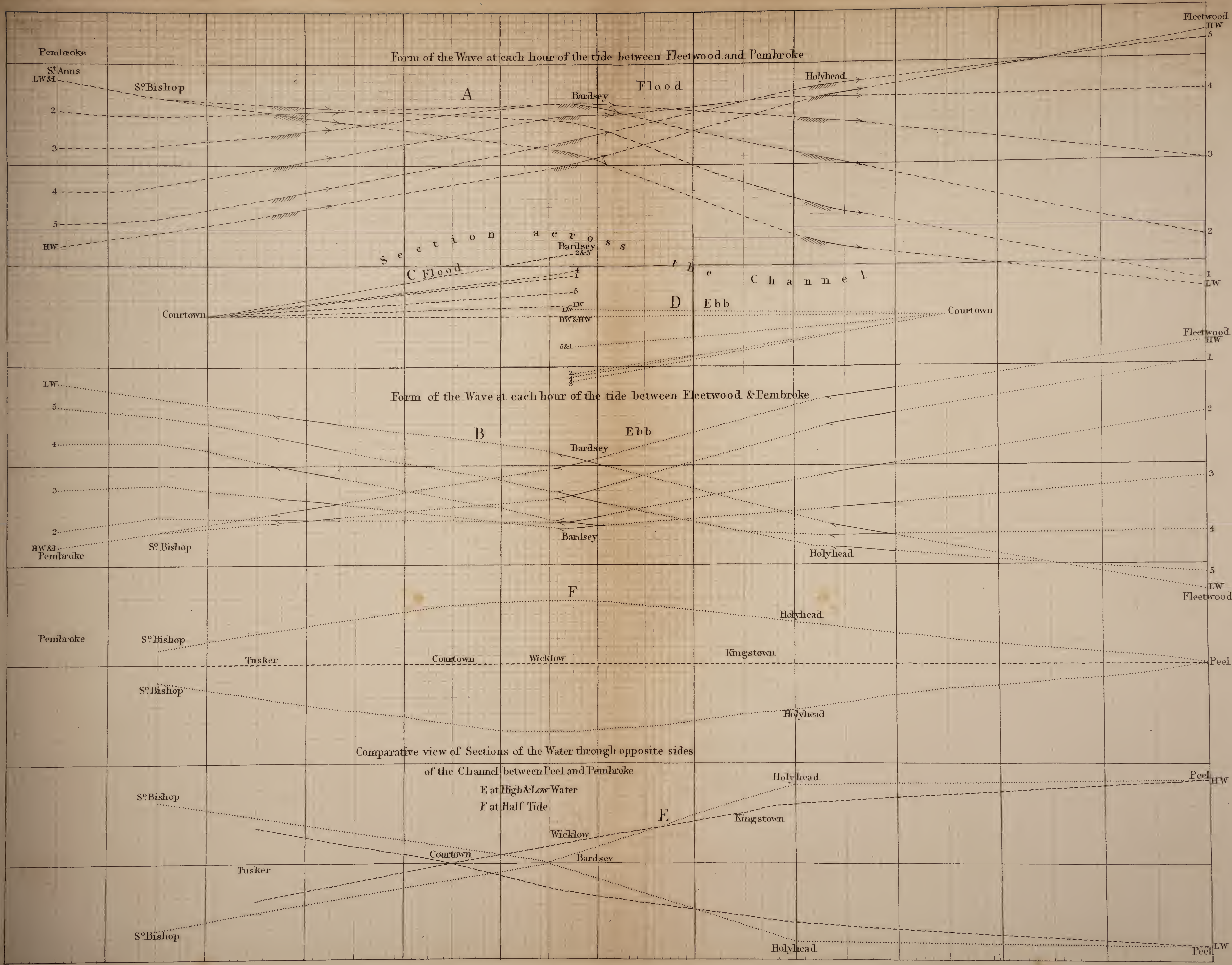
An inspection of the chart (Plate II.) will show that the tide enters the Irish Sea by two channels; of which Carnsore Point and Pembroke are the limits of the southern one, and Rathlin and the Mull of Kintire the boundaries of the northern.

The stream in the southern channel (as before stated) has been ascertained to move *simultaneously in one vast current throughout*; running six hours nearly each way, at an average rate of from two to three knots per hour at the height of the springs, increasing to four knots and upwards near the banks and at the pitch of the headlands; its *times of slack water* corresponding sufficiently near for all practical purposes, with *the times of high and low water for the day at Morecombe Bay*, or more correctly *at Fleetwood*, which is twelve minutes earlier than Liverpool.

Times of slack water throughout the channel.

Course of the central portion of the stream in the south channel.

The *central portion* of the stream of *flood* or *ingoing stream*, runs nearly in a line from a point midway between the Tuskar and the Bishops, to one six miles due west of Holyhead; beyond which it begins to expand eastward and westward, but its main body preserves its direction straight forward for the Calf of Man, which it passes to the eastward with increased velocity as far as Langness Point, and then at a more moderate rate on towards Maughold Head. Here it is arrested by the flood or southern stream from the north channel coming round the Point of Ayre, and


Form of the Wave at each hour of the tide between Fleetwood and Pembroke
A
Flood
Pembroke
St. Anns
LW&1
S^t. Bishop
Bardsey
Holyhead
Fleetwood
HW
Section across the Channel
C Flood
D Ebb
Courtown
Bardsey
Courtown
Form of the Wave at each hour of the tide between Fleetwood & Pembroke
B
Ebb
Bardsey
Bardsey
Holyhead
HW&1
Pembroke
S^t. Bishop
LW
Fleetwood
F
Pembroke
S^t. Bishop
Tusker
Courtown
Wicklow
Kingstown
Holyhead
Peel
S^t. Bishop
Holyhead
Comparative view of Sections of the Water through opposite sides
of the Channel between Peel and Pembroke
E at High & Low Water
F at Half Tide
E
S^t. Bishop
Wicklow
Courtown
Bardsey
Tusker
Holyhead
Kingstown
Peel
HW
S^t. Bishop
Holyhead
Peel
LW

is first swayed round to the eastward by it, and then goes on with it at an easy rate direct for Morecombe Bay.

The *outer portions* of the stream are necessarily deflected from the course of the great body of the water by the impediments of banks on the Irish side of the Channel, and by the tortuous form of the coast on the Welsh. The eastern portion passing Linney Head rushes with great rapidity between the Smalls, Grassholm, and Milford Haven, towards the Bishops, which it passes at a rate of between four and five knots; sets sharply round those rocks in an E.N.E. direction, right over the Bass bank, and into Cardigan Bay; makes the circuit of that bay; and sets out again towards Bardsey at the other extremity of it; then sweeping to the N. by W. past the island and through the sound, it gradually takes the course of the shore, round Caernarvon Bay, filling the Menai Strait as far as Bangor; but the stream still continuing outside towards the South Stack, which it rounds, setting towards the Skerries at a rate of upwards of four knots; and finally, turns sharply round those rocks for Liverpool and Morecombe Bay; completing in its way the high water in the Menai, and filling the Dee, Mersey and Ribble.

Course of the eastern portion of the stream in the south channel.

The *western* portion of the stream, after passing the Saltee, runs nearly in the direction of the Tuskar, sets sharply round it, and then takes a N.E. $\frac{1}{2}$ N. direction, setting fair along the coast, but over the banks skirting the shore, so that vessels tacking near the edge of the sands, have been carried upon them and lost, especially upon the Arklow and Codlin. Abreast of the Arklow is situated that remarkable spot in the Irish Channel, which I have before mentioned as a place where *the tide neither rises nor falls.* The stream notwithstanding sweeps past it at the rate of four knots at the springs, and reaches the parallel of Wicklow Head. Here it encounters an extensive bank recently known; and whilst the outer portion takes the circuit of the bank, the inner sweeps over it, occasioning an overfall and strong rippling all round the edge, by which the bank may generally be discovered; beyond this point the streams unite and flow on towards Howth and Lambay, growing gradually weaker as they proceed, until they ultimately expend themselves in a large space of still water situated between the Isle of Man and Carlingford.

Course of the western portion of the stream in the south channel.

Place of no rise or node of the tide-wave.

Here we have not been able to detect any tide; and here occurs that remarkable phenomenon before mentioned, of the water rising and falling without having any perceptible stream. This space of still water is marked by a bottom of blue mud. Such is the course of the flowing water in the *southern channel.*

Place of no stream.

In the north channel the stream enters between the Mull of Kintire and Rathlin *simultaneously* with that passing the Tuskar into the southern channel, but flows in the contrary direction. It runs at the rate of three knots at the springs, increasing to five knots near the Mull, and to four near Torr Head on the opposite side of the channel. The eastern branch of this stream turns round the Mull towards Ailsa and the Clyde, a portion passing round Sanda up Kilbrannin Sound and Loch Fyne.

Stream in the northern channel.

The *main body* sweeps to S. by E., taking nearly the general direction of the

Course of the main body of the stream.

Remarkable ditch.

channel, but pressing more heavily on the Wigtownshire coast; off which it has scooped out a remarkable ditch, upwards of twenty miles long by about a mile only in width, in which the depth is from 400 to 600 feet greater than that of the general level of the bottom about it. Near the Mull of Galloway the stream increases in velocity to five knots, the eastern portion turns sharply round the promontory towards the Solway, and splits off St. Bee's Head; one portion running up the Solway, and the other towards Morecombe Bay.

Eastern portion of the stream.

Central portion of the stream.

The *central portion* from a midway between the Mull of Galloway and the Copeland Islands, presses on towards the northern half of the Isle of Man, and while one portion of it flows toward the Point of Ayre, the other makes for Contrary Head, and is there turned back at a right angle nearly to its early course. Passing Jurby it reunites with the other portion of the stream, and they jointly rush with a rapidity of from four to five knots round the Point of Ayre, and directly across all the banks lying off there, and catching up the stream from the south channel off Maughold Head, they hurry on together towards that great point of union Morecombe Bay. This bay, the grand receptacle of the streams from both channels, is notorious for its huge banks of sand heaped up in terrible array against the mariner unacquainted with its locality, and also remarkable for a deep channel scoured out by the stream, and known as the Lune Deep, which to the wary navigator is the great hidden beacon of his safety, and serves him, alike in fog or in sunshine, as a guide to his position, and to a harbour of safety in case of need.

Western portion of the stream.

We have now only to speak of the *western limit* of the stream, which we left off Torr Head running at a rate of four knots off the pitch of the point. Hence it strikes directly towards the Maidens, boiling over the Highlander and Russell rocks, and other reefs in the vicinity of that dangerous group; and takes the direction of the coast again from Muck Island to Black Head, at the entrance of the Lough of Belfast, which it fills.

The portion of the stream which sets up the Lough, splits again off Grey Point; one portion flowing up towards Garmoyle, while the other bends back along the shore of Bangor, Grimsport and Orlock, and blends with the general stream which has come on from the Maidens and Blackhead, and passes with it through the sounds of the Copeland Islands. Hence it proceeds along the coast, brushes the South rock, and runs on towards St. John's Point; off which, the stream, like that coming from the southward, expends itself in a large space of still water, which remains undisturbed although pressed upon by streams from various quarters.

Such is a general description of the streams in both channels which attend the *flowing of the water*, or which, for the purpose of distinction, we may designate the *Ingoing stream.*

Ebbing or outgoing stream.

The ebbing or *Outgoing* streams do not materially differ from the reverse of these, except that in the southern channel they press rather more over towards the Irish coast.

Engraved by J.&C. Walker.

These observations do not, however, extend beyond the point where the channels begin to open out, that is beyond a line joining Rathlin and the Mull of Kintire on the north, and the Saltee and Pembroke on the south. Outside these limits, the waters diverge right and left; that on the north joining the stream from Jura, and turning sharp round Rathlin; that on the south, speaking now of the outgoing stream, sweeps past Pembroke into the Bristol Channel on one side, and on the other rounds the Tuskar and passes on to Waterford.

I have now, Sir, endeavoured to convey to you a general idea of the course of the streams throughout the Irish Sea both in the centre and at the sides of the channel, as you will find them represented in the annexed chart, Plate II.; but besides these there are (as usual) at all the points and headlands, when abrupt, counter streams or eddies beginning at about two hours after the offing stream, increasing with the strength of the tide, and occasioning races and overfalls at the places marked on the chart. In the direction of the offing stream there is as little variation of the current at the different hours of tide as will be met with in any sea of similar extent, and indeed it is only with the slackening of the tide that the variations occur; so that, by a due attention to the lines I have given, the navigation may be as certainly conducted here as in any channel with which we are acquainted*.

Half-tide eddies.

During the time these observations on the stream were in progress, others were made upon the *rise* and *fall of the water* at several stations in the channel, and wherever practicable at places in the offing. By combining these observations with the range of tide on the coast of Ireland, published in Professor Whewell's admirable paper on the Tides in the Philosophical Transactions for 1836, Part II., and with observations made by Captains Robinson, Denham, Frazer, Sherringham, Williams, &c., I have constructed a chart of lines of equal range of tide, Plate III., in order that the seaman may ascertain by a simple inspection of the chart, wherever he may be placed in the channel, the amount of spring range to which he has to adapt his soundings.

Chart of lines of equal range of tide.

In this chart the lines denote the range of tide at the places over which they pass, on a day when a spring tide at Liverpool rises thirty feet. In the channel between Holyhead and Tuskar, where it was impossible to get observations of this kind in the offing, the lines have been proportioned according to the known difference of range between the places.

In the sea to the northward of Holyhead the numbers have been compiled from observations made on calm days, at anchor, in connection with those made upon the shore around the places of observation.

The method of obtaining the range at sea was to moor a heavy weight with a small well-stretched line attached to it marked to feet, and to preserve as nearly as possible the same tension of line at the times when the depths were required, a loop was

Method of obtaining the range at sea.

* It is slack water from about forty minutes before to about forty minutes after high or low water at Morecombe Bay.

placed over the hook of a small hand steelyard, and the line brought tight until the index noted a given quantity, which was afterwards considered the standard; and by comparing the observations of several days obtained in this manner with the tide gauges on shore, a near agreement satisfied me that the results were to be depended upon. At the same time it is not pretended that the rise has been determined to any great accuracy, although I think it is within the foot, and quite near enough for the purposes of navigation for which the chart is intended.

Table for the reduction of soundings.

To make this chart useful for the reduction of soundings at intermediate times between high and low water, I have annexed a diagram, Plate VII., constructed from a vast number of observations, and have given directions for using it.

Unequal motion of the upper and lower half of the tide-wave, common to all the tides in Irish Sea and Bristol Channel.

I shall now notice some peculiarities in the tides which have fallen under our observation. In the construction of the Table, Plate VII., for the use of the seaman, in the Irish Sea it was found that the place of the *water at the half-tide interval* did not correspond with that of a mark at *the half range of the wave*, but that it was always below it, showing that the upper half of the wave rose and fell more rapidly than the lower. It was also found that the curve of the Irish Sea tide did not correspond with that of the Bristol Channel tide; that neither followed the law of the sines to corresponding arcs of tidal intervals, upon the falling water especially, as may be seen in Plate VIII., constructed for the purpose of showing the place of the water at corresponding intervals of a falling tide at places distantly situated.

It was also seen that, owing to the unsymmetrical form of the curves, it would be necessary to have tables for the reduction of soundings adapted to rising and falling tides respectively. In Plate VI. I have exhibited the discordance of the curves of the rising and falling wave in the Bristol Channel, where it will be seen that at one stage of the tide, the difference of similar tidal intervals amounts to about four feet.

I may here mention that the Cumberland Basin Gauge (Mr. Bunt's), which has been frequently referred to in the Philosophical Transactions, corresponds very nearly with the Portishead Gauge with which the observations in Plate VI. were made.

Apparent mean elevation of the water.

In connection with the range of tide is that of the apparent mean elevation of the water. All the observations confirm the remark of Professor Airy*, viz. that this mean level is higher at the springs than at the neaps. The mean place of the water however for an entire lunation, during the summer months at least, is tolerably constant, and affords a fair standard to which the reductions used in our nautical surveys may be referred in the event of the gauge being removed by which the observations were made.

As an example, I annex the result of observations made at Holyhead during nearly four entire years.

* Philosophical Transactions, 1845, Part I. p. 31.

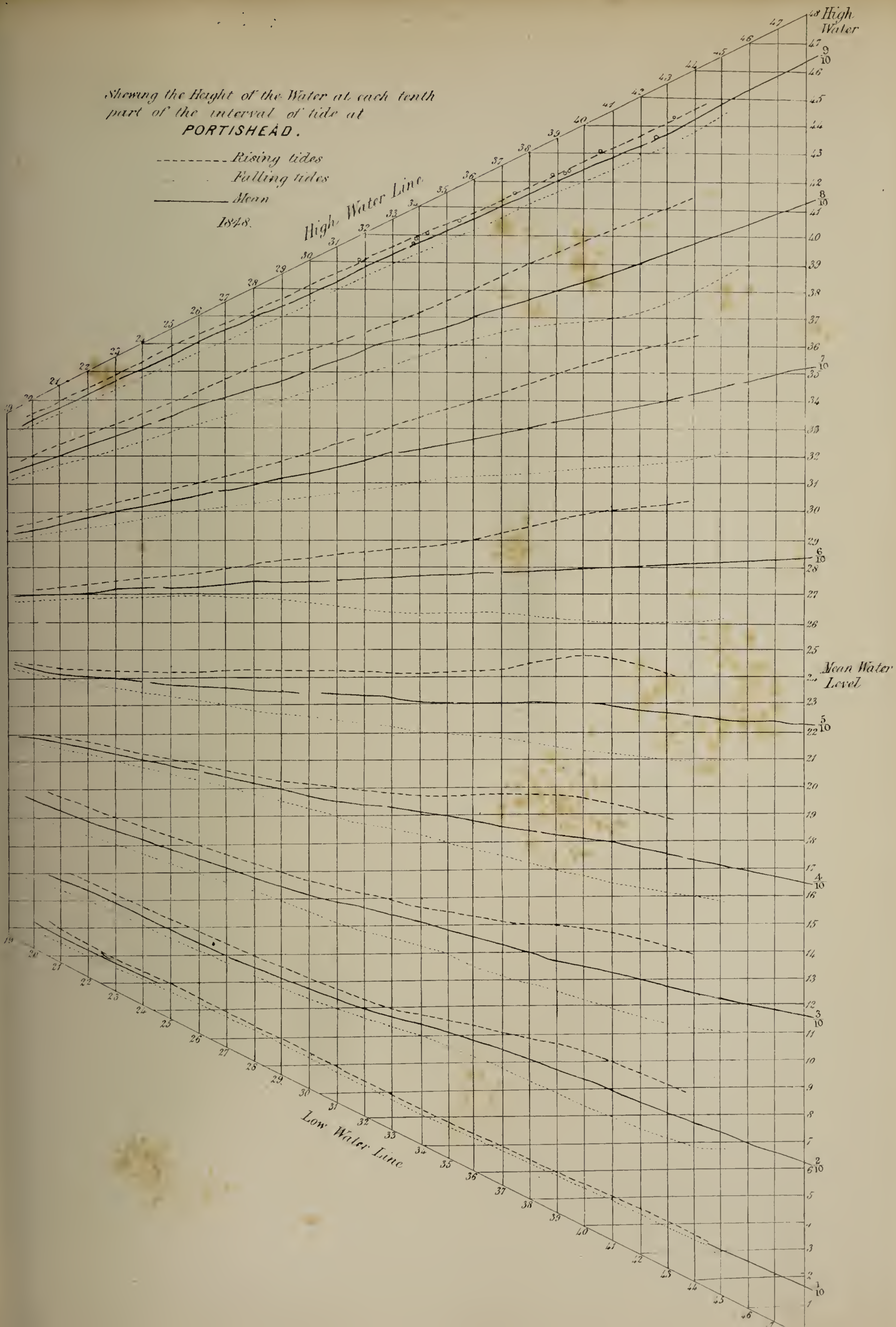

Showing the Height of the Water at each tenth part of the interval of tide at
PORTISHEAD.
Rising tides
Falling tides
Mean
1848.
High Water Line
Low Water Line
High Water
Mean Water Level

Table C.

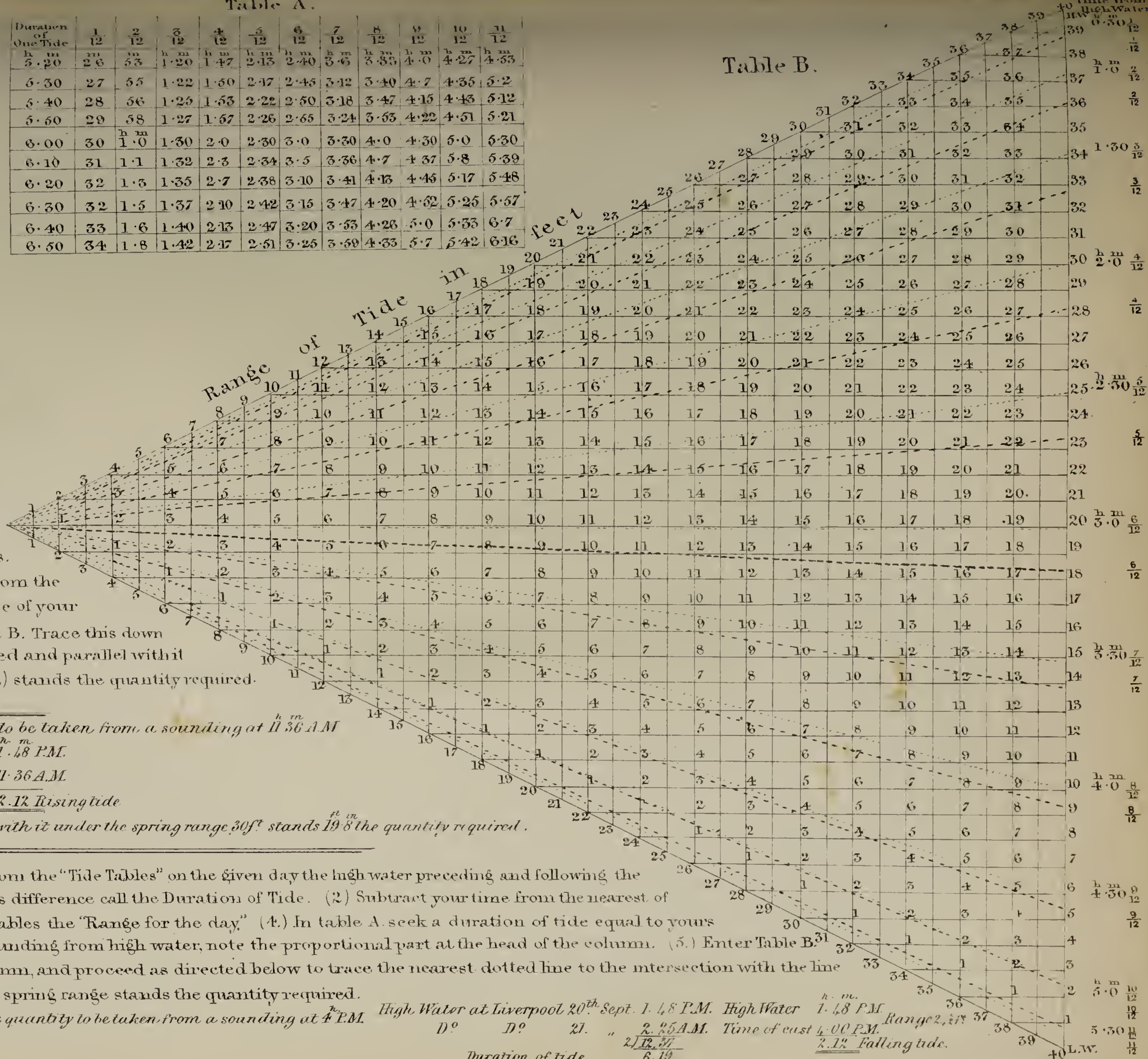
Names of Places.	Range at Springs	Names of Places.	Range at Springs
	feet		feet
Liverpool	30	Killmore	11
Holyhead	20	Loughbar	8
Fleetwood	28	Dunmore	13
Pembroke	22	Ramsey I.M.	22
Kingstown	13	King Will.m Bank	23½
Belfast	11½	Ravenglass	25
Bray	10	Whitehaven	25
Wicklow	8	Donaghadee	15½
Arklow	3	Bardsey I.d	16
Cahore	3	Tusker	9
Blackwater	6	Calf	15¾
Wexford	5½	Peel I.M.	16
Carnsore	8		

For other Places in the Irish Sea see the Tidal Chart Curves.

Table A.

Duration of One Tide	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12
h m 5·20	m 26	m 53	h m 1·20	h m 1·47	h m 2·13	h m 2·40	h m 3·6	h m 3·33	h m 4·0	h m 4·27	h m 4·53
5·30	27	55	1·22	1·50	2·17	2·45	3·12	3·40	4·7	4·35	5·2
5·40	28	56	1·25	1·53	2·22	2·50	3·18	3·47	4·15	4·43	5·12
5·50	29	58	1·27	1·57	2·26	2·55	3·24	3·53	4·22	4·51	5·21
6·00	30	h m 1·0	1·30	2·0	2·30	3·0	3·30	4·0	4·30	5·0	5·30
6·10	31	1·1	1·32	2·3	2·34	3·5	3·36	4·7	4·37	5·8	5·39
6·20	32	1·3	1·35	2·7	2·38	3·10	3·41	4·13	4·45	5·17	5·48
6·30	32	1·5	1·37	2·10	2·42	3·15	3·47	4·20	4·52	5·25	5·57
6·40	33	1·6	1·40	2·13	2·47	3·20	3·53	4·26	5·0	5·33	6·7
6·50	34	1·8	1·42	2·17	2·51	3·25	3·59	4·33	5·7	5·42	6·16

Table B.



To reduce Soundings roughly to low water springs.

Find the range for the day in the "tide tables".* From the time of high water at your station take the time of your sounding and seek a corresponding time in table B. Trace this down to the line of range for the day above mentioned and parallel with it under the range at springs for the place (table C.) stands the quantity required.

EXAMPLE.

Sept. 20th 1845 Off Liverpool. Require the quantity to be taken from a sounding at 11 h 36 m A.M.

Range for the 20th Sept 24½ ft — *High Water 1 h·48 m P.M.*
Time of Sounding 11·36 A.M.
difference 2 h.12 m Rising tide

In table B. under 24½ feet range and 2 h 12 m and parallel with it under the spring range 30 ft stands 19 ft 8 in the quantity required.

To reduce the Soundings accurately. (1.) Take from the "Tide Tables" on the given day the high water preceding and following the time of your sounding and subtract them, half this difference call the Duration of Tide. (2) Subtract your time from the nearest of these times of high water. (3) Find from the tide tables the "Range for the day." (4.) In table A. seek a duration of tide equal to yours above found, and opposite it the time of your sounding from high water, note the proportional part at the head of the column. (5.) Enter Table B. with this proportional part in the right hand column, and proceed as directed below to trace the nearest dotted line to the intersection with the line of range for the day and parallel with it under the spring range stands the quantity required.

EXAMPLE Sept. 20th 1845. off Liverpool, Required the quantity to be taken from a sounding at 4 h P.M.

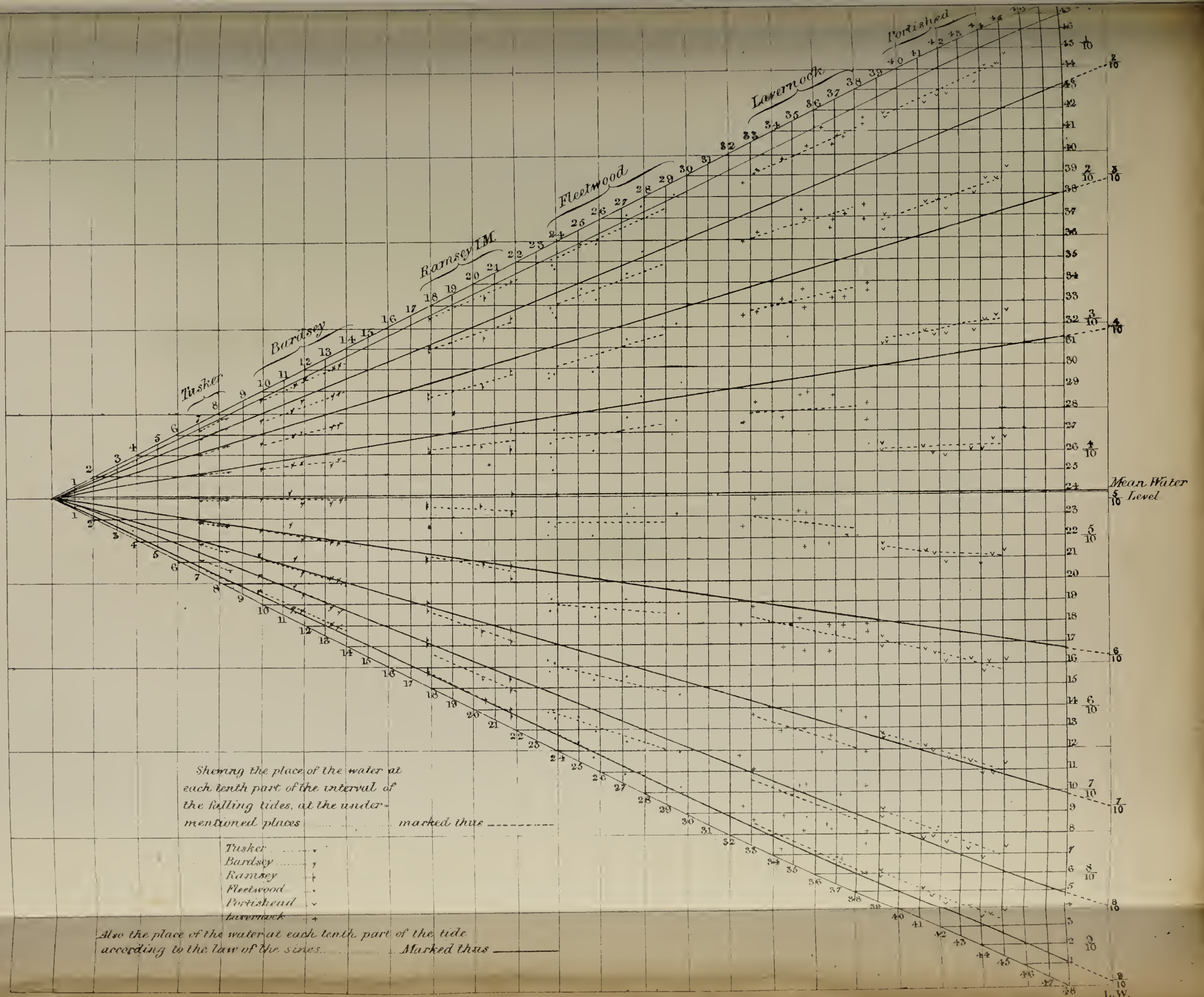
High Water at Liverpool 20th Sept. 1. 48 P.M. — *High Water 1. 48 P.M.*
D° D° 21. " 2. 25 A.M. — *Time of cast 4. 00 P.M.* — *Range 24½ ft*
2) 12. 37 — *2. 12 Falling tide.*
Duration of tide 6. 19

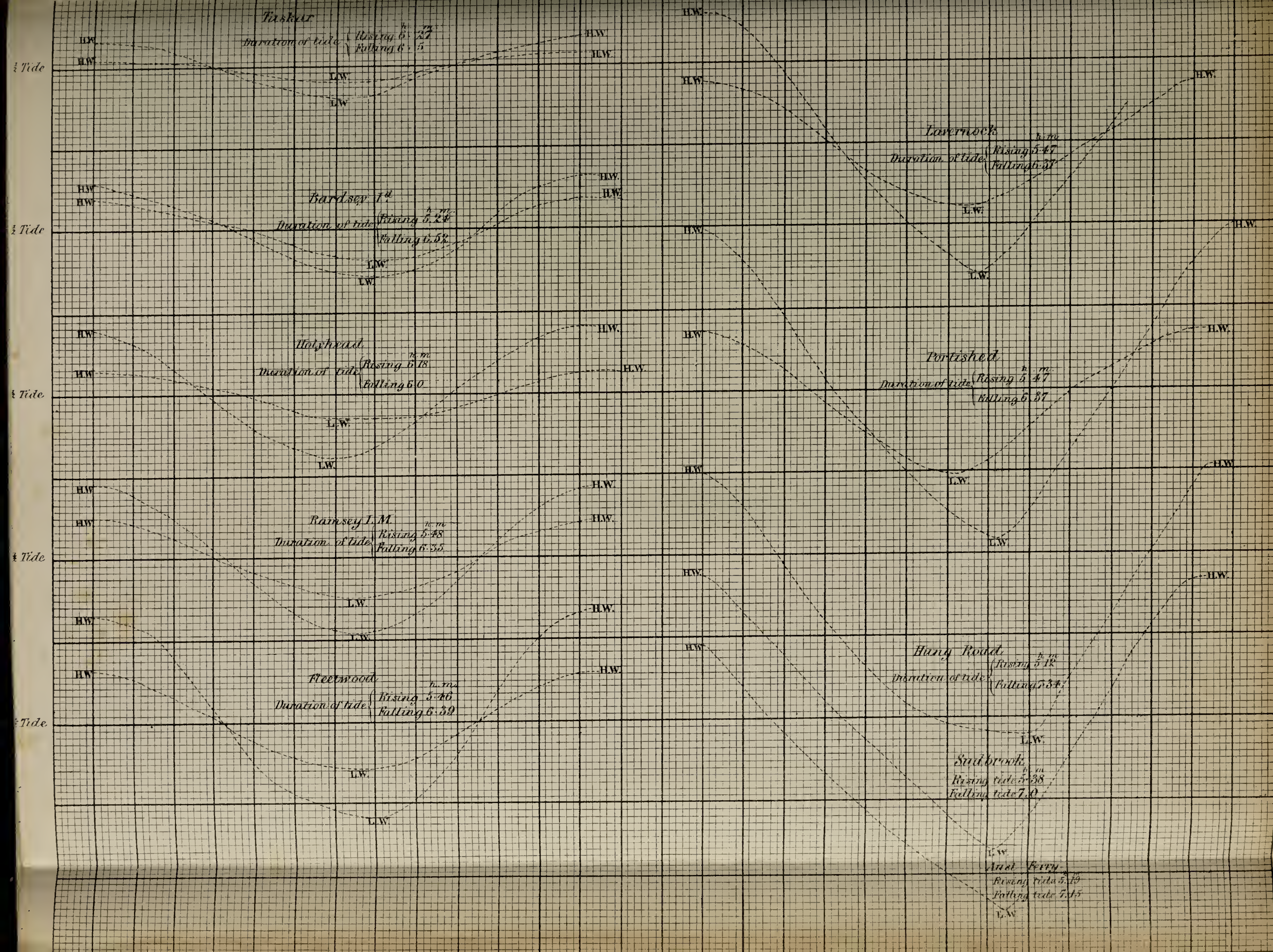
In table A. opposite 6 h·19 m 2 h.12 m falls about half way between 4/12 and 5/12 at the head of the column. In table B. under 24½ feet range of tide and half way between 4/12 and 5/12 and parallel with it under the spring range (from table C.) 30 feet stands 18 ft·9 in the quantity to be taken from the sounding.

* Knowing the range for the day at Liverpool, the range for the day at any other place in the Irish Sea may be obtained approximately by the following rule. As 30 feet, is to the range marked upon the Tide Chart, so is the range for the day at Liverpool, to the range for the day at the place of observation.

EXAMPLE. *Required the range for the day at Bardsey on the 8th September 1847. as 30 : 9 :: 22 ft*
30) 198 / 6·7 range for the day at Bardsey.

N.B. If the tide be falling use the strong dotted lines with the proportional part
If the tide be rising use the light lines with the prop.l part


Mean Water Level
L.W.
Tusker
Bardsey
Ramsey I.M.
Fleetwood
Lavernock
Portishead
Shewing the place of the water at each tenth part of the interval of the falling tides, at the undermentioned places marked thus
Tusker
Bardsey
Ramsey
Fleetwood
Portishead
Lavernock
Also the place of the water at each tenth part of the tide according to the law of the sines. Marked thus


Taskar
Duration of tide { Rising 6 . 27 / Falling 6 . 5
Bardsey I.d
Duration of tide { Rising 5.24 / Falling 6.52
Holyhead
Duration of tide { Rising 6.18 / Falling 6.0
Ramsey I. M.
Duration of tide { Rising 5.48 / Falling 6.35
Fleetwood
Duration of tide { Rising 5.46 / Falling 6.39
Lavernock
Duration of tide { Rising 5.47 / Falling 6.37
Portished
Duration of tide { Rising 5.47 / Falling 6.37
Hung Road
Duration of tide { Rising 5.12 / Falling 7.34
Sudbrook
Rising tide 5.38
Falling tide 7.0
Aust Ferry
Rising tide 5.19
Falling tide 7.15
H.W.
L.W.
h. m.
½ Tide

At Holyhead.

Apparent mean place of the water.

Month.	1838.		1839.		1846.		1847.		Mean of months.	
	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.
January ...	11	$3\frac{1}{4}$	10	6	......		10	7	10	$9\frac{1}{3}$
February ...	10	$6\frac{1}{2}$	10	3	......		9	10	10	$2\frac{1}{3}$
March	10	1	10	2	......		10	4	10	$2\frac{1}{3}$
April	10	1	9	10	......		9	11	9	$11\frac{1}{3}$
May	10	2	9	9	10	$0\frac{1}{2}$	9	$11\frac{3}{4}$	9	$11\frac{3}{4}$
June	......		10	1	10	4	9	$11\frac{3}{4}$	10	$1\frac{1}{2}$
July	10	1	10	$1\frac{1}{2}$	10	$1\frac{3}{4}$	9	$10\frac{3}{4}$	10	$0\frac{3}{4}$
August	10	6	9	10	10	1	9	$10\frac{3}{4}$	10	$0\frac{3}{4}$
September .	10	4	10	7	11	1	10	$3\frac{1}{4}$	10	$6\frac{3}{4}$
October ...	10	$6\frac{3}{4}$	10	$3\frac{1}{3}$	10	8	10	10	10	7
November..	10	7	10	$2\frac{1}{2}$	10	$7\frac{3}{4}$	10	$9\frac{3}{4}$	10	$6\frac{3}{4}$
December..	10	7	11	2	10	6	10	11	10	$9\frac{1}{2}$
Mean of the year...	10	5	10	$2\frac{3}{4}$	10	$5\frac{1}{2}$	10	$3\frac{1}{4}$	10	$3\frac{5}{6}$

Summer months. (April to August)

All the tides of the Irish Sea partake of the nature of river tides in having their ebb longer than their flood, except those of Tuskar and Holyhead, which are the reverse. The respective intervals are given in the order in which the places occur.

Duration of rising and falling tides.

Duration of tide.	Rising. h	m	Falling. h	m
Tuskar	6	27	6	08
Bardsey	5	24	6	52
Holyhead	6	18	6	00
Peel, Isle of Man .	6	00	6	15
Ramsay, Isle of Man	5	48	6	35
Fleetwood	5	46	6	39

All these are the mean of many observations.

Remarkable deviation at Holyhead.

The change at Holyhead is remarkable, and if we follow the durations up to Ramsay we shall see that Peel also, an intermediate station, is affected. The cause of this may possibly be connected with the effort of the water to maintain its level; for in projecting the curve of the wave on paper, this peculiarity, in connection with the very short flood of Bardsey, has the effect of reducing the curve from what it would assume, were Holyhead similarly influenced with other places.

Probable cause.

The next peculiarity to which I shall direct attention is that of the form of the surface of the opposite sides of the channel at the instant of half-tide, and at the times of high and low water respectively at the virtual head of the channel. In Plate IV. I have given these sections of the water for the purpose of comparison.

Form of the surface of the water on east and west sides of the channel at the same instants.

E is a section of the tide-wave at the *instant of high and low water* at Fleetwood; the broken line being the *western* side of the channel, or a line drawn through Courtown on to Peel, Isle of Man; the dotted line being through the *eastern* limit of the channel, or from Peel through Bardsey to Pembroke. F is a section of the water through the same localities at the *third hour* of the tide at Fleetwood; the broken line in this case also representing the *western* portion of the channel, the dotted the *eastern*.

In these sections it will be seen what an unusual depression there is of the water-line between Holyhead and Bardsey, and as this occurs at the time of slack water throughout the channel, we may perhaps connect with it the inversion of the interval of tide before mentioned at the former place.

Curves of tide-waves of various ranges.

I conclude these remarks with directing your attention to another diagram, Plate V., exhibiting the curves assumed by the tide-wave in various parts of the Irish and Bristol Channels. They are given without any corrections having been applied to them.

Importance of the observations being extended.

Having now laid before you the result of our observations upon the tides of the Irish Sea, I wish to call your attention to the great importance of these observations being extended. Independent of the vast advantage of having a correct knowledge of the set of the stream at the entrance of two channels through which the greater part of the trade of this great maritime nation is conducted, I think I shall be able to show that in another point of view also, considerable interest attaches to the subject.

Attempt to trace the course of the stream, and connect the tides of the Irish Sea with those of the English Channel.

That you may meet the question at once, I lay before you the result of an attempt I have made with the aid of the data we already possess, and especially those of Capt. Martin White, R.N., to trace the course of the stream from Pembroke to the Land's End; to connect the tides of the Irish Sea with those of the Bristol and English Channels, and finally with those of the offing; or, in other words, to reconcile and reduce to a system the anomalous and apparently contradictory observations of our naval surveyors in that portion of our coast, by combining them with the information which the recent attention to the tides has already furnished.

Difficulty of so doing.

The first part of this attempt presented but few difficulties as the streams were tolerably regular, but for the last our data, at first sight, seemed to bar all progress. At the mouth of the English Channel especially, opposition presented itself at every step; the observations projected upon the chart exhibited a frieze of arrows pointing in all directions "in happy discordance," and certainly quite useless and unintelligible to the navigator. Relying however upon the general accuracy of the observations, I was encouraged to proceed. After some consideration it seemed evident that the water was influenced by forces acting in opposition nearly to each other, and that there was a tide in the offing whose streams of ebb and flood did not correspond with those of the channels. By applying this idea first to the English Channel, I found the observations responded to it; and carrying it to the offing of the Irish Sea, and considering that channel as comprising the Bristol Channel within its limits, as the English Channel does the Gulf of St. Malo, I had the satisfaction of finding the correctness of the idea confirmed so far as the observations themselves extended. This offing stream appears to be of great extent, setting to the north and south along the coast of Biscay and the British Isles, running six hours nearly each way, and exercising an influence with more or less effect over all the waters of the channels and estuaries it passes in its progress, diverting their courses, and in some

Discrepancies reconciled by the application of an offing stream setting in opposition nearly to that of the channel stream.

Engraved by J & C Walker


TIDE CHART
of the
ENGLISH, IRISH AND BRISTOL CHANNELS
SHEWING THE SET OF THE STREAMS
in each
AT THE SAME INSTANTS OF TIME
of the INGOING tide
BY CAPTAIN F. W. BEECHEY R.N. F.R.S.
1848.
Course of the Stream at At 11 O'Clock ... Slack Water
... 1 ...
... 3 ...
... 5 ... Slack Water
On the Days of Syzygy and at Greenwich time.
IRELAND
WALES
ENGLAND
FRANCE
Very little rise of tide
Slack Water at 5 & 11 Greenwh. Time
Slack Water about 5 & 11 oclock Greenwich Time
Slack water about 5 oclock and 11 oclock Greenwich Time
Rise of Tide very small
Meridian of Greenwich
Shannon River
Limerick
Dingle Bay
Valentia Hr.
Kenmare River
Bantry Bay
Bantry
Cork
Cork Harbour
C. Clear
Waterford
Wexford
Dungarvan
Youghal
Smalls
St. Brides Bay
Milford Haven
Cardigan
Aberystwith
Swansea
Newport
Chepstow
Bristol
R. Severn
Lundy I.
Barnstaple
Bideford
Padstow
Falmouth
Penzance
Landsend
Scilly Is.
Seven Stones
Plymouth
Tor Bay
Southampton
Portsmouth
Brighton
L. of Wight
Needles
Portland
LONDON
R. Thames
Sheerness
Chatham
Margate
Dover
Deal
Ipswich
Harwich
Yarmouth
Lowestoft
Ostend
Calais
Boulogne
R. Somme
Dieppe
Havre
Seine R.
Rouen
C. la Hague
Cherbourg
Alderney
Guernsey
Sark
Jersey
Minquiers
Mt. St. Michel Bay
St. Malo
Granville
Morlaix
Ushant
Brest
Douarnenez B.

cases, when the streams oppose, wholly overpowering or reversing their direction. That you may the more readily comprehend this explanation, I annex two charts, Plate IX. and X., upon which I have traced the course of the stream at *alternate hours* of a spring tide, each line representing the direction of the stream *at the same moment throughout both channels*. From the connection of the observations of the Irish Sea with those of the Bristol Channel, it is clear that the whole of the ebb or outgoing stream of the eastern half of the Irish Channel runs into the Bristol Channel, and forms the flood or ingoing tide of the northern half of that great estuary; and *vice versâ*, the ebb or outgoing stream from the northern half of the Bristol Channel, forms the flood of the Irish Sea, each tide passing to and fro with great rapidity round St. Govan's Head. The centre and southern half of the Bristol Channel receive their waters from the offing and the English Channel, the coast stream bringing the waters up from the Land's End and the English Channel, as the stream on the northern half did those of the Irish Channel, and *vice versâ*.

Influence of offing stream over that of the channel.

Courses of the stream at alternate hours of the tide, from Dungeness to the Irish Sea.

Tides of the Bristol Channel.

The great offing stream at the entrance of the English Channel extends its influence as far up as Cape La Hague, beyond which, owing perhaps to the sudden contraction which there occurs in the Channel, the stream suffers no interruption, but, as in the Irish Sea, passes up and down the Channel six hours nearly each way as far as a line joining Dungeness and Cape Grisnez, the apparent *virtual head* of the tidal channel. Here the influence of the North Sea stream begins to be felt, and here, as in the Irish Channel, again *the time of high and low water at the virtual head of the tide regulates the turn of the up and down stream along the whole channel* as far as the contraction. Beyond this the offing stream being governed by its own high water, and that occurring at about six hours earlier than that of the head of the channel, the offing stream either butts against the returning streams from the channels, or withdrawing its water, solicits their streams and thus alters their course, making them for the most part set across the Channel in curves more or less bent as the spot is more or less removed from the offing; so that there seems to be but one hour's tide each way that passes clean down the Channel from Beachy Head to Scilly, and round the Land's End to Bristol. The outgoing stream from Beachy Head encounters the ingoing stream of the offing tide somewhere about the Start Point, and both are turned down into the great Gulf of St. Malo, which seems to receive the accumulated waters of these opposite tides.

Effect of the offing stream upon the tide in the English Channel.

Meeting of the streams.

Whether or not this influx is instrumental in raising the water here to the extraordinary height of forty-seven feet perpendicular range at springs, or whether it be owing to its form and position as regards the advancing tide-wave, I leave to those who are competent to decide; but it is a coincidence that cannot escape observation, that this spot, like the Bristol Channel, is the concentration of streams from opposite directions; that it has its waters raised to the same extraordinary elevation nearly to a foot, and that its time of high water is nearly the same.

On the change of tide, this great bay, like the Bristol Channel, as it received so it

Gulf of St. Malo tide.

returns its waters in opposite directions; the tide splitting somewhere between Alderney and the Start. But here especially, as also in a similar locality in the Irish Channel, we are in want of observations, and it is very unfortunate that this spot, which from its peculiarity is the most interesting of any in the English Channel, should be so completely deficient; for it seems most evident that the tidal streams meet off here, running together at one time and separating at another.

Such appears to be the explanation of the complicated motion of the waters at the entrances of these two great channels, and of the course of the stream in the English Channel above the contraction of the strait.

In tracing these streams, it was impossible not to be impressed with the many coincidences which assimilate the tidal phenomena of the two channels, so much so as to render it probable that they are subjected to precisely the same laws. As I feel confident that the consideration of this subject will be both interesting and useful, I shall endeavour to trace the resemblance through its several varieties.

Comparison of the tidal phenomena of the English and Irish Channels.

First, I shall consider the Irish Channel to extend, as above mentioned, from a line joining the Land's End and Cape Clear to the end of the tidal flow, which is either at Morecombe Bay or Peel in the Isle of Man; and the English Channel as reaching from a line connecting Ushant with the Land's End, to the end of its tidal flow, or to Dungeness. We shall then see that the English Channel from its outer limit to the end of its tidal stream is 262 geographical miles, and that the Irish Channel from its western limit to the end of its tidal stream, is nearly the same, being about 265 geographical miles. In both channels the stream enters from the south-west, and flows up until stopped by a counter stream. In both channels there is a contraction of the strait almost midway, by the promontories of Cape La Hague in one instance, and St. David's Head in the other, and at very nearly the same distances from the entrance. This contraction is, in both cases, the commencement of the regular stream, which flows six hours nearly each way, the *turn of the stream throughout coinciding with the times of high and low water at the virtual head of the channel*, situated in both cases about 145 miles above the contraction, and that time being very nearly the same, viz. 10^h 50^m at F and C*. Below this contraction, away from the land, the stream in both cases varies its direction nearly every hour, according to the force exerted upon it by the opposing offing stream.

In both cases, between the contraction and the southern horn of the channel, there is situated a deep estuary, the Bristol Channel and the Bay of St. Malo, in which the times of high water coincide, and where, in both cases, the opposing streams meeting in the channel, pour their waters into these gulfs, and where the tides in both places rise to the extraordinary elevation of forty-seven feet at the syzygies. From the Land's End to the meeting of these streams in the Bristol Channel is seventy-five miles, and from Brest to the meeting of the streams off Guernsey the same. A still further coincidence is apparent between the phenomena of these channels. In one,

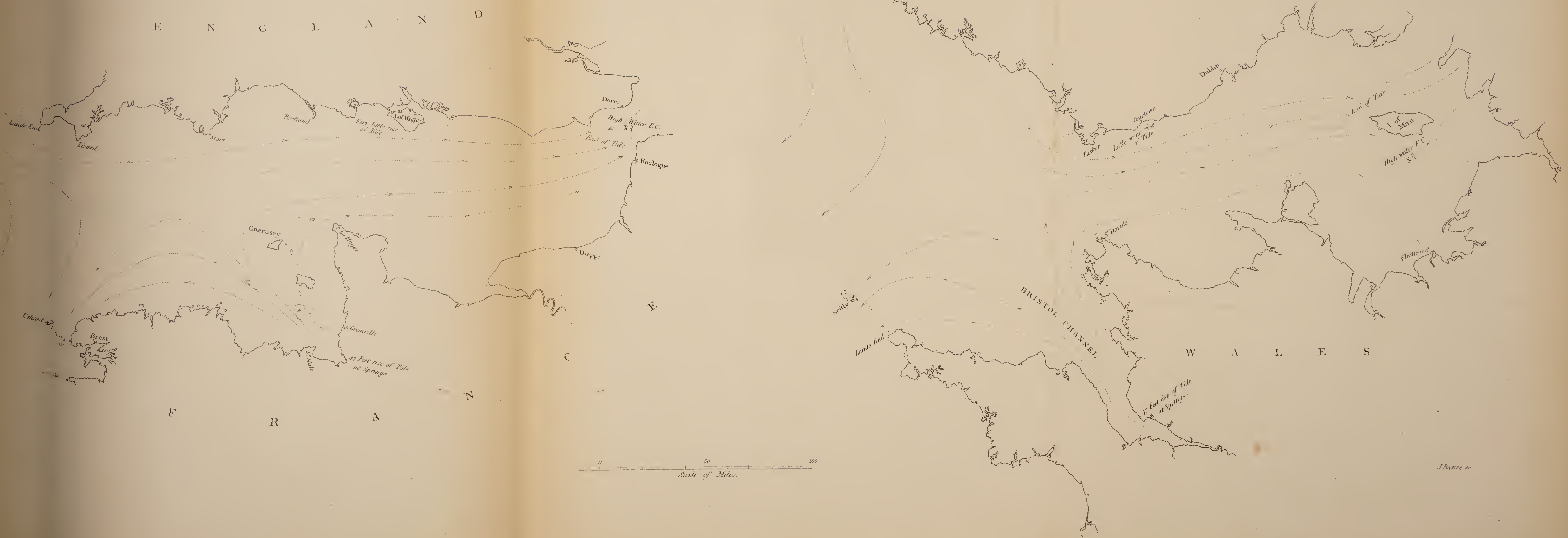
* See Plate XI.

SKETCH
of the Course of the 7 o'Clock stream of Tide
in the
English and Irish Channels.

To accompany a letter to the Hydrographer.

By Captain F. W. Beechey R.N.

1848.



Comparison continued.

at a place called Courtown, a little above the contraction of the strait, and at 150 miles from Cape Clear (its entrance), there is scarcely any rise or fall of the water; and in the other channel (about Swanage), situated also a little above the contraction of the strait, and just 150 miles from the Land's End (the entrance of the channel), there is only five feet rise of the water at a spring range*. In both cases these points of small range of tide are situated on the opposite side of the channel to that of the high elevation above mentioned, and in both cases these spots are the node of the tide-wave (on either side of which the times of high and low water are reversed). And again we trace a similarity in an increased rise of the water on the south-east sides of both channels abreast of the virtual head of the tide: at Liverpool in one case, where the range amounts to thirty-two feet, and at Cayeux in the other, where it is thirty-four feet.

It may also be shown that the progress of the tide-wave along the side of the channels opposite the node is not very dissimilar. Reckoning in both cases from the line which we have before drawn as the outer limits of the channel, we find that in the English Channel from this line to Cherbourg, opposite the small range of tide,—

	Miles per hour.		Miles.
The wave travels	50	and if complete would be in extent	616
In the Irish Channel, from a similar line to Bardsey, it travels	52		649
From Cherbourg to Havre	32		397
From Bardsey to Holyhead	16		193
From Holyhead to end of tide	78		959
Dieppe to the end of the tide	75		922

These numbers are given roughly, merely for the purpose of showing the general resemblance in the character and motion of the wave; and it is probable a more judicious selection of positions and numbers would give a still nearer coincidence. Besides which we are somewhat uncertain as to the establishment at our starting-point. As a comparison, however, the numbers run fairly together. In both cases the retardation of the tide-wave about mid-channel, and the great elongation of the wave towards the end of the strait are remarkable, especially in the Irish Sea.

Lastly, we may notice a singular coincidence in more respects than one, indeed, between the situation of the node placed by Professor WHEWELL† in the North Sea, and a corresponding point of small range and inversion of tide at the back of Kintire‡. The node or hinge of the tide in the North Sea is curiously enough situated as nearly as possible at *the same distance from the head of the tide off Dungeness, as the node at or near Swanage* is on the opposite side of it; and the node at Kintire communi-

* In 1834 it was 3 ft. 10 in.
In 1835 it was 6 ft. 4 in. } 5 ft. 1 in. mean. Philosophical Transactions, 1836, Part II. p. 320.

† Philosophical Transactions, 1835, Part II. p. 298.

‡ I have recently been apprised of this by Captain ROBINSON, R.N., who is surveying the coast of Scotland, but I have not seen the observations.

cated by Captain Robinson, is about the same distance from the meeting of the tide in the Irish Sea as the North Sea node is from the meeting of the waters off Dungeness, and is similarly *situated with respect to the node of Courtown as the North Sea node is with regard to Swanage.*

It appears therefore that the tidal feature of these two channels corresponds in almost every particular.

Further investigation of the tidal phenomena of English Channel urged.

I cannot but consider the identity very remarkable and interesting, and especially so as concerns the relative situations of the nodes of the wave, hinging (as those in the channel appear to do) upon a *single point*, and not upon a negative line across the channel, as may have been imagined. And it seems highly desirable that a critical investigation of the phenomena of this channel should be made; not only for the purposes of science, but especially on account of the navigation of the strait, for already we may trace the cause of vessels entering the channel, being set down upon the French coast about Heaux; and who can call to mind the lamentable loss of our Indiamen on the coast about Boulogne, and not be reminded that these disasters occurred very near the point where the stream may probably be turned down upon that shore by the meeting of the tides off there?

Having now, Sir, placed before you the result of our observations upon the tides of the Irish Sea, and shown the connection which exists between the phenomena of the Irish and English Channels, and the possibility there is of forming into a system the apparently contradictory directions of the stream, at the mouths of those estuaries, which is so very desirable, I have to request that, should you, after the perusal of this letter, approve of the observations being continued round the Land's End and up the English Channel, you will solicit the Lords Commissioners of the Admiralty to furnish the means of so doing.

I have the honour to be, Sir,

Your obedient humble Servant,

F. W. Beechey, Captain.

To Rear-Admiral Beaufort,
&c. &c. &c.,
Hydrographer.

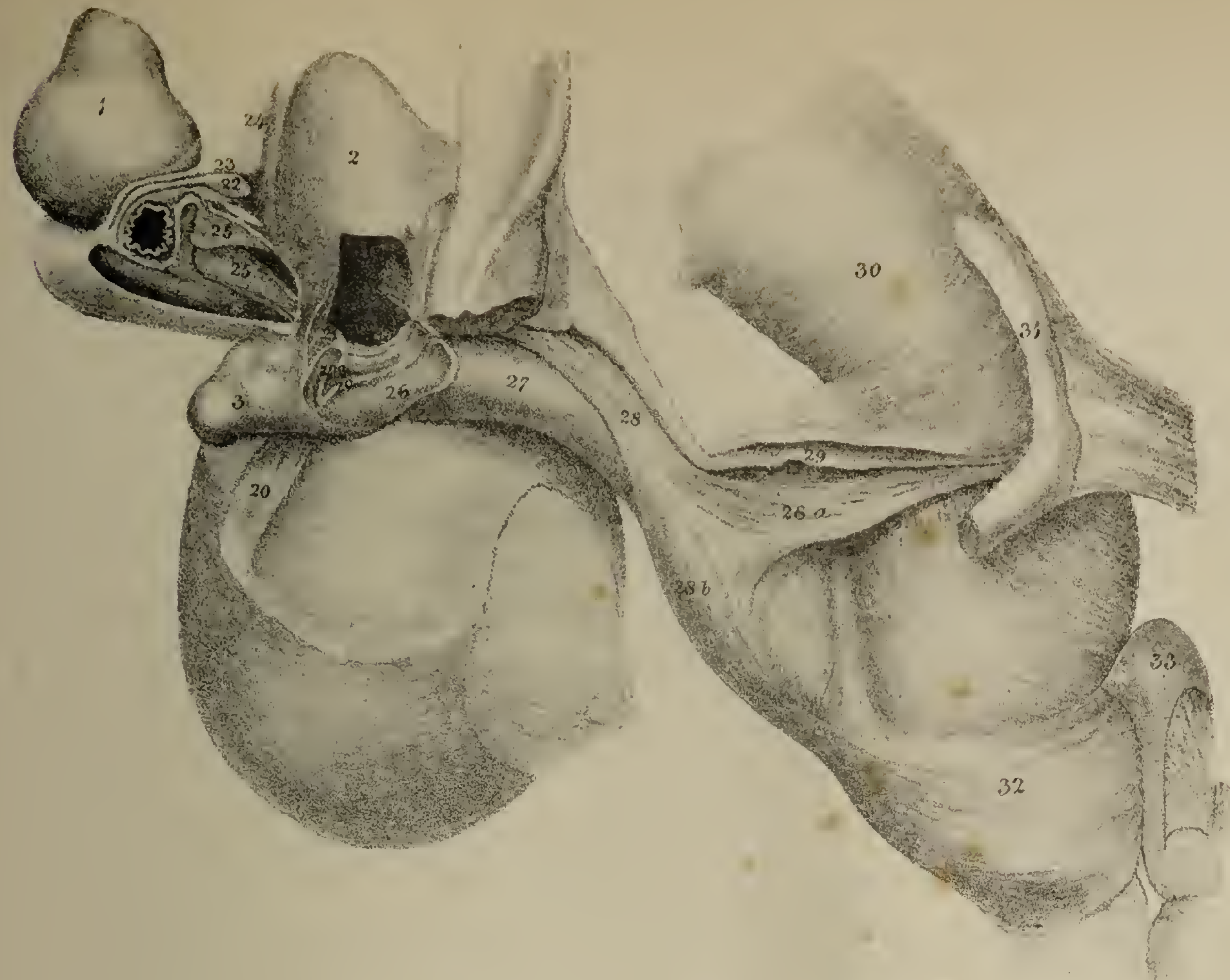


Fig. 1

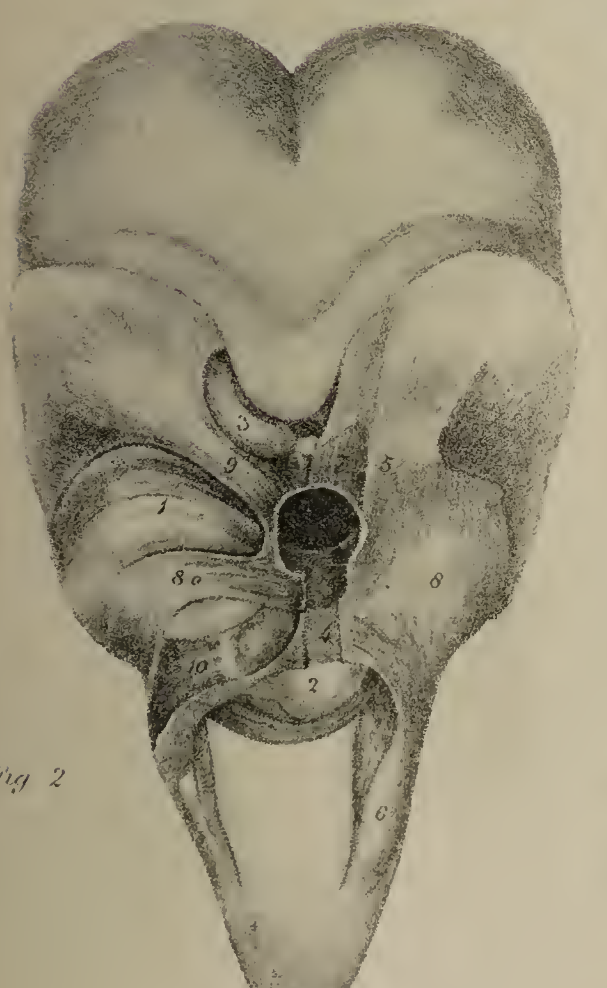


Fig. 2

Fig. 3

VII. *On the Blow-hole of the Porpoise.* *By* Francis Sibson, *Esq.* *Communicated by* Thomas Bell, *Esq., F.R.S.*

Received December 18, 1845,—Read March 12, 1846.

THE Porpoise inspires through the blow-hole which includes the nostrils, and is so placed that the animal can breathe with only a very small portion of its head above the water. A large passage leads down from the external opening to two canals that pass directly downwards through the skull, between the cranial and facial bones; these nasal canals are separated from each other by a thin plate of bone (Plate XII. fig. 1.3–26.27)*. The two canals, after passing down through the skull, coalesce to form a muscular tube that opens at its lower part into the pharynx, by a constricted aperture. Through this aperture the mouth of the larynx rises, after the larynx itself has projected quite through the pharynx, dividing it into two channels.

Connected with the channel that leads from the external opening down to the two bony conduits is a series of pouches; these are composed of aponeurotic walls, folded on themselves in many wrinkles, so as to be capable of great distension†.

The two largest and most dilatable of these pouches are seated on each side and in front of the outer passage, into which they open separately (fig. 1.2–1).

Deeper than, and between these two anterior lateral pouches, is the anterior central pouch, which is placed upon the combined maxillary bones, and communicates with the nasal canal by a large opening guarded by two oblong cartilages; this sac has a muscular coat (fig. 1.2–2).

The openings into the two nasal canals are just behind and below this anterior deep pouch.

Immediately above the apertures of the two bony canals are two other horizontal apertures that communicate with two corrugated dilatable pouches (fig. 1.2–3). These posterior deep sacs are immediately behind the external channel, and are seated between it and the frontal bone; their openings look downwards to the bony canals, their summits rise directly upwards in front of the frontal bones.

The five pouches are under complete muscular control.

When filled, their apertures can be closed, and their contents retained; or the apertures can be thrown open, and their contents pressed out.

The pouches, when distended with air, doubtless act so as to float the external

* Figure 1 must be looked at, not in the direction of the plate, but sideways.

† See Cyclopædia of Anatomy, i. 580, fig. 269, Article Cetacea, by Professor Owen.

opening uppermost, and retain that opening above the surface of the water during sleep and during the act of copulation.

The muscles that open and close the blow-hole, and that act upon the various sacs, form one of the most complicated yet most exquisitely adjusted pieces of machinery that either nature or art presents.

The blow-hole is stretched open, in front, by two muscles (fig. 2–6) that act from the superior maxillary bones, to draw forward the cartilaginous mass in front of the outer opening; behind, by muscular fibres (fig. 2–7) that act from the frontal bones on the posterior and lateral edges of the blow-hole.

To close the blow-hole, two sets of muscular fibres (fig. 2–5) spring from the frontal bone, pass obliquely forwards and inwards, one to each side of the opening, and are inserted into the gristly mass (fig. 2–4) in front of the blow-hole. When they act, they draw this gristly mass forcibly backwards, and close the blow-hole by pressing its anterior and posterior walls together, and compressing them against the posterior pouches and the prominence at the centre of the frontal bone.

A set of muscular fibres (fig. 2–8) arises from the circuit of a depression formed on the cranium to each side of the bony canals; these fibres pass over the anterior lateral pouch, blending with the muscles that close and open the blow-hole. On removing this, a covering of muscular fibres (fig. 2–8 *a*) that cling round the sac is exposed; these arise from the anterior lateral part of the superior maxillary bones, and pass upwards and backwards to form a muscular envelope for the pouch on its inner and posterior surface; the fibres return forwards to be inserted into the outer and anterior walls of the blow-hole.

A fan-like muscle (fig. 2–9) arises from the circuit of the depression to be inserted into the posterior and lateral walls of the blow-hole; this assists in opening it by spreading apart its walls.

On removing this fan-like muscle, a remarkable series of similar fan-like muscles (11.12.13.14.15*) is exposed. These lie one above the other, and arise by muscular fibres from the circuit of the depression; the fibres of each converge to a tendinous aponeurosis thinner than paper.

Four or five of these fan-like thin muscles may be removed by careful dissection, one after another; they act to expand different parts of the outlet of the anterior lateral sac, to draw backwards and expand the posterior walls of the great respiratory canal, and to compress the posterior deep sacs.

On removing these, two remarkable muscles (fig. 1. Archives, fig. 7–22.23) are brought into view that have each a double muscular origin, one from behind forwards from the frontal bone (22 *a*. 23 *a**), meeting another from before backwards and inwards from the superior maxillary bone (22 *b*. 23 *b**). These two origins combine in each to form a tendon; these tendons (22.23) pass forwards in front, and below the neck of the anterior pouch, where they unite with their fellows; each of these tendons

* These numbers refer to drawings that are in the Archives of the Royal Society.

gives off a slip that ascends in front of the neck of the sac, and passes over it to be inserted into the anterior wall of the outlet.

When the posterior or frontal bellies of these muscles act on both sides, they will form a sling which will draw backwards the anterior wall of the breathing canal, and compress it against the posterior wall, thus closing the canal below the openings of the anterior sacs. They will, I conceive, at the same time contract the outlets of the sacs.

The anterior heads, or those from the maxillary bone, will draw forward the sling formed by the combined muscles and open the breathing canal.

These two muscles cross each other, so that in the same muscle the anterior belly is most superficial, while the posterior belly is deepest.

The most superficial of these tendinous slings is blended with the tendinous structure common to a very strong pair of muscles (24*). These muscles arise from the superior maxillary bones to the side of the anterior deep pouch, and ascend over the sides of that pouch to be inserted into the strong gristly tendon (fig. 1–24) at the top of it. The combined muscle forms a long arched musculo-tendinous bridge over the anterior deep sac, that, when it contracts, compresses the sac. The tendinous bridge just described passes backwards underneath the two tendons (fig. 1–22.23) that close the deep portion of the breathing canal; it then turns upwards behind those tendons, and is reflected forwards over them to form the insertion of two other muscles (fig. 2–10) that are superficial to the muscles just described. The combined musculo-tendinous web formed by these two muscles and their common tendon, superficial to the muscles described above, arches over the anterior deep sac; with those muscles it combines to contract the anterior deep sac.

A distinct set of fibres (40*) arises from the side of the inlet of the osseous canal; these fibres pass forwards and upwards over the neck of the anterior deep pouch, to be inserted into the central tendinous web covering that pouch, the outlet of which, this muscle, I conceive, contracts.

Another muscle, (41*) just behind this, seems to have a similar function.

The anterior deep pouch (2.) differs from the other pouches in having no superficial wrinkles, and in having a regular muscular coat; it seems indeed to be rather a pouch-like dilatation of the breathing canal, than one of the regular sacs.

Each posterior deep pouch (fig. 1. 2–3) is situated behind the breathing canal, between it and the frontal bone. Its opening is horizontal, and looks downwards, and is half-surrounded in front by a cartilage (fig. 1–21).

Several fan-like, half-pouch formed muscles (16.17.18.19*), that arise from the frontal bone, contract and empty the posterior pouch.

To contract the mouth of the pouch there are two muscles. One (fig. 1–20) springs from the side of the central protuberance of the frontal bone, and forms a tendon that passes forwards and inwards encircling the inner side and front of the outlet;

* See figures in the Archives.

it splits into two tendons, one of which (20 *a*) is inserted into the outer side of the edge of the bony canal, and the other (20) into the partition of that canal.

The other muscle (21*) arises close to the insertion of the last muscle, takes a sweep round the posterior and inner sides of the outlet of the pouch, and is inserted into the cartilage at the margin of that outlet in front.

It appears that when the blow-hole is closed, the posterior pouches are compressed and emptied, while the anterior pouches are distended with air from the lungs. The anterior pouches will then cause the head and blow-hole to rise to the surface. When the blow-holes are open, the muscles that compress and empty the anterior sacs likewise expand the blow-hole; it is clear that the anterior sacs that may be filled when the blow-hole is closed, will be emptied when it is open; while the posterior sacs that are emptied when the outer passage is closed, may be filled when the passage is open.

The anterior and posterior sacs alternate in their contraction and dilatation, the former being dilated and the latter contracted when the outer passage is closed, and the latter being dilated and the former contracted when the outer passage is open. Thus, first one then the other set of pouches may be dilated with air, to buoy up the head.

The two passages (figs. 1.3–26.27) that descend through the cranium, are each surrounded by a muscular coat; they join below the septum (fig. 3–27), which may be regarded in the light of a vomer, and form an increasing cylindrical pouch, which is analogous to the upper part of the pharynx behind the soft palate. This pouch contracts suddenly, where it opens into the pharynx (fig. 3–34). Through the contracted outlet or communication of the pouch-like channel with the pharynx, the mouth of the larynx rises (35). This outlet is acted upon, and the mouth of the larynx is compressed and retained, by a sphincter (34). The whole tube is surrounded by oblique fibres (fig. 1–27) descending from before backwards. Exterior to this muscular investing coat, is a strong layer of muscular fibres (28) that arise from the superior maxillary bone, in front of the canal, and descend downwards and backwards until they turn the edge of the hard palate; there they part into two sets; one set (28 *b*) passes backwards strengthening the canal behind, another (28 *a*) descends a little forward to the os hyoides.

The epiglottis (36) is long, firm and convex, and ends in an expanded lip. The arytenoid cartilages (37.35) are very large, strong and elongated, being seated immediately below, and contiguous to, the epiglottis.

When the mouth of the larynx is opened, the long arytenoid cartilages are drawn downwards from the epiglottis and apart from each other by the crico-arytenoid muscles (39). The mouth of the larynx is closed by the action of the thyro-arytenoid (38) drawing the arytenoid cartilages over to the epiglottis.

The remarkable apparatus described above forms the breathing passage of the

* See figures in the Archives.

Porpoise. The adjustments of the apparatus are such that the animal can close or open the outer passage either above or below the anterior pouches. When the outer passage is closed, the posterior pouches can be distended and the anterior are emptied; and when the passage is open, the anterior pouches can be distended and the posterior are emptied.

The pouches evidently serve to buoy up the head, so that when the Porpoise rises from the deep, the opening for breathing comes first to the surface; and if the animal remains at the surface, either in sleep or in the act of coition, the air in the pouches will float the blow-hole above the level of the water, when the whole body of the animal is below the level.

The Porpoise, I believe, does not spout. The blow-hole of the whale is in principle the same with that of the Porpoise. I was led, some years since, to think, from the examination of a Rorqual, that the spouting of the whale consists in the regurgitation from the stomach of the sea-water that it must swallow plentifully with its food. I was induced to form this opinion from noticing *the direction of the first stomach*, which may be described as an expanded pouch of the œsophagus, and which is so placed as to send its contents, when it is contracted, directly upwards through the œsophagus, provided the communication be open, *and the form of the larynx* boat-shaped below, and capable of being completely closed.

Apart from all speculation, the blow-hole of the Porpoise, forming a part of its mechanism of respiration, must in itself be regarded as one of the most intricate and nicely adjusted apparatus that has yet been observed.

Description of the Plate.

PLATE XII.

Fig. 1. (Must be looked at, not in the direction of the plate, but sideways.) It represents the cranium of the Porpoise, a section of the blow-hole or nasal canal, and the sacs.

Fig. 2. The cranium of the Porpoise with the muscles that close and open the blow-hole, and also those acting upon and emptying the sacs.

Fig. 3. A section of the pharynx and of the naso-pharyngeal canal leading from the larynx to the nostrils, exhibiting the larynx.

Besides the engraved figures there are five others, deposited in the Archives of the Royal Society, exhibiting, in various stages of their dissection, the muscles that act upon the sacs and the passages of the blow-hole.

The same numbers refer to the same parts in all the figures.

Nos. 1.1. Figs. 1.2 (figs. 4.8. Archives of the Royal Society). The two anterior superior lateral sacs. Membranous.

No. 2. Figs. 1.2 (figs. 7.8. Archives of the Royal Society). The anterior inferior sac. Muscular.

Nos. 3.3. Figs. 1.2 (fig. 8. Archives of the Royal Society). The two posterior sacs. Membranous. The sacs communicate with the nasal canal.

No. 4. Fig. 2. Semicartilaginous mass in front of the blow-hole.

No. 5. Fig. 2. Muscle, arising from the frontal bone, inserted into the semicartilaginous mass (4) in front of the blow-hole; with its fellow it draws that mass backwards to close the blow-hole.

No. 6. Fig. 2. Muscle that, with its fellow, opens the blow-hole by drawing forwards the semicartilaginous mass in front of the blow-hole.

No. 7. Fig. 2. Muscle that dilates the outlet by drawing backwards its posterior wall.

No. 8. Fig. 2. Muscle that compresses and empties the anterior superior sac (1).

No. 8 *a*. Fig. 2. Deeper muscle that envelopes, compresses and empties the anterior superior sac, and dilates the outlet of the blow-hole by drawing forwards and sideways its anterior and lateral walls.

No. 9. Fig. 2. Fan-shaped muscle that dilates the blow-hole by drawing backwards and outwards its posterior and lateral walls.

No. 10. Fig. 2. Muscle that, with its fellow, arches over, compresses and empties the deep anterior sac (2).

Nos. 11.12.13.14.15 (figs. 4.5.6. Archives of the Royal Society). The dilators of the nasal canal. Fan-shaped muscles, that arise like No. 9. from the circumference of the fronto-facial ridge, and are inserted into and expand various portions of the walls of the nasal canal.

Nos. 16.17.18.19 (figs. 6.7.8. Archives of the Royal Society). Compressors of the posterior sacs (3).

No. 20. Fig. 1 (fig. 8. Archives of the Royal Society). Muscle that divides into two tendons (20 and 20 *a*.) and constricts the opening of the posterior sacs (3).

No. 21. Fig. 1. Cartilage in front of the opening of the posterior lateral sac (3), into which is inserted

No. 21 *b* (fig. 8. Archives of the Royal Society). A deep muscle that surrounds and constricts the opening of the posterior sac (3).

Nos. 22.23. Fig. 1. Are sections of the tendons of the two following double-headed muscles 22 *a*. 22 *b*. 23 *a*. 23 *b*.

Nos. 22 *a*. 23 *a* (fig. 7. Archives of the Royal Society). Are the two heads that with their fellows of the opposite side act from behind forwards to tighten the tendons 22.23, and so close the nasal canal below the openings of the anterior superior sacs (1), and, I conceive, constrict the openings of those sacs.

Nos. 22 *b*. 23 *b* (fig. 7. Archives of the Royal Society). Are the two heads that act

from before backwards to open the canal by relaxing the tendons, 22.23 (see p. 32).

No. 24. Fig. 1. Is the tendon of

No. 24 *b* (fig. 7. Archives of the Royal Society). One of a pair of muscles that, like No. 10. fig. 2, arch over and compress the anterior deep sac (2).

Nos. 25.25. Fig. 1. Oblong cartilages, one to each side of the opening of the deep anterior sac (2).

No. 26. Fig. 1. 27. Fig. 3. The vomer separating the two nasal canals.

No. 27. Fig. 1. The upper portion of the naso-pharyngeal canal.

No. 28 *c*. Fig. 3. The naso-pharyngeal canal into which the mouth of the larynx (35) ascends through an opening in the pharynx (34).

Nos. 28.28 *a*. 28 *b*. Fig. 1. Muscular fibres that strengthen the naso-pharyngeal canal and the pharynx.

No. 29. Fig. 1. Muscle that descends to the os hyoides from the palatal bone.

No. 30. Fig. 1. The anterior part of the pharynx.

No. 31. Fig. 1. The os hyoides.

No. 32. Fig. 1. The constrictor of the pharynx.

No. 33. Fig. 1. The thyroid cartilage.

No. 34. Fig. 3. The sphincter of the opening in the pharynx, through which the larynx communicates with the naso-pharyngeal canal, after passing through the pharynx and dividing it into two channels.

No. 35. Fig. 3. The mouth of the larynx.

No. 36. Fig. 3. The epiglottis.

No. 37. Fig. 3. The arytenoid cartilages.

No. 38. Fig. 3. The thyro-arytenoid muscle.

No. 39. Fig. 3. The crico-arytenoid muscle.

Nos. 40.41 (fig. 8. Archives of the Royal Society). Constrictors of the mouth of the deep anterior sac.

VIII. *On the Corrections to be applied to the Monthly Means of Meteorological Observations taken at any hour, to convert them into Mean Monthly values.* *By* James Glaisher, *Esq., of the Royal Observatory at Greenwich.* *Communicated by* G. B. Airy, *Esq., F.R.S. &c., Astronomer Royal.*

Received February 23,—Read March 2, 1848.

ONE of the most useful results of observations made at short intervals during the day and night, and continued for several years, is the knowledge we thus obtain of the diurnal ranges of the different subjects of investigation, and consequently the difference between the mean values of each element, as deduced from observations at one or more hours daily, and the true mean for the period over which the observations are spread.

At the Royal Observatory at Greenwich magnetical and meteorological observations have been taken since the year 1840, as is familiar to the Fellows of this Society. These have been published to the end of the year 1845. The whole of these observations have been made under my immediate superintendence, under the direction of the Astronomer Royal, and I believe that no observations have been made and reduced with greater care or regularity. As the person entrusted with the superintendence of these operations, I have a more perfect knowledge of them than any other person can have; I feel it therefore a duty to communicate their results from time to time, when the doing so promises to be of essential service in promoting the advancement of the subjects of investigation.

I have selected for my present communication some results deduced from the meteorological observations, and I have preferred these to the magnetical, not only on account of the greater accordance in the results year by year, but also because of their more immediate and general use. At the present time I believe that there are a greater number of persons engaged on meteorological researches upon a systematic plan, than have hitherto been so engaged, but, necessarily, these persons can observe only at certain times convenient to themselves, and these times differ from each other. To render their results comparable, it is necessary to apply corrections to every different result depending upon the time, or times of the day at which the observations have been made.

The barometrical and thermometrical observations treated of in this paper have been made at every even hour of Göttingen mean solar time, throughout each of the five years, except on Sundays, Good Friday and Christmas day; the mean of each hour represents the results deduced from about 150 observations; those for each

month represents about 1800 observations; and those for the year represents upwards of 21,000 observations of each element. The hygrometrical observations were not commenced at so early a period as those of the barometer and thermometer; and each of the following yearly tables, relative to the hygrometrical states of the air, represents about 20,000 simultaneous observations of the dry- and wet-bulb thermometers.

The method of formation of the tables was as follows:—

The first step was the formation of tables representing the excess of the mean value of each element at every hour of observation, in every month, above the mean value for the month.

The second step was the formation of tables containing the numbers found in the first process, arranged for the different years. The accordance of these numbers for the different years was found to be very close indeed.

The third step was the taking the mean of the numbers in the second set of Tables, at the same hour in every month.

The fourth process was the laying down these mean values on paper divided into square portions of 144 to one square inch, and considering that every division corresponded to twenty minutes of time; the numbers being laid down as ordinates with "the time of day" for abscissa.

The fifth step was drawing a curve line passing through or near every point in every month at all the hours, giving equal weight to every point. In nearly every case a simple curved line passed through all the points.

The sixth and last step was the measuring the ordinates at every Greenwich hour, and in this way the tables were formed.

The accordance in the results at the same hour in the same month in the different years being found so close and satisfactory, together with the fact of a simple curve passing through all the points, the diurnal march, as shown in the Tables, may be considered as a very close approximation to the facts of nature.

There is a promise at present of systematic observations being taken at many places in England, and by gentlemen of assured competency; these tables will assist such persons very much in determining at what times observations should be taken to determine different atmospherical elements.

As a remark applying to all the Tables in this paper, I may here mention that observers will find that comparatively a very few observations in each day, at hours by no means inconvenient in ordinary life, will furnish a near approximation to the mean and extreme values, as well as to the diurnal and annual variations of atmospherical phenomena.

TABLE I.—Showing the corrections to be applied to the Monthly Mean of the daily readings of the barometer at any hour, to deduce the true mean reading for the month from the observations taken at that hour.

Local mean time.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
Midnight.	0·000	−0·001	−0·002	−0·008	−0·005	0·000	−0·006	−0·010	−0·005	−0·005	−0·011	−0·004
1 A.M.	+0·001	+0·004	+0·013	0·000	+0·002	+0·004	0·000	0·000	0·000	+0·004	−0·005	+0·001
2	+0·002	+0·008	+0·020	+0·007	+0·004	+0·005	+0·003	+0·007	+0·005	+0·010	+0·003	+0·006
3	+0·005	+0·012	+0·023	+0·010	+0·005	+0·004	+0·005	+0·011	+0·010	+0·015	+0·008	+0·010
4	+0·011	+0·014	+0·022	+0·011	+0·005	+0·001	+0·005	+0·014	+0·012	+0·020	+0·013	+0·012
5	+0·015	+0·015	+0·019	+0·011	+0·006	−0·002	+0·006	+0·011	+0·014	+0·022	+0·016	+0·014
6	+0·015	+0·012	+0·012	+0·006	+0·006	−0·006	+0·002	+0·005	+0·010	+0·018	+0·015	+0·011
7	+0·010	+0·007	+0·005	−0·003	+0·006	−0·010	−0·004	0·000	+0·001	+0·008	+0·010	+0·006
8	+0·003	0·000	−0·004	−0·008	+0·003	−0·012	−0·008	−0·007	−0·006	−0·003	+0·003	+0·004
9	−0·008	−0·008	−0·010	−0·011	−0·007	−0·012	−0·010	−0·008	−0·011	−0·009	−0·005	−0·010
10	−0·010	−0·015	−0·015	−0·014	−0·009	−0·011	−0·010	−0·009	−0·013	−0·014	−0·007	−0·015
11	−0·014	−0·016	−0·015	−0·011	−0·006	−0·009	−0·009	−0·008	−0·010	−0·014	−0·005	−0·015
Noon.	−0·005	−0·012	−0·010	−0·008	−0·002	−0·006	−0·006	−0·005	−0·005	−0·010	+0·002	−0·009
1 P.M.	+0·002	−0·006	−0·005	−0·004	0·000	−0·003	−0·003	0·000	0·000	−0·003	+0·007	+0·003
2	+0·005	+0·003	0·000	+0·003	+0·003	+0·003	+0·001	+0·003	+0·004	+0·004	+0·011	+0·008
3	+0·004	+0·006	+0·003	+0·009	+0·006	+0·007	+0·005	+0·005	+0·008	+0·005	+0·010	+0·010
4	+0·002	+0·008	+0·005	+0·004	+0·010	+0·013	+0·009	+0·009	+0·010	+0·003	+0·008	+0·009
5	0·000	+0·006	+0·004	+0·014	+0·014	+0·017	+0·013	+0·011	+0·011	0·000	+0·004	+0·006
6	−0·003	+0·002	0·000	+0·011	+0·015	+0·017	+0·013	+0·011	+0·006	−0·005	0·000	+0·002
7	−0·005	−0·004	−0·006	−0·007	+0·010	+0·014	+0·010	+0·005	0·000	−0·008	−0·006	−0·003
8	−0·006	−0·006	−0·012	−0·005	0·000	+0·008	+0·004	−0·005	−0·005	−0·011	−0·012	−0·006
9	−0·007	−0·008	−0·015	−0·009	−0·006	+0·003	−0·001	−0·010	−0·009	−0·014	−0·017	−0·009
10	−0·005	−0·007	−0·012	−0·012	−0·008	−0·002	−0·005	−0·015	−0·011	−0·012	−0·019	−0·010
11	−0·004	−0·005	−0·010	−0·012	−0·008	−0·002	−0·012	−0·015	−0·011	−0·009	−0·017	−0·009

The sign + denotes that the number is to be added, and the sign — denotes that the number is to be subtracted.

The double sets of + and — signs in every month show that there are two maxima and two minima in the daily variation in the reading of the barometer, and consequently the fact of a triple maximum and a triple minimum at times occurring, is due to accidental circumstances and to irregular daily oscillation.

Four times daily the reading of the barometer is at its mean value; these times in the several months are as follows:—

		h m	h m	h m
In January	......at midnight ;	at 8 0 A.M.;	at 0 40 P.M.;	and at 5 0 P.M.
In February	......at midnight ;	at 8 0 A.M.;	at 1 40 P.M.;	and at 6 20 P.M.
In March	.........at midnight ;	at 7 35 A.M.;	at 1 50 P.M.;	and at 6 0 P.M.
In April	.........at 1^h 0^m A.M. ;	at 6 40 A.M.;	at 1 40 P.M.;	and at 7 20 P.M.
In May	.........at 1^h 0^m A.M. ;	at 8 20 A.M.;	at 1 0 P.M.;	and at 8 0 P.M.
In June	.........at midnight ;	at 4 20 A.M.;	at 1 40 P.M.;	and at 9 20 P.M.
In July	.........at 1^h 0^m A.M. ;	at 6 25 A.M.;	at 1 40 P.M.;	and at 8 45 P.M.
In August	......at 1^h 0^m A.M. ;	at 7 0 A.M.;	at 1 10 P.M.;	and at 7 35 P.M.
In September	...at 1^h 0^m A.M. ;	at 7 30 A.M.;	at 1 0 P.M.;	and at 7 0 P.M.
In October	......at 0^h 25^m A.M.;	at 7 50 A.M.;	at 1 10 P.M.;	and at 5 0 P.M.
In November	...at 1^h 40^m A.M.;	at 8 20 A.M.;	at 11 40 A.M.;	and at 5 45 P.M.
In December	...at 0^h 40^m A.M.;	at 7 40 A.M.;	at 0 45 P.M.;	and at 6 5 P.M.

The diurnal oscillations of the sum of the pressures of air and vapour are plainly very different at different times of the year, the season therefore has an influence over the diurnal oscillation; this circumstance is sufficient to prevent us from being able to determine the mean height by taking the observations at any fixed time throughout the year for the morning mean, and at any fixed time for the evening mean. That mean reading takes place with the greatest degree of steadiness, which occurs between midday and 2^h P.M.; the actual time however varies with the season.

As we can deduce the mean temperature of the air from a few daily observations, to be spoken of presently, so also can we deduce the mean reading of the barometer from a few readings taken daily; and if this be the only element of investigation, that time or times most convenient to the observer may be chosen; but if, in addition, such observations be needed as will serve for studying the irregular oscillations of the readings of the barometer, then several observations should be taken daily, and at such times as appear in the foregoing table to require the largest corrections, both plus and minus. The greater number of observations which are taken daily, will not only serve for investigations of irregular oscillation, but the mean pressure will be more accurately determined, and they will be available in the study of the transmission of waves.

TABLE II.—Showing the corrections to be applied to the Monthly Mean reading of a thermometer placed at the height of four feet above the soil with its bulb freely exposed to the air, but in other respects protected from the influence of radiation and rain, at any hour, to deduce the true mean temperature of the air for the month from the observations taken at that hour.

Local mean time.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Midnight.	+1°·0	+1°·6	+2°·9	+4°·8	+5°·4	+6°·2	+5°·0	+5°·1	+4°·0	+2°·9	+1°·7	+0°·9
1 A.M.	+0·9	+1·8	+3·0	+5·2	+6·0	+7·1	+5·5	+5·5	+4·5	+3·0	+1·8	+1·0
2	+1·2	+2·0	+3·3	+5·7	+6·4	+8·0	+6·0	+6·0	+5·5	+3·4	+2·0	+1·2
3	+1·3	+2·1	+3·6	+6·2	+6·7	+8·7	+6·4	+6·3	+6·4	+3·6	+2·0	+1·3
4	+1·6	+2·3	+3·9	+6·6	+6·7	+9·3	+6·6	+6·5	+6·6	+3·8	+2·1	+1·4
5	+1·8	+2·2	+4·0	+6·7	+6·3	+8·8	+6·2	+6·5	+6·2	+3·8	+2·0	+1·4
6	+1·9	+2·3	+3·9	+6·0	+4·8	+6·4	+4·5	+5·5	+5·3	+3·5	+1·9	+1·4
7	+1·9	+2·1	+3·6	+4·3	+2·6	+3·0	+2·5	+3·3	+4·0	+2·8	+1·7	+1·5
8	+1·5	+1·6	+2·5	+2·0	+0·5	0·0	0·0	+0·9	+2·1	+1·6	+1·0	+1·3
9	+1·0	+0·7	+0·2	−0·9	−2·0	−2·5	−2·0	−1·6	−0·4	0·0	+0·4	+0·9
10	+0·2	−0·5	−1·9	−3·2	−4·0	−4·5	−4·0	−3·5	−3·0	−2·0	−0·6	0·0
11	−1·3	−2·1	−3·5	−5·3	−5·5	−5·8	−5·4	−5·4	−5·0	−3·8	−2·0	−1·3
Noon.	−2·3	−3·2	−5·0	−6·8	−6·7	−7·3	−6·4	−6·5	−6·4	−5·1	−3·1	−2·1
1 P.M.	−2·9	−3·9	−5·8	−7·9	−7·5	−8·1	−6·7	−7·5	−7·1	−5·5	−3·5	−2·4
2	−3·0	−3·9	−5·8	−8·2	−7·7	−8·6	−6·7	−7·7	−7·1	−4·9	−3·6	−2·3
3	−2·5	−3·6	−5·5	−7·7	−7·3	−8·4	−6·5	−7·0	−6·6	−3·7	−3·0	−1·9
4	−1·9	−2·8	−4·5	−6·7	−6·1	−7·4	−5·8	−5·5	−5·5	−2·8	−2·1	−1·3
5	−1·1	−1·6	−3·3	−5·4	−4·8	−6·1	−4·9	−3·6	−4·2	−1·7	−1·2	−0·8
6	−0·6	−0·6	−1·8	−3·5	−3·0	−4·5	−3·5	−2·0	−2·5	−0·8	−0·4	−0·4
7	−0·3	+0·3	−0·4	−1·1	−1·0	−2·4	−1·5	−0·5	−0·6	0·0	+0·1	−0·1
8	+0·1	+0·6	+0·9	+0·7	+0·9	0·0	+0·3	+1·0	+1·0	+0·7	+0·6	+0·2
9	+0·4	+1·0	+1·7	+2·0	+2·3	+1·8	+1·9	+2·4	+1·8	+1·3	+1·0	+0·4
10	+0·6	+1·3	+2·3	+3·2	+3·5	+3·6	+3·3	+3·3	+2·7	+1·9	+1·3	+0·5
11	+0·7	+1·5	+2·6	+4·1	+4·5	+5·0	+4·2	+4·3	+3·4	+2·4	+1·5	+0·8

The sign + denotes that the number is to be added, and the sign − denotes that the number is to be subtracted.

Twice during the day the temperature of the air is at its mean value, and these times are as follows in the several months:—

	h m		h m	
In January	at 10 0	A.M. and again at	8 0	P.M.
In February	at 9 30	A.M. and again at	6 40	P.M.
In March	at 9 10	A.M. and again at	7 20	P.M.
In April	at 8 40	A.M. and again at	7 0	P.M.
In May	at 8 25	A.M. and again at	7 30	P.M.
In June	at 8 0	A.M. and again at	8 0	P.M.
In July	at 8 0	A.M. and again at	8 5	P.M.
In August	at 8 20	A.M. and again at	7 20	P.M.
In September	at 8 55	A.M. and again at	7 20	P.M.
In October	at 9 0	A.M. and again at	7 0	P.M.
In November	at 9 25	A.M. and again at	6 45	P.M.
In December	at 10 0	A.M. and again at	7 20	P.M.

To determine the mean temperature of the air, it might therefore seem that it would be sufficient to take an observation at one of these two periods; but it must be borne in mind that at these times the changes of temperature are rapid, and, consequently, if the observation be made a little too soon or a little too late, very considerable errors might be committed; therefore, observations at these times, unless they are made very accurately with respect to time, are not worthy of implicit confidence.

The better way is to take observations several times during the day, and at such times that the algebraical sum of the corrections is a minimum.

The best plan, however, is to take observations at those hours which are the least liable to interruption by the avocations of the observers, and to apply to their mean results the necessary corrections. Of the hours which are equally convenient, those are preferable about which the least changes are taking place, as then a small error in the time of observation will entail little or no error in the readings.

The mean temperature of the air has hitherto been considered by most observers to be that which is intermediate to the maximum and minimum of the day; and in most places in England the mean temperature has been deduced from these two elements, by taking a simple arithmetical mean between them. The true mean in the summer months is widely different from the half of the sum of the maximum and minimum readings. The mean temperatures, of nearly all places in England, have therefore been estimated too high. At Greenwich this empirical mean has been found always to exceed the true mean; but the amount of the error is variable in the different seasons, but it is found to be the same in the same months of different years. The following table gives the correction for all the months of the year.

Table III.—Showing the corrections to be applied subtractively to the simple arithmetical mean of the maximum and minimum thermometers, to deduce from their readings the true temperature of the air.

January	$0^{\circ}\cdot2$	July	$1^{\circ}\cdot9$
February	0·4	August	1·7
March	1·0	September	1·3
April	1·5	October	1·0
May	1·7	November	0·4
June	1·8	December	0·0

We have thus two easy methods of finding the true mean temperature; first, by taking observations several times a day, and applying corrections to their means from Table II.; and, secondly, by taking the half of the maximum and minimum readings and correcting it by the numbers in Table III.

At all places the form of the diurnal variation is a single progression, having one ascending branch and one descending branch, the maximum occurring early in the afternoon and the minimum occurring at about sunrise; but the amount of the difference of these extremes is variable, depending upon latitude, elevation, locality and geological formation of the country.

If we compare the mean temperatures of places that differ considerably from each other in latitude, we shall find that the mean values are lower as we proceed north.

If we compare the mean temperatures of places having the same latitude, we shall find that the mean value of those situated at the higher level will be less than those at the lower level.

If we compare places having the same latitude, we shall find that the mean temperatures of those places situated inland will be higher in the summer months, and lower in the winter months than those situated in the vicinity of the sea.

If we compare places differing only in their geological formations, we shall find that those places situated upon an arid, dry soil, will have a greater range of temperature than those situated upon a clayey, wet soil.

It is therefore possible that the corrections in Table II. may not be of universal application, but, as the form of the curve described by the daily march is similar at all places, with the exception of being more or less bold, the turning points occurring at nearly the same local time, it is most probable that the amount of the correction applicable to any hour at any place is the same part of the whole monthly mean daily range at that place, as the correction at Greenwich is of the monthly mean daily range at Greenwich; I have therefore computed the following table upon this assumption, to be used at those places where the daily range of the temperature of the air is remarkably small or remarkably large.

Table IV.—Factors to be multiplied into the mean daily range of the reading of a thermometer placed at the height of 4 feet above the soil, with its bulb freely exposed to the air, but in other respects protected from the influence of radiation and rain, to deduce the correction to be applied to the monthly mean reading at any hour, to determine the true mean temperature of the air for the month.

Local mean time.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
Midnight.	+0·123	+0·170	+0·218	+0·286	+0·303	+0·318	+0·289	+0·298	+0·247	+0·266	+0·179	+0·106
1 A.M.	+0·111	+0·192	+0·225	+0·309	+0·337	+0·364	+0·318	+0·322	+0·278	+0·275	+0·190	+0·118
2	+0·148	+0·213	+0·248	+0·340	+0·359	+0·411	+0·347	+0·351	+0·340	+0·312	+0·211	+0·142
3	+0·152	+0·223	+0·271	+0·370	+0·376	+0·446	+0·371	+0·369	+0·395	+0·331	+0·211	+0·154
4	+0·198	+0·245	+0·293	+0·393	+0·376	+0·477	+0·382	+0·380	+0·407	+0·349	+0·222	+0·165
5	+0·222	+0·235	+0·301	+0·399	+0·354	+0·451	+0·358	+0·380	+0·383	+0·349	+0·211	+0·165
6	+0·234	+0·245	+0·293	+0·357	+0·270	+0·328	+0·260	+0·322	+0·327	+0·320	+0·201	+0·165
7	+0·234	+0·223	+0·271	+0·256	+0·146	+0·154	+0·145	+0·193	+0·247	+0·257	+0·179	+0·177
8	+0·185	+0·170	+0·188	+0·119	+0·028	0·000	0·000	+0·053	+0·130	+0·147	+0·106	+0·154
9	+0·123	+0·074	+0·015	−0·053	−0·112	−0·128	−0·116	−0·094	−0·025	0·000	+0·043	+0·106
10	+0·025	−0·053	−0·143	−0·191	−0·225	−0·231	−0·231	−0·205	−0·185	−0·184	−0·064	0·000
11	−0·152	−0·223	−0·263	−0·316	−0·309	−0·298	−0·312	−0·316	−0·309	−0·349	−0·211	−0·154
Noon.	−0·284	−0·340	−0·376	−0·405	−0·377	−0·374	−0·370	−0·380	−0·395	−0·468	−0·327	−0·248
1 P.M.	−0·358	−0·415	−0·436	−0·470	−0·421	−0·415	−0·387	−0·439	−0·438	−0·505	−0·369	−0·282
2	−0·369	−0·415	−0·436	−0·488	−0·433	−0·442	−0·387	−0·487	−0·438	−0·451	−0·379	−0·271
3	−0·309	−0·383	−0·414	−0·458	−0·410	−0·431	−0·376	−0·410	−0·408	−0·340	−0·316	−0·224
4	−0·234	−0·298	−0·338	−0·399	−0·343	−0·379	−0·335	−0·322	−0·340	−0·257	−0·222	−0·154
5	−0·136	−0·170	−0·248	−0·322	−0·270	−0·313	−0·283	−0·211	−0·260	−0·156	−0·127	−0·095
6	−0·074	−0·064	−0·135	−0·209	−0·169	−0·231	−0·202	−0·117	−0·154	−0·074	−0·043	−0·048
7	−0·037	+0·032	−0·030	−0·066	−0·056	−0·123	−0·087	−0·030	−0·037	0·000	+0·011	−0·012
8	+0·012	+0·064	+0·068	+0·042	+0·051	0·000	−0·017	+0·059	+0·062	+0·065	+0·064	+0·024
9	+0·049	+0·106	+0·128	+0·119	+0·130	+0·092	−0·110	+0·140	+0·111	+0·120	+0·106	+0·048
10	+0·074	+0·138	+0·173	+0·191	+0·197	+0·185	−0·191	+0·193	+0·167	+0·175	+0·137	+0·059
11	+0·086	+0·160	+0·195	+0·244	+0·253	+0·257	−0·243	+0·252	+0·210	+0·221	+0·158	+0·095

The sign + denotes that the number is to be added, and the sign − denotes that the number is to be subtracted.

The use of this table is very simple: I will suppose that in January the difference between the mean of the maxima readings and the mean of the minima readings, or the mean daily range be 8°, then the amount of the correction at 4^h P.M. would be

$$-0{\cdot}234\times 8^\circ = -1^\circ{\cdot}9.$$

Again, in the month of June, suppose the mean daily range to be 19°·5, then the amount of the correction at 9^h A.M. would be

$$-0{\cdot}128\times 19^\circ{\cdot}5 = -2^\circ{\cdot}5.$$

I have reason to believe that for most places the corrections in Table II. will be found to be sufficient; and although for some places we may not, by their application, obtain results that are perfectly accurate, they will, nevertheless, be nearer than those which are not corrected.

Table V.—Showing the mean depression of the temperature of evaporation below that of the air, at the height of 4 feet above the soil, at every hour in each month.

Local mean time.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
Midnight.	0°·9	0°·9	1°·1	1°·2	1°·3	1°·7	1°·7	1°·0	0°·6	0°·9	0°·9	0°·9
1 A.M.	1·0	0·9	0·9	1·0	1·0	1·5	1·3	0·9	0·7	0·7	0·9	1·0
2	1·0	0·9	0·8	0·8	0·7	1·2	1·1	0·8	0·6	0·6	0·9	0·9
3	1·0	0·8	0·7	0·7	0·6	1·0	0·8	0·6	0·4	0·6	0·9	0·9
4	1·0	0·8	0·7	0·6	0·6	0·9	0·6	0·5	0·3	0·5	0·8	0·9
5	1·0	0·9	0·7	0·7	0·8	0·9	0·8	0·7	0·3	0·4	0·8	0·8
6	1·0	0·9	0·7	1·0	1·0	1·5	1·4	1·0	0·3	0·4	0·8	0·7
7	1·0	0·9	0·9	1·5	1·5	2·6	2·1	1·4	0·5	0·6	0·7	0·8
8	1·0	1·0	1·4	2·1	2·3	3·7	3·0	2·3	1·2	0·7	0·8	0·9
9	1·0	1·3	2·0	3·0	3·2	4·8	4·0	3·4	2·0	1·2	1·1	1·0
10	1·0	1·5	2·5	4·0	3·9	5·8	5·0	4·4	2·9	1·9	1·5	1·3
11	1·1	1·8	2·9	4·8	4·6	6·8	5·8	5·3	3·8	2·7	1·8	1·8
Noon.	1·4	2·0	3·3	5·6	5·4	7·7	6·5	6·1	4·5	3·3	2·1	2·0
1 P.M.	1·6	2·2	3·7	6·2	6·0	8·3	6·7	6·6	5·1	3·7	2·3	2·1
2	1·8	2·3	3·8	6·5	6·3	8·6	6·7	6·7	5·6	3·8	2·3	2·1
3	1·9	2·5	3·8	6·3	6·2	8·2	6·7	6·6	5·9	3·7	2·0	1·9
4	1·9	2·0	3·6	5·9	5·6	7·1	6·1	6·0	5·3	3·5	1·8	1·6
5	1·7	1·5	3·3	5·5	5·1	6·9	5·9	5·8	4·2	2·7	1·6	1·5
6	1·6	1·2	2·7	4·8	4·5	6·2	5·2	5·0	3·0	2·0	1·4	1·1
7	1·3	1·0	2·0	3·8	3·5	5·3	4·3	4·3	2·1	1·8	1·1	1·0
8	1·0	0·9	1·6	2·9	2·8	4·4	3·5	3·5	1·5	1·4	1·0	0·9
9	1·0	0·8	1·4	2·3	2·3	3·3	2·8	2·5	1·0	1·2	1·0	1·0
10	0·9	0·8	1·2	1·7	1·7	2·5	2·3	1·7	0·7	1·0	0·9	1·0
11	0·9	0·8	1·1	1·4	1·4	2·0	1·9	1·3	0·5	0·8	0·9	1·0

The times at which the least and greatest differences, between the readings of the dry- and wet-bulb thermometers, with the amounts of the differences in every month, were as follows :—

Jan.	{The least difference}	was 0°·8	{And it took place}	at 11h 20m P.M. ;	{The greatest difference}	was 1°·8	{And it took place} from 2h P.M. to 4h P.M.
Feb.	„	{was 0°·8 or 0°·9}	„	during the night hours ;	„	was 2°·5	„ at 3h P.M.
Mar.	„	was 0°·7	„	from 3h A.M. to 6h A.M. ;	„	was 3°·8	„ from 1h 40m P.M. to 3h 0m P.M.
April	„	was 0°·6	„	at 4h A.M. ;	„	was 6°·5	„ from 2h 0m P.M. to 2h 50m P.M.
May	„	was 0°·6	„	from 3h A.M. to 4h A.M. ;	„	was 6°·3	„ from 2h 0m P.M. to 2h 40m P.M.
June	„	was 0°·9	„	from 4h A.M. to 5h A.M. ;	„	was 8°·6	„ from 1h 40m P.M. to 2h 10m P.M.
July	„	was 0°·6	„	from 4h 0m A.M. to 4h 40m A.M. ;		was 6°·7	„ from 0h 40m P.M. to 3h 0m P.M.
Aug.	„	was 0°·5	„	from 3h 40m A.M. to 5h 45m A.M. ;		was 6°·7	„ from 1h 10m P.M. to 2h 50m P.M.
Sept.	„	was 0°·2	„	at 5h 20m A.M. ;	„	was 5°·9	„ at 3h P.M.
Oct.	„	was 0°·4	„	from 5h A.M. to 6h 20m A.M. ;		was 3°·8	„ from 1h 40m P.M. to 2h 50m P.M.
Nov.	„	{was 0°·8 or 0°·9}	„	at all night hours ;	„	was 2°·3	„ from 1h 0m P.M. to 2h 10m P.M.
Dec.	„	was 0°·9	„	at all night hours ;	„	was 2°·1	„ from 1h 0m P.M. to 2h 0m P.M.

The times of least difference in the year are the morning hours in September, and the times of greatest difference are from 1h P.M. to 3h P.M. in June.

Table VI.—Showing the mean depression of the temperature of the dew-point below that of the air at the height of four feet above the soil at every hour in each month.

Local mean time.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Midnight.	2°·5	2°·5	2°·8	2°·8	2°·5	3°·1	2°·8	1°·9	1°·5	1°·7	2°·0	2°·6
1 A.M.	2·8	2·6	2·5	2·4	2·2	2·6	2·2	1·4	1·2	1·4	2·1	2·6
2	2·9	2·6	2·5	2·0	1·9	2·4	1·8	1·2	1·0	1·2	2·1	2·6
3	2·7	2·4	2·4	1·7	1·6	2·0	1·6	1·0	0·8	1·0	2·0	2·5
4	2·6	2·1	2·3	1·6	1·6	1·7	1·5	0·9	0·6	0·9	1·9	2·1
5	2·3	2·0	2·3	1·7	1·6	1·8	1·7	1·0	0·7	0·9	1·8	2·0
6	2·2	2·0	2·4	2·5	2·1	2·7	3·0	1·7	1·0	1·3	1·7	2·0
7	2·1	2·2	2·8	3·7	3·6	5·1	5·1	3·0	1·9	1·9	1·7	2·0
8	2·1	2·3	3·7	4·6	4·5	6·3	6·0	4·2	2·5	2·4	1·8	2·1
9	2·2	2·8	4·7	6·1	5·8	8·1	7·5	5·7	3·7	3·3	2·3	2·3
10	2·5	3·6	5·6	7·8	7·1	9·8	8·8	7·3	5·0	4·1	2·7	2·8
11	3·1	4·3	6·6	9·4	8·5	11·1	9·9	8·8	6·6	5·3	3·1	3·4
Noon.	3·7	4·7	7·5	10·6	9·5	12·1	10·6	9·9	7·9	6·4	3·7	3·7
1 P.M.	3·9	5·2	8·1	11·5	10·3	12·9	10·8	10·7	8·6	7·1	4·3	4·1
2	4·4	5·3	8·6	12·1	10·6	13·3	11·1	10·9	8·9	7·5	4·6	4·2
3	4·6	5·2	8·5	12·0	10·3	13·0	11·0	10·3	8·7	7·3	4·4	4·3
4	4·5	4·8	8·1	11·0	9·5	12·1	10·3	9·7	7·9	6·4	3·8	4·0
5	3·6	4·0	7·5	10·1	8·7	11·1	9·5	8·9	6·7	5·3	3·3	3·7
6	3·1	3·1	6·5	9·0	7·6	10·0	8·5	7·8	5·5	4·4	2·8	3·4
7	2·3	2·5	5·0	6·8	6·4	9·0	7·1	6·3	4·3	3·6	2·6	3·1
8	2·1	2·3	3·9	5·5	5·3	7·5	6·0	4·8	3·1	2·9	2·4	2·8
9	2·0	2·1	3·3	5·0	4·3	6·0	4·8	3·5	2·4	2·4	2·1	2·7
10	2·0	2·0	3·0	4·1	3·6	4·7	3·7	2·6	1·7	2·0	2·0	2·6
11	2·1	2·1	2·6	3·1	2·7	3·5	3·1	1·8	1·3	1·7	1·9	2·4

The times at which the least and greatest excess of air-temperature above dew-point temperature, with the amounts of the differences, were as follows:—

Jan.	The least difference	was 2°·1	And it took place	at 11^h 20^m P.M., and from 6^h A.M. to 8^h A.M.	The greatest difference was 4°·5	And it took place at 3^h 20^m P.M.
Feb.	„	was 2°·0	„	from 5^h 20^m A.M. to 7^h 0^m A.M.;	was 5°·3	„ at 1^h 40^m P.M. to 2^h 40^m P.M.
Mar.	„	was 2°·4	„	from 4^h 0^m A.M. to 5^h 40^m A.M.;	was 8°·5	„ at 2^h 40^m P.M.
April	„	was 1°·6	„	at 4^h 20^m A.M.;	„ was 12°·1	„ at 2^h 20^m P.M.
May	„	was 1°·5	„	at 5^h 0^m A.M.;	„ was 10°·5	„ at 1^h 40^m P.M.
June	„	was 1°·8	„	at 4^h 40^m A.M.;	„ was 13°·3	„ at 1^h 40^m P.M.
July	„	was 1°·5	„	from 3^h 40^m A.M. to 4^h 40^m A.M.;	was 11°·0	„ from 1^h 40^m P.M. to 3^h 0^m P.M.
Aug.	„	was 0°·8	„	from 3^h 40^m A.M. to 4^h 20^m A.M.;	was 10°·8	„ at 1^h 20^m P.M.
Sept.	„	was 0°·6	„	at 4^h 40^m A.M.;	„ was 8°·9	„ at 1^h 20^m P.M.
Oct.	„	was 0°·9	„	at 4^h 20^m A.M.;	„ was 7°·5	„ at 2^h 0^m P.M.
Nov.	„	was 1°·6	„	from 6^h 20^m A.M. to 7^h 20^m A.M.;	was 4°·6	„ from 1^h 40^m P.M. to 2^h 20^m P.M.
Dec.	„	was 2°·0	„	from 4^h 20^m A.M. to 7^h 20^m A.M.;	was 4°·3	„ from 1^h 40^m P.M. to 3^h 0^m P.M.

The time at which the temperatures of the air and of the dew-point were most nearly alike, was in September at 4^h 40^m A.M., and the time at which the greatest difference took place between these temperatures, was in June at 1^h 40^m P.M. The times of the least difference are at about the time of sunrise at all seasons of the year, and the times of the greatest difference are at about the time of the maximum temperature of the air at all periods.

Table VII.—Showing the corrections to be applied to the monthly mean readings of the wet-bulb thermometer placed at the height of four feet above the soil at any hour, to deduce the true mean temperature of evaporation for the month from the observations taken at that hour.

Local mean time.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
Midnight.	+0°·7	+1°·2	+2°·1	+2°·9	+3°·8	+3°·5	+3°·1	+2°·6	+2°·2	+1°·9	+1°·3	+0°·5
1 A.M.	+0·7	+1·4	+2·0	+3·1	+3·1	+4·2	+3·2	+2·9	+2·8	+1·8	+1·4	+ 07
2	+1·0	+1·6	+2·2	+3·4	+4·2	+4·8	+3·5	+3·3	+3·7	+2·1	+1·6	+0·8
3	+1·1	+1·6	+2·4	+3·8	+4·4	+5·3	+3·6	+3·4	+4·1	+2·3	+1·6	+0·9
4	+1·4	+1·8	+2·7	+4·1	+4·4	+5·8	+3·6	+3·5	+4·5	+2·4	+1·6	+1·0
5	+1·6	+1·8	+2·8	+4·3	+4·2	+5·3	+3·4	+3·7	+4·1	+2·3	+1·5	+0·9
6	+1·7	+1·9	+2·7	+3·9	+2·9	+3·5	+2·3	+3·0	+3·2	+2·0	+1·4	+0·8
7	+1·7	+1·7	+2·6	+2·7	+1·2	+1·2	+1·0	+1·2	+2·1	+1·5	+1·1	+1·0
8	+1·3	+1·3	+2·0	+1·0	−0·1	−0·7	−0·6	−0·3	+0·9	+0·4	+0·5	+0·9
9	+0·8	+0·7	+0·3	−1·0	−3·7	−2·1	−1·6	−1·7	−0·8	−0·7	+0·2	+0·6
10	0·0	−0·3	−1·3	−2·3	−3·0	−3·1	−2·6	−2·6	−2·5	−2·0	−0·4	0·0
11	−1·4	−1·6	−2·5	−3·6	−3·8	−3·4	−3·2	−3·6	−3·6	−3·0	−1·5	−0·8
Noon.	−2·1	−2·5	−3·6	−4·3	−4·2	−4·0	−3·5	−3·9	−4·3	−3·7	−2·3	−1·4
1 P.M.	−2·5	−3·0	−4·0	−4·8	−4·4	−4·2	−3·6	−4·3	−4·4	−3·7	−2·5	−1·6
2	−2·4	−2·9	−3·9	−4·8	−4·3	−4·4	−3·6	−4·5	−3·9	−3·0	−2·6	−1·5
3	−1·8	−2·4	−3·6	−4·5	−4·0	−4·6	−3·4	−3·9	−3·1	−1·9	−2·3	−1·3
4	−1·2	−2·1	−2·8	−3·9	−4·4	−4·7	−3·3	−3·0	−2·6	−1·2	−1·6	−1·0
5	−0·6	−1·4	−1·9	−3·0	−2·6	−3·6	−2·6	−1·3	−2·4	−0·9	−0·9	−0·6
6	−0·2	−0·7	−1·0	−1·8	−1·4	−2·7	−1·9	−0·5	−1·9	−0·7	−0·3	−0·6
7	−0·2	0·0	−0·3	−0·4	−0·4	−1·5	−0·8	+0·3	−0·9	−0·1	−0·1	−0·4
8	−0·1	+0·2	+0·6	+0·5	+0·8	0·0	+0·2	+1·0	+0·1	+1·2	+0·3	−0·2
9	+0·2	+0·5	+1·2	+1·2	+1·4	+0·7	+1·1	+1·4	+0·4	+0·6	+0·7	+0·1
10	+0·3	+0·8	+1·6	+1·8	+2·3	+1·7	+2·0	+1·5	+1·0	+1·0	+0·9	+0·2
11	+0·4	+1·0	+1·8	+2·4	+3·0	+2·6	+2·5	+2·1	+1·5	+1·3	+1·1	+0·5

Table VIII.—Showing the corrections to be applied to the monthly mean reading of the temperature of the dew-point at the height of four feet above the soil at any hour, to deduce the true mean temperature of the dew-point for the month from the observations taken at that hour.

Local mean time.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Midnight.	+0°·6	+0°·8	+0°·9	+1°·4	+2°·5	+2°·2	+1°·7	+1°·5	+1°·4	+1°·0	+1°·1	+0°·5
1 A.M.	+0·8	+1·1	+0·7	+1·4	+2·8	+2·6	+1·6	+1·4	+1·6	+0·8	+1·3	+0·6
2	+1·2	+1·3	+1·0	+1·5	+2·9	+3·3	+1·7	+1·7	+2·4	+1·0	+1·5	+0·8
3	+1·1	+2·2	+1·2	+1·7	+2·9	+3·6	+1·9	+1·8	+3·1	+1·0	+1·4	+0·8
4	+1·3	+2·1	+1·4	+2·0	+2·9	+3·9	+2·0	+1·9	+3·1	+1·1	+1·4	+0·5
5	+1·2	+0·9	+1·5	+2·2	+2·5	+3·5	+1·8	+2·0	+2·8	+1·1	+1·2	+0·4
6	+1·2	+1·0	+1·5	+2·3	+1·5	+2·0	+1·4	+1·7	+2·2	+1·2	+1·0	+0·4
7	+1·1	+1·0	+1·6	+1·8	+0·8	+1·0	+1·5	+0·8	+1·8	+1·1	+0·8	+0·5
8	+0·7	+0·6	+1·4	+0·4	−0·4	−0·8	−0·1	−0·2	+0·5	+0·4	+0·2	+0·4
9	+0·3	+0·2	+0·1	−1·0	−1·6	−1·5	−0·6	−1·4	−0·8	−0·3	+0·1	+0·2
10	−0·2	−0·2	−1·1	−1·6	−2·3	−1·8	−1·3	−1·7	−2·1	−1·5	−0·5	−0·2
11	−1·1	−1·1	−1·7	−2·1	−2·4	−1·8	−1·6	−2·1	−2·5	−2·1	−1·5	−0·9
Noon.	−1·5	−1·8	−2·3	−2·4	−2·6	−2·3	−1·9	−2·1	−2·6	−2·3	−2·0	−1·4
1 P.M.	−1·9	−2·0	−2·5	−2·6	−2·6	−2·3	−2·0	−2·3	−2·6	−2·0	−1·8	−1·3
2	−1·5	−1·7	−2·0	−2·3	−2·5	−2·4	−1·7	−2·3	−2·3	−1·0	−1·6	−1·1
3	−0·8	−1·7	−1·8	−1·9	−2·4	−2·5	−1·6	−2·2	−2·0	0·0	−1·2	−0·6
4	−0·3	−1·3	−1·2	−1·9	−2·0	−2·4	−1·6	−1·3	−1·7	0·0	−0·9	−0·3
5	−0·4	−0·9	−0·6	−1·5	−1·5	−2·1	−1·5	−0·2	−1·6	0·0	−0·5	−0·1
6	−0·4	−0·8	−0·1	−0·7	−0·8	−1·6	−1·1	+0·3	−1·1	0·0	−0·2	0·0
7	−0·9	−0·5	−0·2	0·0	0·0	−0·5	−0·5	+0·3	−0·4	0·0	+0·1	0·0
8	−0·7	−0·4	0·0	0·0	+0·8	+0·4	+0·2	+0·3	0·0	0·0	+0·4	0·0
9	−0·5	−0·2	+0·2	+0·8	+0·9	+0·7	+0·6	+0·4	+0·1	+0·1	+0·5	+0·1
10	−0·3	0·0	+0·5	+1·1	+1·7	+1·2	+0·9	+0·4	+0·3	+0·3	+0·7	+0·1
11	−0·1	+0·3	+0·4	+1·0	+1·8	+1·4	+1·2	+0·6	+0·6	+0·5	+0·8	+0·2

The sign + denotes that the number is to be added, and the sign − denotes that the number is to be subtracted.

In every month the largest number in this table to which the plus sign is affixed, indicates the time of minimum temperature of the dew-point; and the largest number to which the minus sign is affixed indicates the time of maximum temperature of the dew-point. The laws which we may deduce from the numbers in this table are as follows:—

At Greenwich the temperature of the dew-point attains its minimum in the first two months, and in the last two months of the year, some hours before the time of sunrise; as the sun rises above the horizon and approaches the meridian, evaporation increases, and the air constantly receives a greater quantity of vapour, consequently the temperature of the dew-point increases till, at about the time of the maximum temperature of the air, the maximum temperature of the dew-point takes place. In summer time the minimum temperature of the dew-point attains its minimum a little before sunrise, and its maximum at about noon. In winter time, after having attained its maximum, the temperature of the dew-point decreases very regularly till next morning. In summer time the value remains very nearly at its maximum value till after the temperature of the air begins to decline; it then very regularly decreases as before, till the following morning.

The fact of the almost stationary temperature of the dew-point during the early afternoon hours in summer is important, and will be referred to again presently.

Table IX.—Showing the corrections to be applied to the monthly mean elastic force of vapour at the height of four feet above the soil at any hour, to deduce the true mean elastic force of vapour for the month from the observations taken at that hour.

Local mean time.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
Midnight.	+0·006	+0·006	+0·008	+0·017	+0·026	+0·031	+0·028	+0·025	+0·024	+0·018	+0·010	+0·009
1 A.M.	+0·011	+0·008	+0·010	+0·021	+0·028	+0·037	+0·031	+0·031	+0·030	+0·020	+0·012	+0·010
2	+0·015	+0·010	+0·011	+0·024	+0·031	+0·043	+0·036	+0·035	+0·035	+0·021	+0·015	+0·010
3	+0·015	+0·011	+0·013	+0·027	+0·032	+0·048	+0·038	+0·039	+0·037	+0·023	+0·017	+0·011
4	+0·015	+0·013	+0·015	+0·029	+0·031	+0·047	+0·037	+0·040	+0·040	+0·025	+0·019	+0·011
5	+0·015	+0·014	+0·016	+0·029	+0·027	+0·037	+0·031	+0·038	+0·040	+0·023	+0·021	+0·011
6	+0·014	+0·015	+0·016	+0·025	+0·019	+0·022	+0·019	+0·029	+0·033	+0·021	+0·021	+0·010
7	+0·013	+0·014	+0·014	+0·016	+0·007	+0·008	+0·007	+0·014	+0·022	+0·018	+0·018	+0·009
8	+0·010	+0·010	+0·010	+0·005	−0·005	−0·004	−0·004	0·000	+0·010	+0·011	+0·012	+0·007
9	+0·007	+0·006	+0·005	+0·005	−0·016	−0·015	−0·014	−0·012	−0·005	+0·005	+0·005	+0·005
10	+0·002	0·000	−0·003	−0·013	−0·024	−0·027	−0·019	−0·021	−0·019	−0·005	−0·004	+0·001
11	−0·004	−0·005	−0·007	−0·020	−0·028	−0·036	−0·025	−0·027	−0·027	−0·009	−0·010	−0·004
Noon.	−0·007	−0·009	−0·012	−0·026	−0·030	−0·042	−0·029	−0·030	−0·030	−0·015	−0·017	−0·007
1 P.M.	−0·008	−0·013	−0·013	−0·027	−0·030	−0·045	−0·033	−0·032	−0·030	−0·018	−0·019	−0·008
2	−0·007	−0·015	−0·013	−0·027	−0·028	−0·043	−0·034	−0·034	−0·029	−0·017	−0·020	−0·008
3	−0·007	−0·012	−0·012	−0·025	−0·026	−0·039	−0·033	−0·031	−0·027	−0·014	−0·016	−0·008
4	−0·007	−0·010	−0·010	−0·020	−0·021	−0·035	−0·028	−0·027	−0·021	−0·009	−0·010	−0·007
5	−0·004	−0·006	−0·006	−0·014	−0·015	−0·025	−0·021	−0·020	−0·017	−0·006	−0·005	−0·005
6	−0·002	−0·004	−0·002	−0·006	−0·010	−0·017	−0·016	−0·015	−0·010	−0·004	0·000	−0·003
7	−0·001	−0·001	+0·002	+0·001	−0·004	−0·007	−0·007	−0·006	−0·003	+0·003	+0·004	−0·001
8	0·000	+0·001	+0·004	+0·005	+0·005	+0·005	+0·004	+0·004	+0·004	+0·005	+0·006	+0·001
9	0·000	+0·003	+0·005	+0·007	+0·013	+0·015	+0·010	+0·010	+0·008	+0·008	+0·008	+0·004
10	+0·001	+0·004	+0·007	+0·010	+0·017	+0·023	+0·017	+0·015	+0·013	+0·011	+0·009	+0·005
11	+0·002	+0·005	+0·008	+0·014	+0·022	+0·029	+0·024	+0·020	+0·018	+0·014	+0·010	+0·006

The sign + denotes that the number is to be added, and the sign − denotes that the number is to be subtracted.

The remarks following Table VIII. apply equally well to this table; I may add, however, that as the temperature increases from the time of sunrise more rapidly than the amount of water necessary to keep the air at the same degree of humidity evaporates, the atmosphere becomes farther and farther removed from a state of saturation; and this is particularly the case in summer time, between the hour of noon and the time of the highest temperature of the air; for, as the temperature increases all this time, whilst the amount of water is stationary or becomes less, it follows that the air is more and more removed from the point of saturation, as is shown by the increasing numbers with a plus sign between these times in Table XI.

TABLE X.—Showing the corrections to be applied to the monthly mean value of the water contained within a cubic foot of air at the height of four feet above the soil at any hour, to deduce the true mean value of the month from the observations taken at that hour.

Local mean time.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.
Midnight.	0·0	+0·1	+0·1	+0·1	+0·2	+0·3	+0·3	+0·2	+0·2	+0·2	+0·1	+0·1
1 A.M.	+0·1	+0·1	+0·1	+0·2	+0·3	+0·4	+0·3	+0·3	+0·3	+0·2	+0·1	+0·1
2	+0·1	+0·1	+0·1	+0·2	+0·3	+0·5	+0·4	+0·3	+0·4	+0·2	+0·1	+0·1
3	+0·1	+0·1	+0·1	+0·2	+0·3	+0·5	+0·4	+0·3	+0·4	+0·2	+0·1	+0·1
4	+0·1	+0·1	+0·1	+0·2	+0·3	+0·4	+0·3	+0·4	+0·3	+0·2	+0·1	+0·1
5	+0·1	+0·1	+0·2	+0·2	+0·2	+0·3	+0·2	+0·3	+0·3	+0·1	+0·1	+0·1
6	+0·1	+0·1	+0·2	+0·2	+0·2	+0·1	+0·1	+0·2	+0·1	+0·1	+0·1	+0·1
7	+0·1	+0·1	+0·1	+0·1	+0·1	−0·1	0·0	+0·1	+0·1	+0·1	+0·1	+0·1
8	+0·1	+0·1	+0·1	0·0	0·0	−0·1	−0·1	0·0	0·0	0·0	0·0	0·0
9	0·0	0·0	0·0	−0·1	−0·1	−0·2	−0·1	−0·1	−0·1	0·0	0·0	0·0
10	0·0	0·0	0·0	−0·1	−0·2	−0·2	−0·2	−0·2	−0·2	−0·1	−0·1	−0·1
11	0·0	−0·1	−0·1	−0·2	−0·2	−0·3	−0·2	−0·2	−0·2	−0·2	−0·2	−0·1
Noon.	−0·1	−0·1	−0·1	−0·2	−0·2	−0·4	−0·2	−0·3	−0·3	−0·2	−0·2	−0·2
1 P.M.	−0·1	−0·1	−0·1	−0·2	−0·2	−0·4	−0·2	−0·3	−0·3	−0·2	−0·2	−0·2
2	−0·1	−0·1	−0·1	−0·2	−0·3	−0·5	−0·3	−0·4	−0·4	−0·2	−0·2	−0·2
3	−0·1	−0·1	−0·1	−0·1	−0·2	−0·4	−0·3	−0·3	−0·3	−0·1	−0·2	−0·2
4	−0·1	−0·1	−0·1	−0·1	−0·2	−0·3	−0·2	−0·3	−0·3	−0·1	−0·1	−0·1
5	−0·1	−0·1	0·0	−0·1	−0·2	−0·2	−0·2	−0·2	−0·2	−0·1	−0·1	−0·1
6	−0·1	0·0	0·0	−0·1	−0·1	−0·2	−0·1	−0·1	−0·1	−0·1	0·0	−0·1
7	−0·1	0·0	0·0	−0·1	0·0	−0·1	0·0	0·0	0·0	−0·1	+0·1	0·0
8	−0·1	0·0	+0·1	−0·1	+0·1	+0·1	+0·1	+0·1	+0·1	−0·1	+0·1	0·0
9	0·0	0·0	+0·1	+0·1	+0·1	+0·2	+0·1	+0·1	+0·1	+0·1	+0·1	0·0
10	0·0	0·0	+0·1	+0·1	+0·2	+0·2	+0·2	+0·1	+0·1	+0·1	+0·1	0·0
11	0·0	+0·1	+0·1	+0·1	+0·2	+0·2	+0·2	+0·2	+0·2	+0·1	+0·1	0·0

The sign + denotes that the number is to be added, and the sign — denotes that the number is to be subtracted.

TABLE XI.—Showing the corrections to be applied to the monthly mean value of the degree of humidity at the height of four feet above the soil at any hour, to deduce the true degree of humidity for the month from the observations at that hour.

Local mean time.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
Midnight.	−0·013	−0·021	−0·063	−0·095	−0·087	−0·105	−0·091	−0·096	−0·080	−0·053	−0·018	−0·011
1 A.M.	+0·002	−0·021	−0·065	−0·106	−0·100	−0·114	−0·095	−0·104	−0·080	−0·059	−0·009	−0·012
2	+0·004	−0·026	−0·066	−0·116	−0·108	−0·125	−0·107	−0·113	−0·085	−0·066	−0·011	−0·017
3	−0·003	−0·033	−0·067	−0·123	−0·113	−0·132	−0·116	−0·117	−0·091	−0·070	−0·020	−0·019
4	−0·013	−0·036	−0·068	−0·126	−0·114	−0·138	−0·120	−0·123	−0·097	−0·075	−0·030	−0·024
5	−0·019	−0·035	−0·066	−0·125	−0·106	−0·139	−0·120	−0·123	−0·098	−0·077	−0·030	−0·024
6	−0·021	−0·034	−0·063	−0·112	−0·085	−0·107	−0·097	−0·107	−0·097	−0·071	−0·033	−0·026
7	−0·020	−0·030	−0·055	−0·080	−0·059	−0·065	−0·055	−0·061	−0·080	−0·058	−0·031	−0·025
8	−0·020	−0·020	−0·035	−0·065	−0·024	−0·015	−0·005	−0·020	−0·047	−0·037	−0·021	−0·018
9	−0·017	−0·007	−0·003	−0·034	+0·018	+0·035	+0·041	+0·030	0·000	−0·009	−0·008	−0·007
10	−0·004	+0·009	+0·031	−0·015	+0·051	+0·078	+0·080	+0·070	+0·042	+0·025	+0·008	+0·008
11	+0·011	+0·028	+0·060	+0·022	+0·083	+0·100	+0·104	+0·102	+0·082	+0·060	+0·027	+0·022
Noon.	+0·031	+0·045	+0·084	+0·070	+0·110	+0·123	+0·114	+0·127	+0·115	+0·088	+0·040	+0·033
1 P.M.	+0·054	+0·058	+0·100	+0·132	+0·126	+0·137	+0·119	+0·142	+0·131	+0·109	+0·050	+0·046
2	+0·059	+0·065	+0·106	+0·151	+0·125	+0·135	+0·123	+0·145	+0·132	+0·113	+0·054	+0·048
3	+0·048	+0·065	+0·104	+0·147	+0·118	+0·123	+0·121	+0·138	+0·126	+0·108	+0·047	+0·036
4	+0·036	+0·053	+0·087	+0·128	+0·108	+0·113	+0·111	+0·120	+0·103	+0·089	+0·032	+0·024
5	+0·021	+0·032	+0·063	+0·110	+0·091	+0·099	+0·095	+0·100	+0·071	+0·055	+0·018	+0·013
6	+0·007	+0·009	+0·038	+0·088	+0·074	+0·078	+0·062	+0·071	+0·044	+0·030	+0·005	+0·004
7	−0·005	−0·010	+0·010	+0·059	+0·052	+0·049	+0·025	+0·036	+0·009	+0·007	−0·005	−0·003
8	−0·014	−0·023	−0·010	+0·020	+0·022	+0·010	−0·015	0·000	−0·015	−0·011	−0·012	−0·005
9	−0·016	−0·029	−0·032	−0·030	−0·018	−0·025	−0·040	−0·038	−0·040	−0·025	−0·017	−0·007
10	−0·019	−0·030	−0·048	−0·058	−0·050	−0·060	−0·068	−0·067	−0·058	−0·039	−0·020	−0·008
11	−0·018	−0·036	−0·060	−0·080	−0·075	−0·085	−0·080	−0·085	−0·071	−0·048	−0·020	−0·009

The sign + denotes that the number is to be added, and the sign — denotes that the number is to be subtracted.

When evaporation commences in the morning with the increase of temperature, the vapour seems to accumulate on the surface of the soil, till the air in its vicinity becomes heated, and the daily ascending current of air commences. It seems likely that this stratum of vapour neither attains a great thickness nor spreads upwards till the ascending current takes place, and that it ascends and spreads as long as the ascending current continues.

In summer time, between the hour of noon and that of the maximum temperature of the air, the temperature of the dew-point remains nearly stationary*, and the degree of humidity becomes less in value notwithstanding that evaporation is the most rapid. It is evident, therefore, that all the water evaporated must at once pass upwards. The strength of the ascending current of air being at a maximum at the same time that the stream of vapour is the most rapid, it would seem that the rapidity of motion of the latter is dependent on the former. Towards the evening hours, when the temperature of the air is decreasing rapidly, the ascending current will decrease in force, and ultimately cease altogether, giving place to the descending current of night; then the vapour again accumulates on the surface of the earth, not only from evaporation, but also from the vapour flowing down with the descending current, the air more and more nearly approaches to a state of saturation, and the degree of humidity of the air arrives at its maximum.

* See remarks following Table VIII.

Therefore, there is a rapid increase of vapour and decrease of the degree of humidity during the day, and a rapid diminution of vapour and increase of the degree of humidity during the evening and night hours. These remarks are to be understood to apply to a point at the height of four feet only from the earth.

At present I cannot give any information relative to the distribution of vapour at distances greater than four feet. However, from the fact, that whilst evaporation is the most rapid, the air at the height of four feet becomes relatively drier than it would be in consequence of the increase of temperature alone, it is plain that much vapour must pass upwards.

In the higher strata of the atmosphere the changes of temperature are less than in the lower strata; and at a point not very distant from the earth, but varying in different seasons, the temperature must be stationary during the twenty-four hours of the day; we may readily infer that at a certain point above the earth, the air becomes relatively more moist during the course of the day whilst the ascending current continues, and less so at night whilst the vapour flows downwards with the descending current; being exactly the reverse of the facts which take place at the height of four feet. At some certain point above the earth, the temperature of the dew-point, like that of the air, must be stationary during the twenty-four hours of the day. Observations to determine the absolute quantity of water mixed with the air, and the relative humidity at different distances from the earth, are much needed.

Table XII.—Showing the corrections to be applied to the weight of a cubic foot of air, under the average temperature, humidity and pressure, at any hour, to deduce the true weight for the month from the observations taken at that hour.

Local mean time.	January.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.	gr.
Midnight.	−0·5	−1·7	−2·7	−5·3	−5·3	− 6·1	−5·1	−5·1	−4·2	−2·9	−1·8	−1·0
1 A.M.	−1·1	−2·0	−3·0	−6·0	−5·9	− 7·7	−5·8	−6·2	−5·2	−3·1	−1·7	−1·4
2	−1·6	−2·0	−3·2	−6·6	−6·5	− 9·0	−6·4	−6·5	−5·6	−3·5	−1·8	−1·5
3	−1·8	−2·2	−3·5	−7·1	−7·1	−10·0	−6·8	−6·8	−6·0	−3·8	−1·8	−1·7
4	−2·0	−2·3	−3·7	−7·6	−7·1	−10·1	−7·0	−7·0	−6·2	−4·0	−2·0	−1·6
5	−2·0	−2·5	−3·7	−7·7	−6·9	− 8·9	−6·7	−6·9	−6·0	−4·0	−1·9	−1·6
6	−2·0	−2·5	−3·7	−6·5	−5·5	− 6·5	−5·0	−5·9	−4·9	−3·8	−2·0	−1·6
7	−1·9	−2·5	−3·4	−4·5	−3·0	− 3·5	−2·7	−3·9	−3·0	−3·4	−2·0	−1·6
8	−1·7	−2·0	−2·5	−2·3	−0·7	− 0·0	−0·3	−1·0	−0·5	−2·5	−1·6	−1·5
9	−1·1	−0·9	−0·4	+0·3	+1·7	+ 2·4	+1·9	+1·5	+2·4	−1·0	−0·7	−0·9
10	−0·2	+0·4	+1·7	+2·6	+3·8	+ 4·5	+4·0	+3·7	+4·7	+1·5	+0·6	+0·1
11	+1·3	+1·8	+3·6	+5·4	+5·7	+ 6·2	+5·8	+5·9	+6·4	+4·0	+2·0	+1·5
Noon.	+2·4	+3·3	+5·2	+7·8	+7·2	+ 7·7	+7·3	+7·3	+7·5	+5·8	+3·5	+2·6
1 P.M.	+3·1	+4·4	+6·1	+9·3	+7·9	+ 8·7	+8·3	+8·1	+8·0	+6·6	+4·3	+3·1
2	+3·3	+5·2	+6·1	+9·2	+8·3	+ 8·8	+8·4	+8·3	+7·8	+6·6	+4·3	+3·1
3	+3·0	+5·0	+5·5	+8·7	+8·2	+ 8·8	+8·3	+8·0	+6·9	+6·1	+3·8	+2·7
4	+2·4	+3·8	+4·5	+7·5	+7·3	+ 8·2	+7·4	+6·8	+5·5	+4·9	+2·7	+1·7
5	+1·6	+2·5	+3·2	+5·8	+5·5	+ 6·8	+5·7	+5·1	+3·8	+3·3	+1·5	+1·1
6	+0·8	+1·2	+1·7	+4·0	+3·5	+ 4·5	+3·0	+3·0	+1·8	+1·7	+0·6	+0·6
7	+0·4	+0·4	0·0	+1·8	+1·1	+ 1·5	+1·8	+1·0	−0·2	+0·4	−0·3	+0·2
8	+0·1	−0·4	−1·1	−0·5	−1·0	− 0·5	0·0	−1·0	−1·5	−0·7	−1·0	−0·2
9	0·0	−1·2	−1·7	−2·3	−2·7	− 2·5	−1·8	−2·8	−2·4	−1·5	−1·4	−0·5
10	−0·1	−1·5	−2·4	−3·6	−4·3	− 4·0	−3·3	−4·0	−3·3	−2·0	−1·6	−0·7
11	−0·4	−1·9	−2·8	−4·7	−5·2	− 5·0	−4·4	−5·0	−4·1	−2·7	−1·8	−0·9

The sign + denotes that the number is to be added, and the sign − denotes that the number is to be subtracted.

It was a matter of considerable interest to determine the extent of country to which these corrections would apply, and to this purpose I have had recourse to the observations made at different parts of the country and furnished to the Registrar-General. Some of the results of this investigation I have already mentioned in the remarks following Table III. The general result was found to be, that for all places situated inland, the values contained in these tables may be adopted at once. For places situated near the sea the hygrometrical values may not be strictly true, but in the absence of any series of observations taken in these localities from which the corrections can be deduced, we must arrive at approximate mean values by means of the observations at the Royal Observatory at Greenwich. In the reduction of the observations for the Registrar-General I have so done; and I have found the tables to be of great assistance, not only in the reduction of the observations, but also in the detection of errors, and pointing out the place where such existed.

I have merely to remark that I have not formed tables of corrections for longer periods than a month, as the doing so would have extended this paper to a great length, and such can be readily formed from the tables themselves; neither have I spoken of the mean or other values, as I hope soon to have the honour of presenting to this Society some of the meteorological results deduced from the observations taken at the Royal Observatory between the years 1840 and 1845.

Greenwich, Feb. 10, 1848.

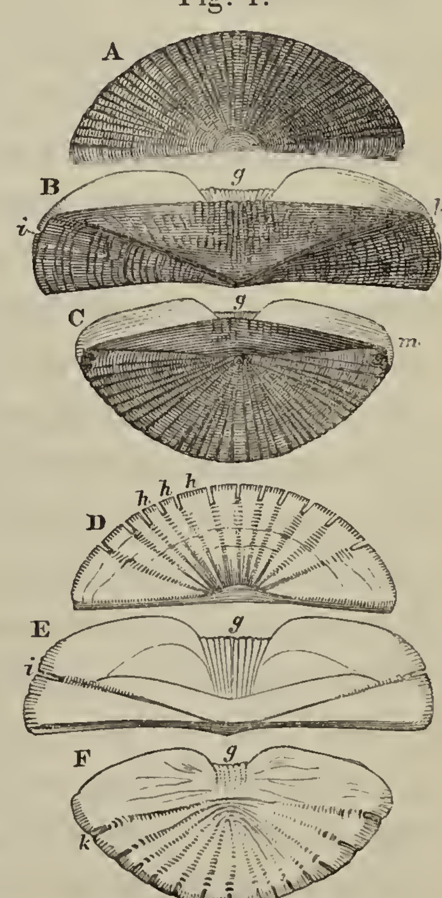
IX. *On the Structure of Chitons.* *By* J. E. Gray, *Esq., F.R.S., F.Z.S., &c.*

Received June 17,—Read June 17, 1847.

LINNÆUS and most of his successors arranged the *Chitons* with the *Pholades* and the *Balani*, as multivalve shells. Adanson, with his usual tact, placed them with the *Patellæ*, and the anatomy of the animal, published by Poli and Cuvier, has shown the propriety of this position. M. de Blainville separated the Chitons from the other Mollusca, with which they had always been placed on account of their possessing a series of imbricated shelly valves arranged along the central line of their back, and placed them with the *Cirripedes* in a peculiar subclass, which he called Articulated Mollusca, and considered as intermediate between the two divisions of the animal kingdom. I need scarcely observe that this division has not been adopted, the Cirripedes having now been proved to be true Crustacea.

There are a few peculiarities in the internal structure of these animals not found in other Mollusca. The shells also offer some differences, which have not, as far as I am aware, been previously noticed, and which it is the object of this paper to describe.

Fig. 1.



Chiton striatus.

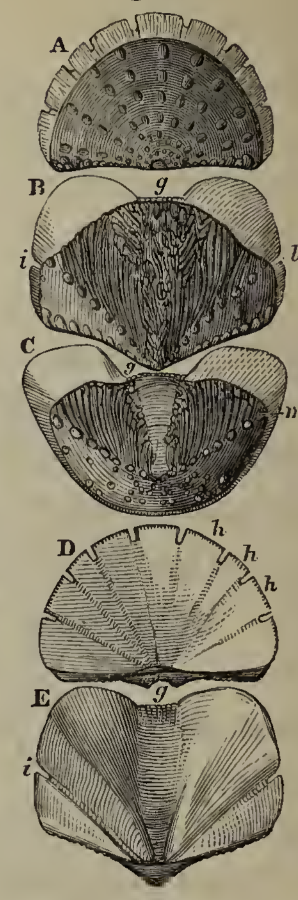
The *Chitons* at first sight differ from all the other Mollusca in being provided with a series of imbricated valves, forming a line down the centre of the back, and in this character they differ essentially from the *Patellæ*, to which they are most nearly allied in the general form, the position of the gills, &c. I have therefore thought it desirable to examine these valves with reference to one another, and to determine which most resemble the valves usually found on other Mollusca, and which are the additional or supplementary valves.

The posterior terminal valve (fig. 1, C and F, fig. 2, C) of the more normal *Chitons*, as in the restricted genus *Chiton* for example, most nearly resembles the conical form which the valves of Mollusca generally assume, and the other valves are only modifications of the same form.

The seven anterior valves are formed like the posterior one, but with the greater part of the hinder half deficient, and with the front edge of insertion somewhat enlarged. In the front valve (figs. 1 and 2, A and D) the anterior

wings of insertion found fully developed in the intermediate valves (figs. 1 and 2, B and E) are reduced, and the teeth (figs. 1 and 2, *g*), which are generally to be seen in the middle portion of the front edge of the posterior and intermediate valves between the wings of insertion, are greatly enlarged, forming an uniform edge extending along the entire outer and anterior boundary of the valve.

Fig. 2.



Enoplochiton niger.

The lobes on the margin of the front valve are generally to be seen distinctly developed, but of a smaller size on the front edge of the central portion of the posterior and medial valves (figs. 1 and 2, *g*); and they are generally most developed on the valve next to the front one; from the notches (figs. 1 and 2, *h*) between these lobes in the edge of the front valve, there issue series of pores; and similar series of pores, but more crowded and interrupted, are to be observed in the middle of the inner surface of the intermediate valves (figs. 1 and 2, *g*), showing the truth of this comparison. The notch (figs. 1 and 2, *i*) on the hinder part of each wing of insertion of the intermediate valves is evidently the same as the front notch on the edge of insertion of the hinder valve (fig. 1, *k*); the raised edges (figs. 1 and 2, *l*) on the medial valves which separate the front of the valves into what has been called the lateral and central area, are analogous to the more or less distinct keel (figs. 1 and 2, *m*) which passes from the apex to the front edge of the margin of the hinder valve.

In the more abnormal *Chitons*, as in the genus *Katharina*, which have the apex of the posterior valves lower down, and produced beyond the margin, arising from the hinder portion of the posterior valve being more or less abortive, while the front portion is more than usually developed, the hinder valve loses its peculiar character and more nearly resembles the usual form of the intermediate valves, and the posterior edge of the hinder valve has only a small simply striated or smooth edge of insertion in the place of the lobed plates of insertion of the other genera.

The result of this examination of the structure of the valves of the *Chitonidæ* is exactly what we might have expected *à priori*, though it has not, that I am aware, been observed by any preceding author, and did not occur to me until after the comparison of the valves with one another. That is to say, the posterior valve which is placed over the more important organs is generally the most fully developed, and is the homologue of the shell of the *Patella*, while the others which are arranged in front of it are more imperfect, and the front one is the most rudimentary of the series. The *Chitons* may therefore be considered as normal Gasteropodes, with a series of more or less rudimentary valves in front of the usual shell. These additional valves appear to have no relation to the second lateral valve found in the normal or bivalve *Mollusca*, or to its representative in the Gasteropodes, where in some genera

it assumes the form of an operculum, but is, as in the *Chitons*, often wanting, especially in the adult state.

Besides having this multiplication of the valves, the parts of the cartilaginous mantle not covered by the valves are generally more or less closely covered with calcareous scales or subcylindrical spines, which sometimes are so fine as to assume the form of more or less flexible bristles or hairs. I cannot call to mind any mollusca which are protected in a similar manner; the scales and bristles more resemble those on the peduncles of certain Cirripedes belonging to the family *Pollicipedidæ*, and must be regarded, as the scales are in those genera, as rudimentary valves; their form and disposition afford very good zoological characters for the distinction of the species. In some, as in the *Chiton amiculatus*, where the valves are hidden, these spines are nearly transparent, and resemble *spicula*. In others, on each side of the mantle, there is a symmetrical series of pores, each armed with a tuft of spicula. The spicula may be only modifications of the spines which are found on the surface of the mantle in the other species; but this is a subject worthy of more minute inquiry, and I am not aware of any similar spicula being found in any other molluscous genus, unless they are to be compared to the spicula which are to be found imbedded in the mantle of *Phyllidia*, an allied genus; but the latter are more like the spicula of Radiata and Sponges.

The valves of those kinds (as of *Chiton amiculatus* of Pallas) which are entirely imbedded in the mantle and hidden from external view, are formed much like the valves of other mollusca, of numerous layers of calcareous matter, and they increase in size by the addition of new material deposited on the inner edge. Their structure is compact, hard, heavy and very brittle, and the calcareous matter takes the form of crowded perpendicular laminæ, placed side by side, diverging from the apex towards the edge. As the animal increases in size, these valves are thickened by the addition of numerous layers of hard shelly matter, deposited on the inner side, forming a hard, glassy concretion.

But the greater number of species have a part of the valve which is not covered by the mantle, but exposed. This exposed part consists of a perfectly distinct external coat, peculiar I believe to the shells of this family. The outer coat of these valves is separated from the lower or normal portion by a small space, filled by a cellular calcareous deposit, which is easily seen in a section of the valves, and also on the edge of the valves, occupying the space between the inner and outer plate of insertion, where it looks like a series of circular holes or tubes; the space between these coats and the cellular internal layer enlarges in thickness and length as the valves increase in size and thickness.

This cellular or tubular structure, as the shell increases in size, also fills up, as far as the margin of the outer coat, the notches on the edge of the inner coat which separate into lobes the inner part of the valves which are inserted into the cartilaginous mantle. The filling up of these fissures with this porous matter is peculiar to

this family; for the fissures on the margin of the shells of the genus *Emarginula*, of *Pleurotoma*, of *Pleurotomaria*, and of the holes formed by the notches in the edge of the lip of *Haliotis*, are filled up by layers of shelly matter of the same structure as the rest of the shell, and deposited in the same manner. These fissures, filled with this porous substance, may be seen on the inner surface of the valves forming lines of pores diverging from the tip, and increasing in width as they approach the edge, to the notches above referred to: they are useful in a zoological point of view, as showing the number of lobes into which the inner plate of insertion is divided.

The valves of the species whose shells are covered by the mantle have a plain edge, with only one or two notches.

Those of which the shells are partly external have the inner coat of the valve produced beyond the outer coat, thus forming what is called the plate of insertion; for the valves of these animals do not simply cover the mantle as with a case, but have their edge inserted into the cartilaginous mantle, another character peculiar to this group.

The inner plate of insertion, besides being divided into lobes by the fissures or slits above mentioned, has the edges of the lobes divided into deep grooves or pectinations.

The edge of the outer coat, which is never slit or lobed, is similarly but not so strongly pectinately divided, where it is inserted into the mantle.

This kind of edge is probably produced by the perpendicular radiating laminæ of which the two coats are formed, the number of teeth appearing to agree with the number and thickness of the plates, the teeth and laminæ being thicker and more numerous on the outer than on the inner coat.

This kind of edge does not occur, as far as my observation extends, in any other mollusca, for it is very unlike the grooves on the edge of many bivalves which are formed by the processes on the surface of the mantle, and more resembles the plates between the tubes in the substance of the valves of some Barnacles (*Balani*), but has no real resemblance to them in structure.

The more prominent peculiarities of this family appear to be,—

1. That instead of having a single valve, as is the case with most Gasteropodes, they have a series of more or less perfect valves placed in front of the normal valve, the front one being the most imperfect, all imbricated the one over the other.

2. Besides this increase in the number of valves, the surface of the mantle is covered with numerous rudimentary valves assuming the form of scales or spines.

3. These spines are sometimes placed in tufts symmetrically dispersed on the sides of the body.

4. The valves of the more normal *Chitons*, which are partly exposed, are furnished with two additional coats, of the size of the exposed part, not found in the shells of any other mollusca, the intermediate coat being of a porous texture; and this coat fills up the symmetrical slits usually found in the innermost coat.

5. The valves of these shells, instead of being simply placed on the surface of the mantle and attached to the animal by muscles, are inserted by their edge into the substance of the cartilaginous mantle.

Hence we may conclude, that though it is impossible to adopt M. de Blainville's views with regard to the systematic arrangement of *Chitons*, yet they offer many particulars not found in other mollusca; and that in the structure of the edge of the valves, where they are inserted into the mantle, and in the formation of the central cellular coat, which is doubtless formed by small processes of the mantle, like the tubes in the substance of the valves of the coronal Cirripedes, they offer an analogy to the shells of those Crustacea which has not before been observed.

X. *An Investigation on the Chemical Nature of Wax.*

By Benjamin Collins Brodie, *Esq.*
Communicated by Sir Benjamin C. Brodie, *Bart., F.R.S., &c.*

Received March 2,—Read March 30, 1848.

I. *On Cerotic Acid, a new Acid contained in Bees'-Wax.*

In the summer of 1845, while studying at Giessen, in the laboratory of Professor von Liebig, I undertook, at the request of that distinguished chemist, the analysis of certain waxes which were the results of an experiment made by Herr Gundlach of Cassel, of feeding bees upon different kinds of sugar. It is not my intention to give those analyses here, and I mention them now only for the purpose of stating that it was this circumstance which first turned my attention to the inquiry of which I now offer the results to the Royal Society, and that it was in Professor von Liebig's laboratory that this investigation was begun.

Various chemists have before me undertaken a similar inquiry. The chemical history of a substance so abundant in nature and so useful to man as wax was always a curious question. Of late it has acquired a peculiar interest from our knowledge, derived from repeated experiments, that wax is formed in the organs of the bee, and that in the body of that insect that remarkable change of sugar into wax takes place, the knowledge of the true conditions of which would, we may hope, throw light upon the formation of fatty bodies, and on the way by which out of vegetable products the continual repair of the animal structure is effected. The first step to such a knowledge must be the accurate study of the chemical nature of those substances which are thus produced.

But little progress however has been made in this inquiry. I may sum up in a few words those results already known which, by my own experiments, I am able to confirm as true. It has been ascertained that wax is separable by alcohol into two portions, which have been called cerin and myricin; that, by the action of potash upon wax, an acid or acids may be obtained, and also an unsaponifiable body, cerain; and that by the distillation of wax we obtain volatile oils, solid hydrocarbon, and an acid which has been surmised to be margaric acid, from its resemblance to that substance.

I say that these are the ascertained facts. The high atomic weight of these bodies, and the unavoidable errors of analysis, have rendered it easy to find formulæ for them, and to speculate as to their nature. If, however, the views which, in the fol-

lowing pages, I offer to the Royal Society are correct, their true chemical relations and constitution have been undiscovered.

It would be useless, and it is by no means my intention, to comment upon all the ideas which other chemists have entertained upon this matter. There is however one theory, which has been advanced by certain chemists in France, of which it would be unbecoming in me to take no notice, both because it has a certain apparent amount of fact to support it, and because the originators of it, and others also, believe that they have finally settled the question of the true place which wax should hold in our classification of chemical substances*. M. Lewy has stated that cerin, that portion of the wax which is the more soluble in alcohol, is converted by oxidation by means of lime and potash into stearic acid. The method he pursued was that used by Dumas with such success for the conversion of alcohol into acetic acid; of potatoe oil into valerianic acid; and for other similar transformations. He has explained the reaction by giving to the cerin the formula $C_{68}H_{68}O_4$. On this hypothesis, the cerin, by conversion into stearic acid, loses three equivalents of hydrogen and takes up three equivalents of oxygen into its constitution, the reaction being expressed by the equation

$$C_{68}H_{68}O_4+3HO=C_{68}H_{68}O_7+H_3=\left[\begin{array}{c}\text{stearic acid}\\ +H_3\end{array}\right],$$

the cerin being considered the aldehyde of stearic acid. M. Gerhardt has taken up and extended this idea. Proceeding on the belief that the myricin and cerin are isomeric—a belief, I may observe, not justified by experiment, although almost universally adopted by chemists—he has stated that the myricin, of which by far the larger portion of the wax consists, corresponds to the metaldehyde of the same acid, and in a paper which he entitles, "Faits pour servir à l'histoire de la cire des abeilles†," has explained on this hypothesis the origin of the products of the dry distillation of that substance. The theory of M. Lewy agrees sufficiently well with his analyses of the substances in question and with the relations of his formulæ. I must however observe, that any person who will take the trouble of reckoning out those analyses on which the formula of stearic acid depends, according to the atomic weight of carbon now almost universally adopted by chemists, carbon 6, hydrogen 1, will see that this formula itself demands a new inquiry before we can accept it. These results of M. Lewy are in many ways at variance with my own experiments; but I confess that if the agreement of the analysis and of the melting-point of the substance he obtained by oxidation of the cerin with the analysis and the melting-point of stearic acid were in truth a sufficient proof of the identity of the bodies, this difference in our results would be to me very difficult to explain. Wax certainly stands in a remarkable relation to fat, but I do not believe that relation to be of the nature M. Lewy has conjectured, and I cannot but think that, in

* Annales de Chimie, vol. xiii. p. 439; and Jahres-Bericht, Berzelius, vol. xxiv. p. 468.

† Ibid. vol. xv. p. 236.

his desire to establish this relation, he has rested content with insufficient proofs of his theory. Any person who has had a little experience in these inquiries, must know how deceptive this melting-point and analysis are as criteria of the purity or identity of bodies. The separation of this class of substances by crystallization is difficult, and often the composition per cent. of two substances of entirely different chemical formulæ will agree within the unavoidable errors of the method of analysis. The reasons will hereafter appear why I am justified in saying that, in the cerin which M. Lewy analysed, he did not work on a pure chemical substance; and that consequently any theory founded on the reactions of this body must fall to the ground; I believe that by more careful inquiry he might have procured that substance of a different melting-point and constitution to that which he found. I have also in vain searched the papers which M. Lewy and M. Gerhardt have published upon this matter, to find some account of the preparation and the analysis of a salt of this so-called stearic acid, an easy and satisfactory experiment, and one which alone, in the case of acids of such high atomic weight, can justify the chemist in pronouncing on their constitution. Until such a salt is made, I cannot but consider that the evidence is insufficient, that the remarkable oxidation in question has ever been effected.

I propose to give to the Society, in three papers, the results of an investigation on the nature of wax. The present paper will contain an inquiry as to the constitution of the so-called cerin; I mean that portion of the bees'-wax which is the more soluble in boiling alcohol. The second paper will treat of the chemical constitution of a wax from China, a substance which, although it considerably differs in its appearance and properties from bees'-wax, in the form in which it comes before us in nature, is nevertheless, chemically speaking, closely analogous to that body. In a third paper I propose to consider the nature of myricin, the other constituent of the bees'-wax itself. I may here state, that to ensure the purity of the wax used in the following experiments, I prepared it myself from the comb. It was made by bees in the county of Surrey in the years 1845 and 1846. This wax I have always used for the first preparation of a substance. For further experiment I have sometimes used wax procured in other ways.

Cerotic Acid.

If wax melting at about 62° or 63° Centigrade be treated with boiling alcohol, a considerable portion will be dissolved. If this operation be repeated, the quantity of substance dissolved the fifth or sixth time will be evidently less than that dissolved in the first operation. But however often this operation be repeated, there will always be a portion of wax dissolved. This fact alone might lead us to suspect that any absolute separation of these two portions of the wax, by boiling with alcohol and subsequent crystallization out of that liquid, was impossible.

A partial separation can, however, be readily effected, and by this method a sub-

stance can be obtained melting at 70° and even 72° Centigrade, much harder than the residue of the wax, brittle, and slightly crystalline in its structure. If this substance be boiled with a solution of caustic potash, it is saponified with the greatest facility. If the soap be decomposed by an acid, a baryta salt formed of the acid produced, and this salt be dried and washed out with ether, a certain portion of a substance will be taken up by the ether, which is but very slightly acted on by potash. This is that substance which has been called cerain. I remarked that while from the residue of the wax this matter could be procured in considerable quantities, but a very small portion was obtained from the cerin; a portion very small in relation to the acids formed, and the quantity of which diminished with the purification of the substance. The analysis also of the unsaponified cerin agreed closely with that of the acid as separated from the baryta salt, and the melting-points only differed by four or five degrees Centigrade. Further experiment showed me that the same cerin, when boiled with caustic baryta, entered with the greatest facility into combination with that substance. These facts led me to suspect that the formation of the unsaponifiable body was due to the presence of a certain portion of the other substances of the wax, and was no true product of the decomposition of the cerin itself; but that this substance was in truth no other than the acid itself existing, in a free state, in the wax. The experiments which follow are inconsistent with any other hypothesis.

To prepare this acid bees'-wax is to be boiled out with strong alcohol, and the hot solution poured off from the undissolved residue. This operation may be repeated with advantage three or four times. The precipitates which are formed on the cooling of the alcohol are to be collected together and treated repeatedly with alcohol in the same manner as the wax itself, until the melting-point of the precipitate is raised to about 70° C., after which point the further purification of the body by this method of crystallization is extremely difficult. This substance is now to be dissolved in a large quantity of boiling alcohol, care being taken perfectly to effect the solution. To this solution is to be added a boiling solution of acetate of lead in alcohol, by which a voluminous precipitate is produced, which does not redissolve in the boiling mixture. The solution is to be boiled and filtered hot, by means of a hot water apparatus, from the precipitate. The precipitate while moist is to be taken from the filter, placed again in the flask, and boiled out with strong alcohol, and afterwards with ether. This operation is to be repeated several times until the fluid which passes through the filter carries no substance with it. The first portions of alcohol will contain considerable portions of a wax matter having a lower melting-point than the cerin, and having the appearance of the residue undissolved by the hot alcohol.

The lead salt is to be decomposed by very strong acetic acid. The substance which separates, after having been well-washed with boiling water, is to be dissolved in absolute alcohol, and the solution, while hot, filtered. On the cooling of the alcohol a substance will crystallize out in fine granular crystals. This substance melts at about 78° C. It is highly crystalline on cooling from the melted state. Very

carefully prepared, it gave to analysis the following numbers. The analysis was made with chromate of lead.

0·26 grm. of substance gave 0·753 grm. of carbonic acid and 0·307 grm. of water.

These analyses give in 100 parts—

Carbon	78·98
Hydrogen . . .	13·12
Oxygen	7·90
	100·00

This corresponds to the formula $C_{54} H_{54} O_4$.

	Atomic weight.	Calculated in 100 parts.
C_{54} . . .	324	79·02
H_{54} . . .	54	13·17
O_4 . . .	32	7·81
	410	100·00

It is very difficult to wash this lead salt perfectly out. Even in the present case, after the greatest care, I obtained a trace of a substance, soluble in ether, on attempting further to purify the substance by combining it with baryta and washing out its baryta salt. This makes no difference in the numbers or the melting-point. But it is advisable to prepare the pure acid by boiling the acid as separated from the lead salt with caustic potash, precipitating the baryta salt by a large excess of chloride of barium, and carbonate of soda ; washing this well out with ether ; decomposing with an acid, and crystallizing the resulting substance repeatedly out of alcohol and ether. The acid prepared in this manner melted at 78°, 79° C., being the same melting-point as the acid separated from the lead salt; it gave to analysis the following numbers :—

I. 0·263 grm. gave 0·7583 carbonic acid and 0·3085 water.

II. Another preparation. 0·2615 grm. gave 0·7558 carbonic acid and 0·307 water.

III. 0·2612 grm. gave 0·7562 carbonic acid and 0·308 water.

These analyses correspond in parts per cent. to—

	I.	II.	III.
Carbon	78·63	78·82	78·95
Hydrogen . . .	13·04	13·04	13·10
Oxygen	8·33	8·14	7·95
	100·00	100·00	100·00

The silver salt of this acid was made by precipitating the ammoniacal solution of the acid in alcohol with nitrate of silver. It is desirable to take an excess of ammonia, and the solution must be precipitated boiling by a hot alcoholic solution of the nitrate. The salt is to be washed with water and dried, first *in vacuo*, then in a water-bath.

I. 0·469 grm. of this salt gave 1·067 carbonic acid and 0·4315 water.

II. 0·4663 grm. of this salt gave 1·0675 carbonic acid and 0·4273 water, giving in 100 parts,—

	I.	II.
Carbon	62·04	62·43
Hydrogen	10·22	10·18
Oxygen and silver . .	27·74	27·39
	100·00	100·00

I. 0·6566 grm. of this salt gave 0·1413 grm. of silver.

II. 0·6508 grm. of this salt gave 0·1388 grm. of silver.

III. 0·6147 grm. of this salt gave 0·1292 grm. of silver.

IV. 0·6641 grm. of another preparation gave 0·1396 grm. of silver.

V. 0·665 grm. of the same preparation gave 0·1396 grm. of silver.

These determinations correspond in 100 parts to—

	I.	II.	III.	IV.	V.
Silver	21·52	21·33	21·02	21·02	20·99

The above analyses agree with the formula $C_{54} H_{53} O_3$, AgO.

	Atomic weight.	Calculated in 100 parts.
C_{54} . . .	324·0	62·66
H_{53} . . .	53·0	10·25
O_4 . . .	32·0	6·19
Ag . . .	108·1	20·90
	517·1	100·00

Cerotic Ether.

The combination of this acid with the oxide of ethyle is readily obtained by dissolving the acid in absolute alcohol, and passing hydrochloric acid gas through the solution. This substance has the appearance of a soft wax, and melts at 59°–60° C.

I. 0·2628 grm. gave 0·765 carbonic acid and 0·3094 water.

II. 0·2638 grm. gave 0·7695 carbonic acid and 0·3105 water, giving in 100 parts,—

	I.	II.
Carbon	79·38	79·55
Hydrogen	13·08	13·08
Oxygen	7·54	7·37
	100·00	100·00

To obtain these numbers the action must be long continued. They correspond to the formula of the ether, $C_{58} H_{58} O_4 = C_{54} H_{53} O_3 + C_4 H_5 O$.

	Atomic weight.	Calculated in 100 parts.
C_{58}	348	79·45
H_{58} . . .	58	13·24
O_4	32	7·31
	438	100·00

Chlor-Cerotic Acid.

Cerotic acid is readily acted on by chlorine if melted in that gas. Hydrochloric acid is formed, and a substitution of chlorine for hydrogen in the substance takes place. The following substance was made by passing a current of chlorine over the acid, melted in a small flask in a water-bath. The action must be continued for several days. I considered the action as terminated when no more vapour of hydrochloric acid could be perceived. The substance had now undergone a remarkable transformation in appearance, a transformation similar to which may be effected in all the wax substances with which I have experimented.

It was perfectly transparent, slightly yellow, and of the consistency of a very thick gum, more than viscid, and yet capable of being drawn out in threads.

I. 0·382 grm. of this substance gave 0·5581 carbonic acid and 0·1752 water.

II. 0·401 grm. of this substance gave 0·5848 carbonic acid and 0·184 water.

These analyses give in 100 parts—

Carbon	39·82	39·77
Hydrogen	5·10	5·09
Chlorine and oxygen . .	55·08	55·14
	100·00	100·00

I. 0·486 grm. by the usual method of analysis gave 1·012 grm. chloride of silver, containing 0·249 chlorine.

II. 0·6715 grm. by the usual method of analysis gave 1·3915 grm. chloride of silver, containing 0·344 chlorine.

These analyses give as the per-centage of chlorine—

	I.	II.
Chlorine	51·40	51·28.

These analyses lead us to the formula

$$C_{54}\begin{cases} H_{42} \\ Cl_{12} \end{cases} O_4.$$

	Calculated in 100 parts.
C_{54}	39·45
H_{42}	5·10
Cl_{12}	51·50
O_4	3·95
	100·00

Chlor-Cerotic Ether.

This last substance has the properties of an acid, and gives with soda a salt almost insoluble in water. I prepared this salt but did not analyse it, as the substance was lost in attempting to dry it on a water-bath, in which case it is decomposed. The compound ether, however, of the acid may be obtained in a similar way to that in which is obtained the compound ether of the cerotic acid itself. The numbers which this ether gave to analysis, taken with those of the chlor-cerotic acid, and of the cerotic acid, are conclusive as to the formulæ of these bodies. The appearance of this ether is similar to that of the acid from which it is derived.

0·454 grm. gave 0·6815 carbonic acid and 0·214 water,
which analysis gives in 100 parts—

Carbon	40·94
Hydrogen	5·24
Oxygen and chlorine	53·82
	100·00

0·650 grm. of the substance gave 1·332 grm. of chloride of silver, which contains 0·3294 grm. of chlorine, and corresponds to chlorine 50·68 parts per cent.

The formula of the ether $C_{58}\begin{cases}H_{46}\\Cl_{12}\end{cases}O_4$ requires in 100 parts—

	Calculated.
C_{58}	41·11
H_{46}	5·40
Cl_{12}	49·99
O_4	3·50
	100·00

which agree remarkably with the numbers found; it being remembered that the body cannot be crystallized or indeed in any way purified.

Distillation of Cerotic Acid.

Cerotic acid is volatile. If the acid as separated from the lead salt by acetic acid be distilled, the acid separated from the distillate and purified by boiling with potash and washing the baryta salt with ether, it will again be procured with a melting-point nearly corresponding to that of the original acid.

0·251 grm. of this substance gave 0·7254 carbonic acid and 0·2977 water, giving in 100 parts—

Carbon	78·80
Hydrogen	13·19
Oxygen	8·01
	100·00

being precisely the same numbers as the substance gave before distillation.

A question suggests itself, If it be true that this acid is volatile, and it be also true that it exists in the wax in an uncombined state, how has it happened that it has never been discovered among the products of the distillation of wax which have been so often examined?

In truth, it is a remarkable fact, that while this acid, when heated in a nearly pure state, as separated from the lead salt, distils over nearly unaltered, the same substance, when distilled in an impure state, as separated, namely, by alcohol from the wax, and mixed, as in this case, with other wax matter, which is decomposed by heat, itself suffers decomposition. In the course of this investigation other examples have occurred to me of similar facts.

About 3 oz. of the cerin, melting at 70° C., from which this acid can be precipitated by acetate of lead, was distilled. The first portions of the distillate consisted entirely of oil. Towards the end a small quantity of solid matter appeared. The oil, on standing, separated into two portions: the lower part was withdrawn by a pipette, being but a small portion, say one-twentieth of the whole. The upper portion was boiled with potash, only a very small portion of acid however was separated in this manner. The soap was drawn off by a syphon, and the oil washed out with water. On being washed with strong alcohol the oil dissolved, almost entirely, leaving only a small portion of solid matter of the appearance of paraffin. This oil consists of a mixture of oils of different boiling-points, which were separated as far as possible by rectification, but during the distillation the boiling-point kept constantly rising, and I could obtain no oil in which it was absolutely constant.

Three of these oils, of the respective boiling-points of 210° to 220° C., 230° to 250° C., and 250° to 274° C., were analysed. By far the greater portion of this whole distillate went over between 230° and 250° C. The results of these analyses were—

In the case of the first oil,

(1). 0·2676 grm. gave 0·826 carbonic acid and 0·3307 water.

In the case of the second oil,

(2). 0·2629 grm. gave 0·8127 carbonic acid and 0·3325 water.

In the case of the third oil,

(3). 0·2697 grm. gave 0·8443 carbonic acid and 0·341 water,

giving in 100 parts—

	I.	II.	III.
Carbon	84·17	84·30	85·37
Hydrogen . . .	13·73	14·05	14·05
Oxygen	2·10	1·65	0·58
	100·00	100·00	100·00

The amount of oxygen therefore diminishes as the boiling-point rises in these oils, the last oil being nearly pure hydrocarbon. It is useless to reckon out formulæ for them: I give however the analyses to point out the source of the oils in the wax distillate, and to account for the disappearance of the cerotic acid in that product.

The products of this distillation prove also that we must look to some other body than the cerin for the source of the margaric acid, or the acid resembling that substance which has been found in the wax distillate, and for the origin also of the paraffin, of which only traces can be found even on the distillation of the impure cerin.

The existence of a free acid in wax is a fact to which we have no parallel in the constitution of any known fat, and although the reactions, which I have given, left little doubt upon my mind that in the wax the acid was in this condition, it yet appeared to me desirable to procure it from the wax by simple crystallization. By patience this may be accomplished. The cerin analysed by M. Lewy melted at 62°·5 C. I have stated that by means of alcohol this substance may be procured of a melting-point of 72°. If this substance of 72° melting-point be repeatedly dissolved and crystallized out of a large quantity, not of alcohol, but of ether, the melting-point can be raised to 78°, in which state the substance is highly crystalline, and has all the appearance of the acid as procured by other means.

0·25625 grm. of this gave 0·7435 CO_2 and 0·3005 HO,

which gives in 100 parts—

Carbon	79·13
Hydrogen . . .	13·20
Oxygen	7·67
	100·00

Both the melting-point and the analysis perfectly agree with the melting-point and analysis of cerotic acid, which have been already given, and with its formula.

It is certainly a strange fact that the presence of this body in the wax should so long have escaped the notice of chemists. The wax however is a complex substance, and the cerotic acid to be obtained in purity has to be separated from many other bodies which disguise its nature and reactions.

Should any chemist be induced to verify the results which I have given, I must beg him also rigidly to observe the methods I have laid down for the preparation and purification of the substances, for errors which are slight in the analysis of the substance and which neither the analysis nor melting-point detect, become of great importance when the transformations of the substance are investigated, and its atomic weight is to be determined.

The alcoholic extract, out of which the cerotic acid has crystallized, contains, although in very small quantities, yet another acid. If to the solution an alcoholic solution of acetate of lead be added, a precipitate of a lead salt is produced: this salt is readily distinguished from the salt of the cerotic acid as it is dissolved on boiling the alcoholic solution, out of which, on cooling, it will separate in crystalline grains. This substance is contained in very small quantity in the wax. It resembles in appearance margaric acid. I have analysed this acid and one of its salts. These

experiments showed that it approached margaric acid in its constitution: but I have never been able to procure the quantity necessary to its sufficient purification, and the analyses led to no conclusions as to its formula which deserve to be recorded.

It is a matter of interest to know in what proportions, relative to its other constituents, the cerotic acid is contained in the wax. This may readily be determined by precipitating by acetate of lead the solution of a known quantity of wax and determining the quantity of the lead salt produced. It is necessary however, in order to know to what quantity of acid in the wax the found quantity of lead salt corresponds, to know the atomic weight of the lead salt itself. There is great difficulty in washing out this lead salt so as perfectly to remove from it the myricin. In order therefore to effect any satisfactory purification of it, I was obliged to operate on a smaller quantity of the salt than I should otherwise have chosen. My experiment, however, was sufficient to show that the salt is the neutral salt containing one equivalent of lead.

0·1445 grm. of the lead salt gave when ignited 0·032 grm. of solid residue. This residue, extracted with acetic acid and dried, left 0·0065 grm. of residue, and lost 0·0255 grm., which, reckoned as oxide, contains 0·0236 grm. lead. This gives 0·0296 as the whole quantity of lead in the salt. Reduced to parts per cent. this gives 20·48 as the per-centage of lead. The formula $C_{54}\,H_{53}\,O_3+PbO$ requires 20·26 per cent.; this therefore is the formula of the salt.

To determine the proportion of the acid in the wax itself, a portion of pure yellow wax was dissolved in ether and filtered from adhering impurities, then dissolved in naphtha-ether and precipitated by acetate of lead dissolved in alcohol; an additional portion of ether was afterwards added to ensure the entire precipitation of the salt, the solution was filtered hot, and the lead salt was carefully washed out on the filter.

1·0905 grm. of wax treated in this manner gave 0·3015 grm. of the lead salt, which, assuming it to be the neutral salt, is equivalent to 0·24 grm. of the acid $C_{54}\,H_{54}\,O_4$. This reduced to parts per cent. gives 22·0 of the cerotic acid in every 100 parts of bees'-wax.

Although this was the proportion of cerotic acid in the bees'-wax I had prepared, it by no means followed that other specimens of wax when examined would give a similar result. Various chemists have stated that the proportions of the cerin varied in different specimens of wax; and although the estimate of the cerin was a mere matter of guess, no criterion of its presence or absence being known, yet nevertheless the great difference in the results of the different chemists who have attempted to determine this point led me to suspect that the quantity of the cerotic acid in the wax did in truth vary; and that it might be possible, if so, to find a wax which even did not contain this body. I have found this acid in all the bees'-wax made in this part of the world, bleached or otherwise, which I have examined*. It occurred to me

* I should except a wax made by wild bees in Wiltshire, which I tested with acetate of lead for this acid, but could find none. The quantity of the wax however was so small that I was unable to make many experiments with it.

however that wax made under very different conditions of climate and vegetation to ours might possibly have a different constitution. With this view I procured, through the kindness of a friend who was resident in Ceylon, some bees'-wax from that island. The wax as I received it was white, having in all respects the appearance of English wax, and melting at 63° C. In other points also, as I shall show in another paper, its chemical nature was the same as that of English wax. The cerotic acid however was entirely absent. When extracted in the boiling alcohol a portion of the wax dissolves; but on the addition of acetate of lead to the alcoholic solution hardly a trace of precipitate is formed.

Such a variation in the nature of an insect-secretion under different conditions of life is a remarkable fact, which we may place by the side of the curious difference in the nature of the constituents of butter in different years, discovered by LERCH. In his excellent investigation on the nature of the volatile acids of butter, this chemist discovered* that the butyric and caproic acids of one year were in another year replaced by vaccinic acid, an acid from which, by deoxidation, the two former acids might readily be formed.

$$\overbrace{C_{20}\,H_{20}\,O_7}^{\text{Vaccinic acid.}} = \overbrace{C_8\,H_8\,O_4}^{\text{Butyric acid.}} + \overbrace{C_{12}H_{12}\,O_4}^{\text{Caproic acid.}} - O.$$

The cerotic acid, $C_{54}\,H_{54}\,O_4$, although far removed in the series of fatty acids from these volatile acids of butter, yet nevertheless belongs to the same chemical series of bodies, to that series, namely, which contain carbon and hydrogen in equal equivalent proportions, and which, theoretically at least, are capable of being produced by deoxidation from sugar or from starch.

Any fact is of importance which can throw light upon the law by which these substances are truly convertible, one into the other, and it would be highly interesting to investigate, in those waxes where the cerotic acid is not found, by what bodies, if any, it can be replaced.

* Annalen d. Ch. und Pharm. vol. xlix. p. 230.

13 *Albert Road, Regent's Park,*
February 28*th*, 1848.

XI. *An Investigation on the Chemical Nature of Wax.*

By Benjamin Collins Brodie, *Esq.*

Communicated by Sir Benjamin C. Brodie, *Bart., F.R.S., &c.*

Received March 30,—Read May 11, 1848.

II. *On the Chemical Nature of a Wax from China.*

The wax which is the subject of the following investigation, is a substance which was imported into this country from China as an article of commerce. Its appearance closely resembles that of spermaceti. It is, like spermaceti, white and, in large masses, highly crystalline, but differs from it by being harder, more brittle, and of a more fibrous character of crystallization. The melting-point of the wax is about 83° C. It is but very slightly soluble in either alcohol or ether, but dissolves with great facility in naphtha, out of which fluid it may be crystallized. This substance is generally spoken of as a vegetable wax: on looking however into such facts as I can gather which throw any light on its origin, it seems more than probable that, like bees'-wax, it is the secretion of an insect. Sir George Staunton, in his "Embassy to China*," gives an account of a wax of insect origin, which there formed an article of trade, and in his work may be seen a drawing of the insect and of the tree on which the insect lives. Other writers on China give a similar account. In the Comptes Rendus for 1840† is a paper by M. Stanislas Julien, who gives an account of this tree wax, and states it to be the work of an insect: where may be found also a great number of extracts from Chinese writers on agriculture, giving an account of the insect itself and of the trees suitable for its food; one of these trees is the *Rhus succedaneum*. This same gentleman, M. Stanislas Julien, gave to M. Lewy, who was engaged in an investigation on these wax substances, a specimen of the wax from this very plant, which is therefore in all probability this insect-wax. The melting-point, the appearance and the analysis of this wax, as given by M. Lewy, agree so exactly with those of the wax which I have examined, that I cannot but believe them to be the same chemical substance, and that this wax also is of insect origin.

The existence of any other wax-making insect, such as this *Coccus ceriferus*, besides the bee, is a point of considerable interest in relation to the question as to the origin of the wax in that insect, and the possibility of the chemical transformations by which it is produced.

* Vol. i. p. 352, edition 1797.

† Vol. x. p. 619. The title is "Nouveaux renseignements sur la cire d'arbre, et sur les insectes qui la produisent." See also in the same volume, M. Virey, Sur les insectes qui produisent la substance appelée par les Chinois, "Cire d'arbre."

The Chinese wax, as it appears in commerce, is a substance nearly in a state of chemical purity. By alcohol small portions of a greasy matter may be separated from it, and on distillation it affords traces of acrolein, which is not a product of the distillation of the pure wax. The impurities however are unimportant.

I have spoken of this substance as a wax; and in truth, although to the eye it more nearly resembles spermaceti or stearine than ordinary bees'-wax, the substance, nevertheless, which, even in appearance, it more nearly resembles than any other, is the purified cerin, that is, that cerotic acid of which, mixed with certain other waxy matter, in a former paper, I have shown the bees'-wax to consist. The accurate investigation of the chemical nature of the Chinese wax has brought to light certain curious chemical relations which exist between these bodies, and led to the discovery of the alcohol of cerotic acid.

Cerotin.

Chinese wax may be boiled for a long time either with dilute or with concentrated potash with hardly any signs of saponification. If, however, it be melted with the hydrate of potash, it is readily decomposed. This decomposition is best effected in an iron basin over a large gas flame or gentle fire. The mass, after the action, is soluble in boiling water, giving a milky solution. From this solution two substances may be procured; an acid which may be combined with baryta by precipitation of the soap with chloride of barium; and a wax-substance of another nature which is obtained by washing out the baryta salt with any suitable solvent, such as alcohol, ether, or naphtha. The soap, after precipitation by chloride of barium, becomes perfectly clear, and, to obtain the wax-substance which is not combined with the baryta, the baryta salt is first to be separated by filtration from the fluid, then dried and pulverized. It is convenient to effect, at first, a partial separation of the salt from the other matter by washing it out with a large quantity of boiling alcohol, and filtering the solution hot through linen. After this operation has been two or three times repeated, the substance, which passes through the filter, is to be redissolved in the same alcohol and the solutions filtered, in a similar manner, through paper, so as to separate the small portions of the baryta salt which unavoidably pass through the linen. The solution is much facilitated by the addition of a small quantity of naphtha to the alcohol. If the substance be purified by crystallization out of ether and absolute alcohol, its melting-point will be raised to 79° C. Previous to analysis the substance is to be dissolved in absolute alcohol and ether, and the solution filtered. This is a necessary precaution, as the naphtha dissolves traces of the baryta salt. The substance was analysed.

I. 0·258 grm. gave 0·7715 CO_2 and 0·327 HO.

II. The numbers have been mislaid.

III. 0·2602 grm., another preparation, gave 0·7785 CO_2 and 0·334 HO, which give in 100 parts,—

	I.	II.	III.
Carbon . .	81·55	81·76	81·59
Hydrogen . .	14·08	14·25	14·26
Oxygen . .	4·37	3·99	4·15
	100·00	100·00	100·00

These numbers give the formula $C_{54} H_{56} O_2$.

	Atomic weight.	Calculated in 100 parts.
C_{54} . . .	324	81·81
H_{56} . .	56	14·14
O_2 . . .	16	4·05
	396	100·00

This is the formula of an alcohol. Other experiments decided that this was in truth the class of bodies to which this substance belonged, and led me to adopt the particular formula for it which I have given. This alcohol I call cerotin. If this substance be heated with lime and potash according to the method of Dumas, hydrogen gas is given off, and if the experiment be conducted with care there will hardly be traces of any other volatile product. In the tube is found an acid. The experiment requires considerable heat, and I have found that it is best made in a long combustion-tube, suspended by means of two corks in a large tube of porcelain, which forms an air-bath. The apparatus is heated by charcoal in a combustion-trough. In this manner the heat can be regulated with the greatest precision. The acid having been purified in the usual manner, which it is unnecessary again to refer to, is a substance highly crystalline in its texture and perfectly resembling in its sensible properties the cerotic acid from bees'-wax, with which acid analysis shows it to be identical. The melting-point of this preparation was about a degree higher than that of the cerotic acid from wax, namely 81° C.

0·259 grm. gave 0·754 CO_2 and 0·309 HO,

which correspond in 100 parts to

Carbon	79·39
Hydrogen . . .	13·28
Oxygen	7·33
	100·00

This agrees with the formula $C_{54} H_{54} O_4$.

	Calculated in 100 parts.
C_{54} . . .	79·02
H_{54} . .	13·17
O_4 . . .	7·81
	100·00

I prepared also the silver salt of this acid. The method used for its preparation was the same as that used in the case of the cerotic acid from bees'-wax.

I. 0·3775 grm. of this salt gave 0·862 CO_2 and 0·349 HO.

II. 0·3625 grm. of this salt gave 0·833 CO_2 and 0·3385 HO, which correspond in 100 parts to—

	I.	II.
Carbon	62·27	62·74
Hydrogen	10·27	10·38
Oxygen and silver	27·46	26·88
	100·00	100·00

I. 0·654 grm. of the same gave 0·135 grm. of silver.

II. 0·629 grm. of the same gave 0·1305 grm. of silver.

These correspond in parts per cent. to—

	I.	II.
Silver	20·64	20·74

The formula of the silver salt of cerotic acid requires—

	In 100 parts.
C_{54}	62·66
H_{53}	10·55
O_4	6·19
Ag	20·90
	100·00

Sulphate of the Oxide of Cerotyle.

When cerotin is treated in the cold with concentrated sulphuric acid, it is only acted on by the acid partially and at the surface. If the action be increased by heat, a reddening of the mass and decomposition take place. If however the cerotin be taken in a state of fine granular division, as obtained by crystallization out of ether, and in this condition acted upon by the acid, all decomposition is avoided and a perfect combination takes place of the acid with the cerotin. The granular crystals are to be dried between blotting-paper, and then digested with the sulphuric acid, in the cold, for two or three hours, sufficient acid being added to render the mixture a rather fluid paste. The mass is to be thrown into cold water, and washed out with the same on a filter. As long as the solution is acid the wash-water will go through perfectly clear, but when the acid is washed out it becomes slightly turbid. After the adhering acid has been thus removed, the substance may be dried *in vacuo*, and, when perfectly dry, dissolved in ether and crystallized out of that fluid. In this condition it is perfectly soluble even in water, and dissolves with the greatest facility in water to which the smallest quantity of alcohol has been added. When the solution in water has been evaporated to dryness at a low temperature, it remains in the form of a soft wax.

I. 0·333 grm. of this substance gave 0·912 CO_2 and 0·3915 HO.

II. 0·3317 grm. of this substance gave 0·9025 CO_2 and 0·3865 HO,

corresponding in 100 parts to—

	I.	II.
Carbon	74·67	74·20
Hydrogen	13·06	12·95
Oxygen and sulphur	12·27	12·85
	100·00	100·00

Owing to the loss of a portion of the substance I was not able to make a separate determination of the sulphur. These numbers, however, so correspond to the formula SO_3, $C_{54}H_{55}O+HO$, and so exclude all other probable formulæ, as to determine the constitution of the compound.

	Atomic weight.	In 100 parts.
C_{54}	324	74·31
H_{56}	56	12·84
O_5	40	12·85
S	16	
	436	100·00

Adopting therefore the usual chemical language as to such compounds, this substance is to be regarded as the sulphate of the oxide of cerotyle, containing one equivalent of water, cerotyle being $C_{54}H_{55}$, the hypothetical radical of the alcohol.

Chlor-cerotic Aldehyde—Chlor-cerotal.

The action of chlorine on cerotin gives us a proof of the strong analogy of chemical constitution between alcohol and that substance, notwithstanding the wide interval by which they are separated in the alcoholic series. A body is formed analogous to chloral; two equivalents of hydrogen are removed without substitution, the substance passing, apparently, through the condition of an aldehyde, before chlorine is substituted. The chlorine produces a similar change in the appearance of the substance to that which is produced by the action of chlorine on cerotic acid. The character of wax is entirely lost, and the substance converted into a perfectly transparent slightly yellow body, possessing the appearance and consistency of a gum-resin, and which becomes highly electric by friction. The experiment was made by passing the chlorine over the substance kept melted in a flask by means of a water-bath. The gas was dried. The action is slow, and the experiment takes several days. When no more action was perceived, the substance was boiled with water, dried in a water-bath, and analysed.

I. 0·394 grm. of the substance gave 0·5435 CO_2 and 0·169 HO.

II. 0·4404 grm. of the substance gave 0·6120 CO_2 and 0·186 HO,

giving in 100 parts—

Carbon	37·62	37·89
Hydrogen	4·77	4·70
Oxygen and chlorine	57·61	57·41
	100·00	100·00

I. 0·51075 grm. of the substance gave 1·141 grm. of chloride of silver, equivalent to 0·2814 chlorine.

II. 0·7035 grm. of the substance gave 1·574 grm. of chloride of silver, equivalent to 0·388 chlorine.

III. 0·667 grm. of the substance gave 1·489 grm. of chloride of silver, equivalent to 0·3673 chlorine.

giving in 100 parts—

	I.	II.	III.
Chlorine	55·11	55·10	55·07

From these data we may calculate for the substance the formula $C_{54}\left\{\begin{matrix}Cl_{13\frac{1}{4}}\\H_{40\frac{3}{4}}\end{matrix}\right.O_2$.

Calculated in 100 parts—

C_{54}	38·12
$H_{40\frac{3}{4}}$	4·79
$Cl_{13\frac{1}{4}}$	55·20
O_2	1·89
	100·00

It is very difficult to tell with certainty when the action of the chlorine on the substance ceases, and I have therefore written the fractional equivalents, which agree rather more closely with the analyses than the whole numbers, $C_{54}\left\{\begin{matrix}Cl_{13}\\H_{41}\end{matrix}\right.O_2$.

The analysis determines with certainty that the substance no longer belongs to the alcohol type; for the addition of two equivalents of hydrogen to the formula would require above 0·3 per cent. more hydrogen than the quantity found, a kind of error which is highly improbable.

Cerotic Acid from the Saponification of the Wax.

The perfect washing out of the baryta salt from which the cerotin has to be separated, is attended with considerable difficulty. It is best effected by naphtha-alcohol or naphtha-ether. The wax acid, after separation from the baryta salt, is to be purified, first by long boiling with water in an open vessel, to get rid of all traces of naphtha, and then by repeated crystallization out of ether. In this way the melting-point may be raised to 78°, 79° C. In appearance the acid perfectly resembles the cerotic acid, with which it has also the same melting-point. The substance, after long repeated crystallizations, was analysed.

I. 0·2632 grm. gave 0·765 CO_2 and 0·3035 HO.
II. 0·2631 grm. gave 0·7626 CO_2 and 0·3095 HO.
III. 0·232 grm. gave 0·6695 CO_2 and 0·274 HO.

These analyses give in parts per cent.—

	I.	II.	III.
Carbon	79·26	79·04	78·70
Hydrogen . . .	12·81	13·07	13·12
Oxygen	7·93	7·89	8·18
	100·00	100·00	100·00

The formula $C_{54} H_{54} O_4$ requires

C_{54}	79·02
H_{54}	13·17
O_4	7·81
	100·00

I prepared the silver salt of this acid.

I. 0·656 grm. of substance gave 0·1335 silver.

II. 0·6635 grm. of substance gave 0·1355 silver,

which gives in 100 parts—

	I.	II.
Silver	20·35	20·42

I. 0·4675 grm. of the salt gave 1·072 CO_2 and 0·431 HO.

II. 0·4655 grm. of the salt gave 1·0767 CO_2 and 0·4295 HO,

which give per cent.

	I.	II.
Carbon	62·53	63·08
Hydrogen	10·24	10·25
Oxygen and silver .	27·23	26·67
	100·00	100·00

The formula $C_{54} H_{53} O_3 + AgO$ requires

C_{54}	62·66
H_{53}	10·25
O_4	6·19
Ag	20·90
	100·00

There is a difference between the calculated and found amount of silver of about 0·5 per cent. which I cannot doubt arises from traces of cerotin still adhering to the acid, notwithstanding all the pains I took to wash out the salt; for I found that the amount of silver increased with the purification of the acid by crystallization, even after I could find not the slightest variation in the melting-point. The silver salt made from an acid which was a part of the same original preparation as the above and of exactly the same melting-point, but which had not been so often crystallized, gave in two determinations 20·07 and 20·09 per cent. silver. It will be seen that the formula I have given for the acid is confirmed by the analysis of the acid obtained from distillation of the wax.

Distillation of Cerotin.

Cerotin requires a high temperature for its distillation. The operation is accompanied with little explosions, owing to the formation of a small quantity of water. The distillate is perfectly colourless to the last, and solid, giving, when pressed with blotting-paper, hardly a trace of oil; it resembles in its general appearance the cerotin itself, but has a lower melting-point, about 70° C.

The distillate also contains a larger per-centage of carbon than the substance. A portion, melting at 73° C. and purified by crystallization, gave to analysis carbon 83·20 and hydrogen 14·22 per cent.; but it is very difficult to raise the melting-point much beyond 73° C. At first, from the constancy of the melting-point, I was led to think that a new oxygen combination had been formed. I found, however, afterwards that the melting-point could be raised to 79° C.; that is, the melting-point of the alcohol, a substance of a lower melting-point remaining behind; and there can be little doubt but that part of the cerotin distils over unaltered, while another portion decomposes into solid hydrocarbon and water. I found, in fact, that the amount of carbon diminished as the melting-point was raised.

Distillation of Chinese Wax.

The Chinese wax is decomposed by heat. When distilled, the distillate consists of two portions; a wax acid which forms the earlier portions of the distillate, and a portion which is not acted on by potash, and from which the acid portion may be separated by saponification. The soap requires to be drawn off by a syphon. I will first give the analysis of the acid, which is undoubtedly the same acid as that procured by saponification of the wax itself. The melting-point of the acid may, by the methods pursued in other cases of separation from the soap, purification and crystallization out of ether, be raised to 80°, 81° C., which is probably the true melting-point of the cerotic acid.

0·2613 grm. of the acid gave 0·7555 CO_2 and 0·3075 HO, which corresponds in parts per cent. to—

Carbon	78·85
Hydrogen	13·08
Oxygen	8·07
	100·00

I give here again, for the sake of comparison, the numbers of the formula.

	Calculated.
C_{54} . . .	79·02
H_{54} . . .	13·17
O_4 . . .	7·81
	100·00

The silver salt of the acid, prepared as in other cases, gave to analysis the following results:—

0·3625 grm. of the salt gave 0·827 CO_2 and 0·332 HO, giving in 100 parts,—

Carbon	62·21
Hydrogen	10·18
Oxygen and silver .	27·61
	100·00

I. 0·649 grm. of the salt gave 0·139 grm. of silver.

II. 0·626 grm. of the same gave 0·13375 grm. of silver.

These determinations correspond in parts per cent. to—

	I.	II.
Silver . . .	21·42	21·19

The formula $C_{54} H_{53} O_3 + AgO$ requires—

	Calculated.
C_{54} . . .	62·66
H_{53} . . .	10·25
O_4 . . .	6·19
Ag . . .	20·90
	100·00

Ceroten.

The portion of the distillate from which the soap has been separated by decantation and by repeated washings and boiling out with water, consists chiefly of a solid hydrocarbon, one of those substances which have been comprehended and confounded under the general name of paraffin. It is mixed with a certain quantity of oil, from which it may be almost entirely separated by pressure in a press between folds of blotting-paper. If this substance be crystallized out of naphtha-alcohol and then out of ether, it may be obtained of a melting-point of 57°, 58° C. In this state it is highly crystalline on cooling, and presents the general appearance of the substance called paraffin.

I. 0·2555 grm. gave 0·802 CO_2 and 0·331 HO.

II. 0·2593 grm. gave 0·810 CO_2 and 0·332 HO,

which give in 100 parts,—

	I.	II.
Carbon	85·60	85·20
Hydrogen . . .	14·39	14·23
	99·99	99·43

These analyses correspond to the formula

		Calculated.
C_{54} . . .	324	85·71
H_{54} . . .	54	14·28
	378	99·99

This substance may be called ceroten. After the discovery of the alcohol, there was a strong presumption that the hydrocarbon and the cerotin would be related in the manner expressed by the formula, the hydrocarbon being the olefiant gas of the wax alcohol. It was, however, very desirable to find some method for the determination of its formula. I investigated, with this view, the action of chlorine on the substance.

Chlor-Ceroten.

If moist chlorine be passed over the melted ceroten in the manner before described in the case of the other wax substances, it is readily acted upon by the gas. It loses its wax character, becomes gum-like, and is ultimately converted into a transparent resin; the substance becoming harder and harder with the increase of the chlorine substituted, at length becoming extremely hard, and cracking in all directions, on cooling, on the surface of the glass on which it has been melted. The action goes on very slowly, but more rapidly with moist than with dry gas. It was continued for several weeks, until no more traces of hydrochloric acid formed could be observed.

During the operation portions of the substance were taken out at intervals and analysed.

0·303 grm. of the first portion thus examined gave 0·3495 CO_2 and 0·0965 HO, corresponding in 100 parts to—

Carbon	31·46
Hydrogen	3·54
Chlorine	65·00
	100·00

These numbers agree with the formula $C_{54}\left\{\begin{matrix}H_{35}\\Cl_{19}\end{matrix}\right.$, which requires in 100 parts,

C_{54}	31·31
H_{35}	3·39
Cl_{19}	65·30
	100·00

After an interval of about four days the substance was again analysed.

I. 0·376 grm. of the substance gave 0·4060 CO_2 and 0·9085 HO.

II. 0·362 grm. of the substance gave 0·3915 CO_2 and 0·1005 HO.

These analyses give in 100 parts,—

	I.	II.
Carbon	29·45	29·49
Hydrogen	2·91	3·09
Chlorine	68·64	67·42
	100·00	100·00

The formula $C_{54}\left\{\begin{matrix}H_{33}\\ Cl_{21}\end{matrix}\right.$ requires

C_{54} . . .	29·43
H_{33} . . .	2·99
Cl_{21} . . .	67·58
	100·00

After a further interval of several days the substance was again analysed.

I. 0·4434 grm. of the substance gave 0·464 CO_2 and 0·115 HO.

II. 0·309 grm. of the substance gave 0·323 CO_2 and 0·08 HO,

which give in 100 parts,—

	I.	II.
Carbon . . .	28·54	28·51
Hydrogen . . .	2·79	2·88
Chlorine . . .	68·67	68·61
	100·00	100·00

Notwithstanding that between these and the last analyses the action of the chlorine had been prolonged for a considerable time, the formula shows a difference of only one equivalent of chlorine.

The formula $C_{54}\left\{\begin{matrix}H_{32}\\ Cl_{22}\end{matrix}\right.$ requires

C_{54} . . .	28·76
H_{32} . . .	2·84
Cl_{22} . . .	68·40
	100·00

These analyses determine with certainty the ratio of the hydrogen to the carbon in the ceroten, and leave no doubt as to the nature of the hydrocarbon. M. Lewy attempted to take the density of the vapour of paraffin from bees'-wax. He found, however, that this could not be effected, as the substance was altered, in process of conversion into vapour, with the formation of a small quantity of hydrocarbon gas; the paraffin however remaining white, and the analysis showing no variation in composition*. It has been also remarked by others that if bees'-wax be repeatedly distilled, the solid hydrocarbon disappears from the distillate. These observations point to the source of the oil in the distillation of the Chinese wax, viz. the transformation of the ceroten itself into isomeric hydrocarbons. In fact, I found, if the ceroten be distilled and redistilled in a closed tube of the form annexed, that by effecting the distillation in this manner under pressure, after about two distillations the distillate becomes liquid and the solid matter entirely disappears. The experiment after about six distillations was put an end to by the bursting of the heated end of the tube, when a large quantity of combustible vapour

* Ann. de Chimie, Series III. vol. v. p. 398.

was given off. The oil which had collected at the other end of the tube was a mixture of hydrocarbons of various boiling-points, from 75° C. to above 260° C. No trace of solid matter was to be seen.

If from the products of decomposition we turn to the analysis of the Chinese wax itself, we find numbers which are perfectly consistent with the idea that the chemical position of this body is among the class of compound ethers, where its reactions also would lead us to place it.

To purify the substance, it is to be crystallized out of naphtha and alcohol; washed with ether to remove the naphtha; boiled with water and crystallized again out of absolute alcohol, in which it is soluble, although with difficulty. Its melting-point is 82° C.

I. 0·2644 grm. gave 0·798 CO_2 and 0·323 HO.

II. 0·2622 grm. gave 0·79 CO_2 and 0·3205 HO,

which give in 100 parts—

	I.	II.
Carbon . . .	82·31	82·16
Hydrogen . . .	13·57	13·58
Oxygen . . .	4·12	4·26
	100·00	100·00

These numbers agree with the formula

C_{108} . . .	648	82·23
H_{108} . . .	108	13·71
O_4 . . .	32	4·06
	788	100·00

This formula affords us a simple solution of the decompositions of this substance by saponification and by heat.

In the former case,

$$C_{108}\,H_{108}\,O_4+KO,\ HO=\overbrace{C_{54}\,H_{53}\,O_3,\ KO}+C_{54}\,H_{56}\,O_2.$$

In the latter,

$$C_{108}\,H_{108}\,O_4=C_{54}\,H_{54}\,O_4+C_{54}\,H_{54}.$$

It is my intention shortly to offer to the Society another communication, on the nature of myricin from bees'-wax; but I will now take the opportunity of stating that I have discovered in the investigation two wax substances of the formulæ $C_{60}\,H_{62}\,O_2$ and $C_{92}\,H_{92}\,O_4$.

PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXLVIII.

PART II.

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MDCCCXLVIII.

Adjudication of the Medals of the Royal Society for the year 1848 by the President and Council.

The Copley Medal to John Couch Adams, Esq., for his "Investigations relative to the disturbances of Uranus, and for his application of the inverse problem of perturbations thereto."

The Royal Medal in the department of Astronomy, to Thomas Galloway, Esq., for his Paper "On the Proper Motion of the Solar System," published in the Philosophical Transactions for 1847.

No recommendation of the Royal Medal in the department of Physiology having been received, this Medal was awarded to Charles James Hargreave, Esq., for his Paper entitled "On the Solution of Linear Differential Equations," published in the Philosophical Transactions for 1848.

The Rumford Medal to M. V. Regnault, for his "Experiments to determine the Laws and the numerical data which enter into the calculation of Steam-Engines."

The Bakerian Lecture for 1848 was delivered by The Rev. William Whewell, D.D., F.R.S., and entitled "On the Tides of the Pacific, and on the Diurnal Inequality."

CONTENTS.

PHILOSOPHICAL TRANSACTIONS.

XII. *Observations on some Belemnites and other Fossil Remains of Cephalopoda, discovered by Mr.* REGINALD NEVILLE MANTELL, *C.E. in the Oxford Clay near Trowbridge, in Wiltshire.*

By GIDEON ALGERNON MANTELL, *Esq., LL.D., F.R.S., F.L.S., Vice-President of the Geological Society.*

Received March 2,—Read March 23, 1848.

THE group of argillaceous deposits of the Oolite or Jurassic formation, termed the Oxford Clay, has yielded some of the most interesting and instructive examples of the fossil remains of Cephalopoda hitherto discovered in England. Christian-Malford, in Wiltshire, is especially celebrated for the very perfect specimens of the soft parts of certain extinct forms of this class of molluscous animals; examples having been obtained in which the body and muscular tunic or mantle, the cephalic arms with their uncinated acetabula, the capsule or external tunic of the eye-ball, the ink-bag, and the phragmocone, are preserved, and in some instances but slightly displaced from their natural relative positions and connections. The remarkable fossils described in the Memoir on the Belemnite* by Professor OWEN, were procured by the Marquess of NORTHAMPTON, Mr. CUNNINGTON, and Mr. PRATT, from this locality.

A branch line from the Great Western Railway to Trowbridge in Wiltshire, now in progress, in some parts passes over, and in others cuts through, the usual series of oolitic strata of that part of England; namely, the Kimmeridge Clay, Oxford Clay, Kelloway Rock, and the Great Oolite with its subordinate beds of Cornbrash, Forest Marble, Bradford Clay, &c.† My son, Mr. REGINALD NEVILLE MANTELL, who is engaged on this work under the eminent engineer I. K. BRUNEL, Esq., availed himself of this favourable opportunity of collecting a very extensive suite of the fossils brought to light by the various cuttings and excavations required in the construction of the railway. This collection comprises many hundred specimens of the shells and

* Philosophical Transactions for 1844, p. 65.

† Wonders of Geology, 6th edition, p. 502.

other organic remains which usually abound in this division of the Oolite formation, and among them are several unique and exquisite examples of Ammonites, Belemnites, &c. Imbedded with the animal remains were large quantities of drifted wood, and stems and branches of trees: some of these specimens are in the state of bog-wood and lignite; others are transmuted into limestone, and have the original structure well-preserved. Trunks and branches of coniferous trees, from ten to twenty feet in length, and from a few inches to upwards of a foot in diameter, were abundant; a few traces of the foliage of Cycadeaceous plants, and of Araucariæ, were likewise met with.

The geological character of the beds of Oxford Clay exposed along this railway, is that of a fluvio-marine formation; that is, an accumulation of deep sea and littoral shells, promiscuously intermingled with the debris of terrestrial vegetables brought into the sea from distant lands by the agency of streams and rivers, and transported by marine currents into the bed of the ocean. It would be highly interesting, but irrelevant to my present purpose, to dwell on the geological phenomena presented by the sections laid bare by the operations of the engineer along this tract of ten or twelve miles. The great quantity of the shells of mollusks referable to species which dwell in the profound abyss of the ocean, collocated with those which can exist only in waters of moderate depths, and the intercalation of drifted trees and plants, formed a striking illustration of the nature of the bed of the ancient oolitic sea, strewn with the spoils of the land, and the exuviæ of the animals with which the waters of that ocean were densely inhabited*.

As the Oxford Clay traversed by the Trowbridge line is a continuation of the beds that were cut through at Christian-Malford, whence all the specimens of Cephalopoda collected by Mr. Buy, the well-known fossil dealer of Chippenham, were procured, my son's attention was particularly directed to the discovery of examples that would tend to elucidate the nature of the soft parts of the animal to which the Belemnite belonged; for notwithstanding the memoir above referred to, doubts were entertained by several competent observers as to the validity of the arguments which led Professor Owen to assign the fossil Cephalopod, termed *Belemnoteuthis antiquus* by the late Mr. Channing Pearce, and described by him in 1842†, to the same genus as the true Belemnite, and to the species named *B. Owenii* by Mr. Pratt.

Care was therefore taken to remove, when practicable, the Belemnites, Ammonites, &c. with a large portion of the surrounding clay; and this, when hardened by drying,

* My son's collection comprises very fine specimens of *Ammonites Königi, A. Calloviensis, A. sublævis, A. athleta*, &c.; beautiful examples of a boat-like ammonite with a sharp keel, *A. Chamusseti*; a large depressed ammonite with a flat back and a single row of nodular tubercles on the wreaths; several kinds of *Nautilus*; numerous small shells of the genera *Rostellaria, Terebra, Turritella, Trochus*, &c.; *Ostrea deltoidea, Gryphæa dilatata, Terebratula*, &c.; bones of *Ichthyosauri, Plesiosauri, Teleosauri, Cetiosauri*, &c.; and a few scales and teeth of fishes.

† Geological Proceedings for 1842, vol. ii. p. 593; but not referred to in Professor Owen's memoir.

was diligently examined to ascertain whether there were any remains, or traces of the imprints, of the soft and perishable parts, of the bodies of the original animals. Although these researches were not rewarded by the discovery of any good examples of the muscular tunic, arms, &c. of the Cephalopoda, whose hard and durable relics are scattered in profusion through the strata, there are in the collection my son transmitted to me, a few specimens which present characters hitherto unobserved, or at least unnoticed, by any author, and which appear to me of sufficient importance to be placed on record, as interesting additions to our knowledge of the structure of the animal of the Belemnite.

In the following remarks I shall restrict myself to the description of the fossils, of which accurate figures by Mr. Joseph Dinkel, executed under my immediate inspection, are subjoined; and the bearing of the facts described on the still mooted question as to whether the Belemnoteuthis and the Belemnite belong to the same genus; in other words, whether the soft parts of Cephalopoda found in the Oxford Clay of Wiltshire, and figured and described by Mr. Channing Pearce, Professor Owen, and Mr. Cunnington, belong to the Belemnites geologically associated with them, but with which they have never yet been found in organic connection.

The late Mr. Channing Pearce, whose early death every British palæontologist must deeply regret, was the first who noticed and described the muscular mantle, phragmocone, uncinated arms, &c. of certain Cephalopods found in the Oxford Clay at Christian-Malford, and which he referred to a new genus under the name of *Belemnoteuthis*. According to the observations of this gentleman, and of subsequent authors, the body of this Cephalopod was of an elongated form, and contained a large internal conical shell, which is chambered and siphonated at its apical or distal extremity to the extent of about one-half (?) the length of the entire cone, and terminates anteriorly, or at its basal part, in a capacious chamber or cavity, in which the ink-bag, and probably other viscera, were placed. The external surface, which is of a brown colour, generally possesses a glossy smoothness, as if produced by its immediate contact with the secreting surface of the mantle. The outer integument of this conical shell consists of a thick corneo-calcareous layer (which for convenience I will call the epidermis), investing a nacreous, iridescent substance. Two large sessile eyes have been detected, and in several specimens the cephalic arms are more or less perfectly preserved; there are likewise indications of a pair of long tentacula, superadded to the eight shorter arms, as in the existing *Decapods**. The arms were furnished with acetabula or suckers, the horny hoops of which were beset with curved spines or hooks, as in the living *Onychoteuthis*. Traces of a pair of pallial fins have been detected by Professor Owen, to whose memoir I would refer for minute details of structure, which are not within the scope of the present communication†.

* Professor Owen, Philosophical Transactions, 1844, p. 78.

† Ibid. p. 79.

The outline, Plate XIV. fig. 1, is intended to convey an idea of the form of the animal of the *Belemnoteuthis*, so far as the fossils hitherto discovered warrant the restoration; of course this sketch is only approximative, and is given as a mere diagram of what is *actually known* as to the shape and structure of the original, and not as an accurate delineation of the living animal; for there are strong reasons to conclude that the phragmocone was wholly internal, or at least was covered by the skin. The above is, I believe, a correct description of the *Belemnoteuthis*, and of the form and relations of the several parts of its structure: it is based on the specimens described and figured by Mr. CHANNING PEARCE, Professor OWEN*, and Mr. CUNNINGTON†, and others in my possession.

Now in order to connect the body of the Cephalopod above described with the elongated conical fossil commonly known as the *Belemnite*, and termed by naturalists the *guard* or *osselet* of the same, it has been assumed that the terminal chambered cone of the former (Plate XIII. figs. 2, 3, 5) was originally implanted in the alveolus or cavity of the latter; like the unquestionable phragmocone of a Belemnite, represented in Plate XV. figs. 4, 5, *a*, *a*. It follows, if we admit the correctness of this interpretation, that every example of the chambered conical shell of a Cephalopod, found in the Oxford Clay, is the phragmocone of a Belemnite that has slipped out of the alveolus of the guard, and been compressed into its present shape by the weight of the superincumbent strata. Upon the correctness of this supposition the generic identity of the *Belemnoteuthis* and the *Belemnite* entirely rests; for no specimen has yet been met with, in which the guard of a Belemnite is in *natural contact* with the muscular mantle, and other soft parts. If the phragmocone of the Belemnite, and the chambered shell of the Belemnoteuthis be not identical, then we know nothing whatever, *from actual observation*, of the soft parts of the animal to which the Belemnite belonged: to this point therefore I would first solicit particular attention.

Phragmocone of the Belemnoteuthis.—As the figures in the Philosophical Transactions, 1844, Plate III., &c., and in the London Geological Journal, Plate XV. XVI., show the phragmocone in natural connection with the muscular tunic or mantle, &c., I need not adduce further examples of this undisputed fact, and will therefore only remark, that such specimens are of rare occurrence: in most instances, the detached chambered shell, more or less flattened, is found imbedded in the clay, with but few, if any, vestiges of the soft parts, as in Plate XIII. figs. 1, 2, 3, 5. In this state, these fossils occurred by hundreds on the newly-exposed surface of the Oxford Clay near Trowbridge. My son describes some areas when first laid bare by his labourers, as being literally spangled over with the nacreous cones of Belemnoteuthes, and the splendid iridescent shells of Ammonites; while here and there Belemnites of large size, with their phragmocones attached, were lying in relief. Upon drying, the thin

* Philosophical Transactions, 1844, p. 77–80, Plates III. IV. V. VI.

† London Geological Journal, Plates XV. XVI.

nacreous shelly covering cracked to pieces, and flaked off from the clay: the illustrative examples presently to be described, were preserved by imbuing them with a thick solution of gum, and allowing them to dry, before attempting to dig them up. From a great number of specimens I have selected four (Plate XIII. figs. 1, 2, 3, 5.) in illustration of the form and structure of the phragmocone of the Belemnoteuthis. Figs. 2, and 3, exhibit, as I believe, the perfect shell, with the peristome, or mouth of the large anterior or basal chamber, entire. Although both these fossils are much compressed, the cavity of the receptacle filled with clay is distinctly seen in fig. 2: in fig. 3, the margin of the peristome (*a*) is only visible on one side, owing to the position of the shell in the clay. In both specimens the distal extremity is perfect; it terminates in a blunt point, having a longitudinal ridge on the side exposed, which probably indicates the dorsal aspect: in other instances, the corresponding space on the opposite or ventral side is occupied by two elevated, flat, slightly divergent longitudinal bands, as in Plate XIII. fig. 5; in fig. 2, the thick corneo-calcareous epidermis covers the lower part of the cone, but is in a great measure removed from the upper portion, and the pearly or nacreous layer of shell which lies beneath is exposed. The epidermis in fig. 3, though much cracked by pressure, extends over the greater part of the shell.

The descriptions of the phragmocone of the Belemnoteuthis, as given by Mr. Channing Pearce and Mr. Cunnington, accord in every respect with my own observations. Mr. Cunnington particularly dwells on the constant and uniform occurrence of the two strongly-marked ridges which extend upwards from the apex in a nearly parallel direction. "These," he observes, "are shown in Plate II. fig. 6. of Professor Owen's paper, and are regarded by that gentleman as resulting from the accidental crushing of the shell. I have, however, before me thirteen specimens, in which these ridges, though in some instances displaced by the compression they have undergone, are yet so distinct, and so constantly exhibit the same relative proportions and distances, that it is impossible not to regard them as resulting from original structure. A transverse section shows that these ridges are formed by an elongation of the fibres, and consequent thickening of the shell. Moreover, they are always exceedingly distinct in those specimens where the apex is solid—as is sometimes the case to the extent of half an inch—and has resisted the force that has compressed the rest of the shell. *It is obvious that this structure could never have fitted into the circular cavity constituting the alveolus of the Belemnite*.*" My son long since pointed out to me the marked difference in the form of the respective shells; that of the Belemnite being an elongated cone (Plate XV. figs. 4, 5, *b*), and that of the Belemnoteuthis more obtuse (Plate XIII. figs. 4, 5); and he remarked that no amount of pressure could reduce the former into the same angle as the latter.

[In the cabinet of the late Mr. Channing Pearce, there is a portion of a phragmocone of a Belemnoteuthis which is less compressed than any other specimen I have

* London Geological Journal, pp. 98, 99.

seen, and is also the only instance in which the true external surface of the epidermis is preserved. Through the kindness of Mr. Charlesworth I have been favoured by Mr. Pearce, Senior, with the loan of this fossil, which is represented in Plate XIII. fig. 1. The external pellicle, though cracked, is almost entire, but is removed in a few places (*a*, *a*, *a*), so as to expose the brown corneo-calcareous layer which in most specimens appears as the outer case, as in Plate XIII. figs. 2, 3. The surface is very finely striated with delicate interrupted elevated lines, disposed in a longitudinal direction, as shown in the slightly enlarged drawing of part of the same at fig. 1 *a*. This structure appears to be analogous to that observable on the back of the sepiostaire of the Cuttle-fish, and on the guards of certain species of Belemnites; but I have never observed any traces of such a texture on the surface of the phragmocones of the latter, which invariably presents the characters delineated in Plate XV. fig. 4, *a*.]*

According to the facts at present known, the chambered cone of the Belemnoteuthis appears to me to approximate most nearly to the *Beloptera*†, which has a horny calcareous conical shell with transverse septa, and is destitute of a guard.

The Belemnite.—To avoid unnecessary details, I beg to refer to Dr. Buckland's Bridgewater Essay‡, Professor Owen's Hunterian Lectures§, and my Medals of Creation||, for descriptions and figures of the form and structure of the Belemnite as indicated by the specimens then known; and I now proceed to describe the most illustrative example from Trowbridge, which is represented (somewhat less than half the size of the original) in Plate XV. fig. 3. This fossil comprises the following parts:—

1. *The Capsule*¶ or *periostricum*; this external investment (c', c', c'), which consists of a thin shelly or corneo-calcareous integument that closely embraces the guard, and gradually enlarging upwards, finally surrounds the peristome of the phragmocone, constituting the thin horny laminated sheath or receptacle (*c*, *c*), has been described by all previous observers as an extension of what they termed the sheath or capsule; within this receptacle the ink-bag and other viscera were probably contained.

2. The *Guard* or *Osselet* (Plate XV. figs. 1, 2), which is the fossil known to collectors as the Belemnite. This is an elongated conical body, of a fibro-calcareous spathose structure, diminishing in size towards its distal extremity, and terminating in a point. At the basal, or opposite end, it is truncated, and has a deep conical cavity termed the *alveolus*, which contains the siphonated apical portion of the phragmocone.

3. *The Phragmocone* (Plate XV. figs. 3, 4, 5, *a*, *a*): this is a thin shelly inverted cone; the distal part containing a series of from twenty to thirty air-chambers or cells,

* *Received March* 16, 1848.—S. H. C.

† See Dr. Buckland's Bridgewater Essay, Plate 44′, fig. 15.

‡ Ibid. Section VII. p. 371. § Vol. i. p. 333. || Vol. ii. p. 462.

¶ I would restrict the term sheath, or *capsule*, to this outer corneo-calcareous case or integument, for the phragmocone cannot properly be said to have a capsule, its conical shell being inseparably connected with the transverse septa; it would be quite as correct to designate the involuted part of the shell of a Nautilus by this term, as the conical shell of the phragmocone of a Belemnite.

formed by concavo-convex transverse septa or plates. This apparatus has been aptly compared by Dr. Buckland to a pile of watch-glasses, gradually diminishing in size towards the apex; the septa are perforated by a tube or siphunculus, which is situated on the ventral margin. The phragmocone enlarges upwards, and anteriorly to the siphonated part constitutes a large chamber, from the margin of which are produced two, or more, long, upright, shelly or calcareous processes, as shown in Plate XV. fig. 3, *b*, *b′*; these were probably for the support of the soft parts, or for the attachment of muscles. Whether there were similar processes on the opposite margin of the chamber cannot be determined, as the peristome is imperfect.

The figure in outline, Plate XIV. fig. 2, is intended to define these several parts more clearly.

a, a, a. The *Capsule, periostricum,* or external sheath of the Belemnite, now first described.

b. The *Guard* or *Osselet.*

c. The apical chambered part of the phragmocone which is situated in the *alveolus* or cavity of the *guard.*

d. The visceral or basal chamber of the phragmocone.

e, e. Two elongated processes produced from the margin of the peristome of the visceral chamber; observed for the first time.

In the specimens delineated, Plate XV. figs. 1 and 2, the guard is in great part covered by the external integument or shelly capsule (*c′*, *c′*, *c′*), which is very thin, and resembles in colour and in its glossy appearance, the outer coat or epidermis of the phragmocone of the Belemnoteuthis (Plate XIII. figs. 2, 3). The capsule is seen adherent to the crushed phragmocone beyond the upper part of the guard in Plate XV. fig. 2; and in fig. 3 it is shown to be continuous with the horny receptacle into which it ultimately expands (*c″*).

Professor Owen detected on the exterior surface of some spathose guards obtained from the Oxford Clay, "traces of a more immediate investment of a thin friable layer of white calcareous matter, analogous to that of the outer layer of the sheath of the phragmocone;" but the specimens were evidently too imperfect to demonstrate the nature of this *periostricum,* and its identity with the expanded capsule or horny receptacle above; for the phragmocone is erroneously described as having a sheath "continued backwards to line the alveolar cavity, and forwards from its basal outlet to form the visceral chamber anterior to the phragmocone*."

The appearance of the surface of the phragmocone situated in the alveolus, when first exposed by breaking away a portion of the investing guard, is represented, Plate XV. fig. 4, *a*; and the same specimen with part of the shell removed, to show the edges of the plates of the air-chambers, in fig. 5, *a*. The annular risings and depressions on the surface of the shell, fig. 4, *a*, indicate the situation of the concavo-convex transverse septa. The phragmocones of the Belemnites in the Oxford

* Philosophical Transactions, 1844, p. 69.

Clay of Wiltshire are not symmetrical; but the apex of the cone inclines to one side in consequence of the position of the axis of the guard, and terminates in a very fine point, as in the example, fig. 4. In this specimen the cells are filled with calcareous spar, as is frequently the case with that portion of the phragmocone which is implanted in the alveolus; a fact doubtless connected with the similar mineral state of the osselet itself. The latter is almost invariably saturated as it were with crystallized carbonate of lime of a brown colour and radiated structure; for notwithstanding the microscopical observations of Dr. Carpenter and Professor Owen, I think the evidence is in favour of the conclusion of the Dean of Westminster, that this crystalline condition has resulted from sparry infiltration, subsequently to interment, into the cellular and radiated calcareous texture of which the osselet was originally composed*; in like manner as the crustaceous coverings and spines of the Echinoderms in the Oolite and Chalk, are so saturated with calc-spar as to have acquired the hardness and peculiar oblique fracture of the crystalline mineral matter.

The facts above described appear to me to confirm the opinion advanced by Mr. C. Pearce and Mr. Cunnington, that the Belemnoteuthis is generically distinct from the Belemnite. In addition to the discrepancies in the form and structure of the phragmocones of the respective Cephalopods, pointed out by these observers, the specimen represented, Plate XV. fig. 3, demonstrates that in the Belemnite there are two other essential characters of which no traces have been detected in the Belemnoteuthis; namely, the periostricum, capsule, or external corneo-calcareous investment (c', c, c); and the processes that extend from the basal margin of the visceral chamber (b, b'), the peristome of the Belemnoteuthis being entire (Plate XIV. figs. 3, 4). It is worthy of remark, that the surface of the capsule of the Belemnite, and that of the phragmocone of the Belemnoteuthis, are alike smooth and glossy; as if in the one the osselet, &c., and in the other the chambered shell, were originally wholly enveloped by the mantle. But the exterior of the siphonated phragmocone of the Belemnite is devoid of polish, and is in such close contact with the walls of the alveolus, that it is very rarely the shell can be displayed entire as in the specimen, Plate XV. fig. 4.

If it be contended that upon physiological grounds the phragmocone of the Belemnoteuthis must have been implanted in the alveolus of a guard or osselet, I would reply that none of the species of Belemnites with which these chambered cones are associated in the Oxford Clay of Wiltshire, can have belonged to them, for the reasons already assigned; and a guard has still to be discovered that shall meet the exigencies of the case. If, therefore, with our present scanty information, we affirm that the Belemnite and the Belemnoteuthis belong to one and the same genus, I respectfully submit that we are not only reasoning in advance of the data hitherto obtained, but contrary to known facts.

From a tolerably extensive knowledge of the fossil Cephalopoda of the argillaceous strata of the Oolite and Lias, I am led to conclude that the Oxford Clay of Wiltshire

* Bridgewater Essay, p. 372.

contains the remains of at least three genera of naked Cephalopods ; namely, the Belemnite, Belemnoteuthis, and a true Calamary with a horny dorsal gladius or pen. In the two last, vestiges of an ink-bag, or of its inspissated contents, are generally manifest ; but I have never detected the slightest trace of the existence of such an organ in natural connection with any part of a Belemnite ; and I learn from M. Van Breda, that in the very large collection of Belemnites of the late Count Munster, there was not one unequivocal instance of this kind. If in the specimens figured in the Bridgewater Essay, Plate 44′, fig. 7, the ink-bag be in its natural position, and not merely in accidental apposition with the basal chamber of the phragmocone of a Belemnite, of course the question as to whether this genus of Cephalopods was furnished with such an organ, is decided in the affirmative.

In conclusion, I would state, that although I am of opinion that the body and soft parts, and consequently the true characters of the animal of the Belemnite, have yet to be discovered, I do not question the soundness of the views of the correlation of cephalopodal organization, and of the physiological inductions resulting therefrom, enunciated in the memoir on the Belemnite by the Hunterian Professor. That the unknown animal of the Belemnite closely approached the Belemnoteuthis, both in form and structure, is highly probable, but proofs are yet required to confirm the inferences of the physiologist.

19 *Chester Square, Pimlico,*
February 1848.

Description of the Plates.

All the figures are from specimens in my possession, and of the natural size, with the exception of fig. 3, Plate XV., which is much reduced, the original specimen being twenty-two inches long.

PLATE XIII.

Figs. II. III. V. Phragmocones of *Belemnoteuthis antiquus.*

a. The basal or upper or anterior part, forming a chamber for the ink-bag, and probably other viscera.

b. The apical or distal termination of the shell.

Fig. I. The apical portion of the phragmocone of a Belemnoteuthis invested with the external integument; at *a, a, a,* is seen the corneo-calcareous layer which lies beneath.

Fig. I^a. Part of fig. 1, slightly magnified to show the striated surface; *a.* marks a denuded space, exposing the internal layer.

Fig. II. Shows the perfect form of the peristome, *a*; though much compressed, the cavity of the chamber, partially filled with clay, is well-defined.

Fig. III. This specimen is also perfect, but in consequence of its position in the clay, only one-half of the basal or upper part of the margin of the peristome (*a*) is visible.

Fig. IV. *Ammonites Jasoni*, from Trowbridge, with two elongated processes extending from the margin of the aperture, bearing some analogy to the shelly prolongations from the peristome of the Belemnite, Plate XV. fig. 3, *b*, *b'*.

Fig. V. A phragmocone showing the ventral (?) aspect of the chambered apex, with two parallel longitudinal bands extending upwards (*b*).

PLATE XIV.

Fig. I. A restored outline of the animal of the Belemnoteuthis, so far as at present known.

a. The cephalic arms.
b. Remains of a pair of long tentacula.
c. The eyes.
d. The pallial fins.
e. Ink-bag.
f, *f*. The mantle.
g. The phragmocone; the letter indicates the visceral chamber of the same.
i. Apical or distal part of the phragmocone.
k. Two longitudinal bands on the ventral aspect of the same.

Fig. II. Outline of the known parts of the Belemnite.

a, *a*. The capsule or *periostricum*.
a', *a'*. The horny expansion of the same forming the receptacle which surrounds the basal chamber and peristome of the phragmocone.
b. The guard or osselet, one-half being removed to show the radiated structure, and the alveolus with the apical part of the phragmocone, at *c*.
c. The chambered distal extremity of the phragmocone.
d. The visceral chamber of the same.
e, *e*. Two calcareous processes arising from the margin of the peristome at *f*, *f*.

Fig. III. Phragmocone of Belemnoteuthis.

a. The visceral chamber.
b. Ridge on the dorsal side of the apex.

Fig. IV. A phragmocone of Belemnoteuthis seen on the ventral aspect.

b. Two nearly parallel elevated longitudinal bands.

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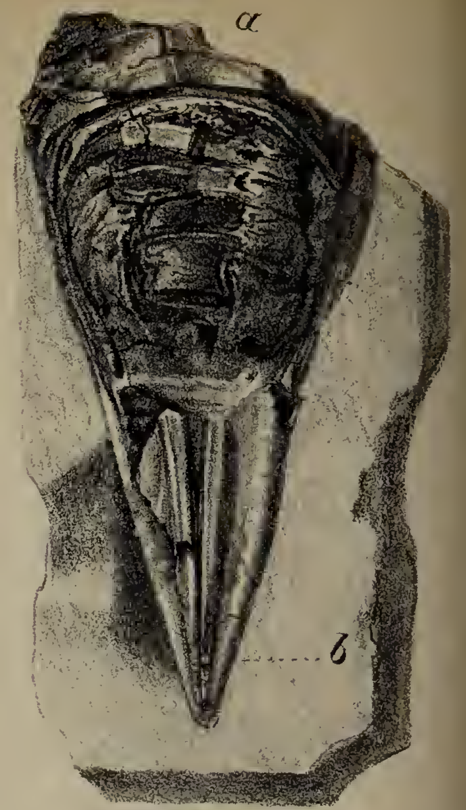

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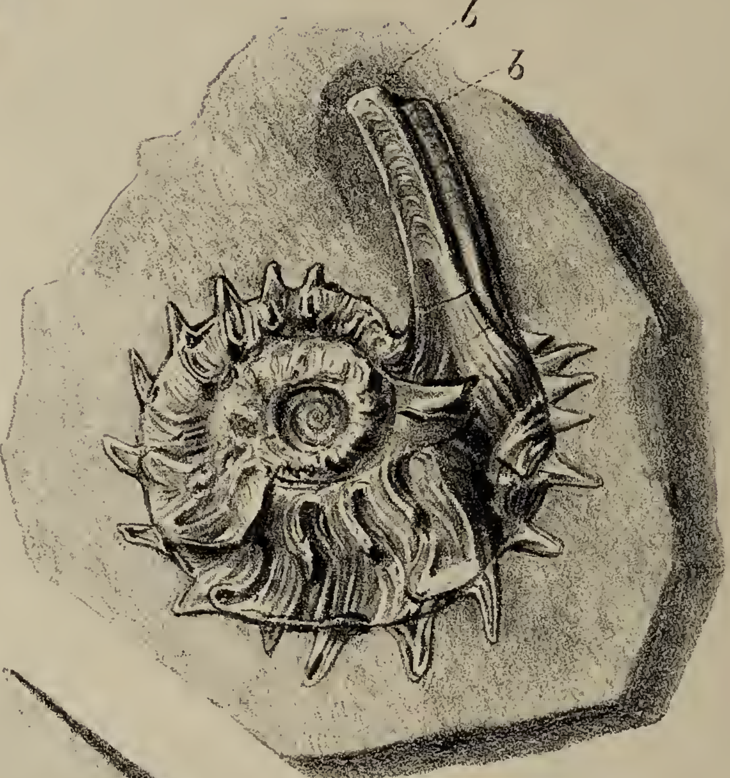

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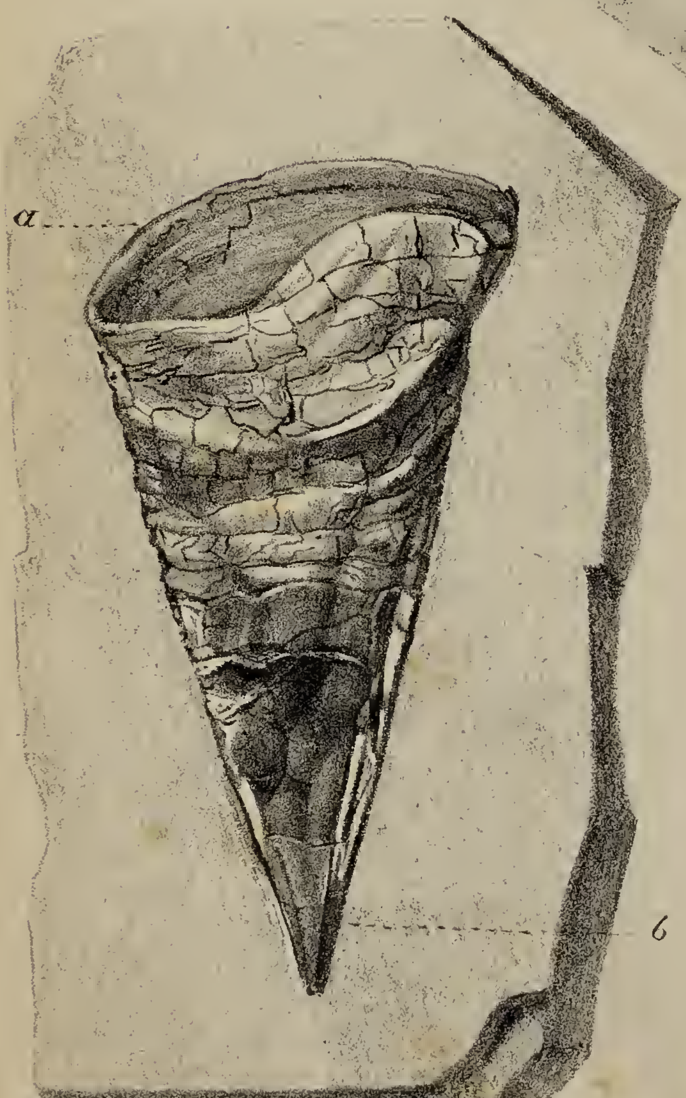

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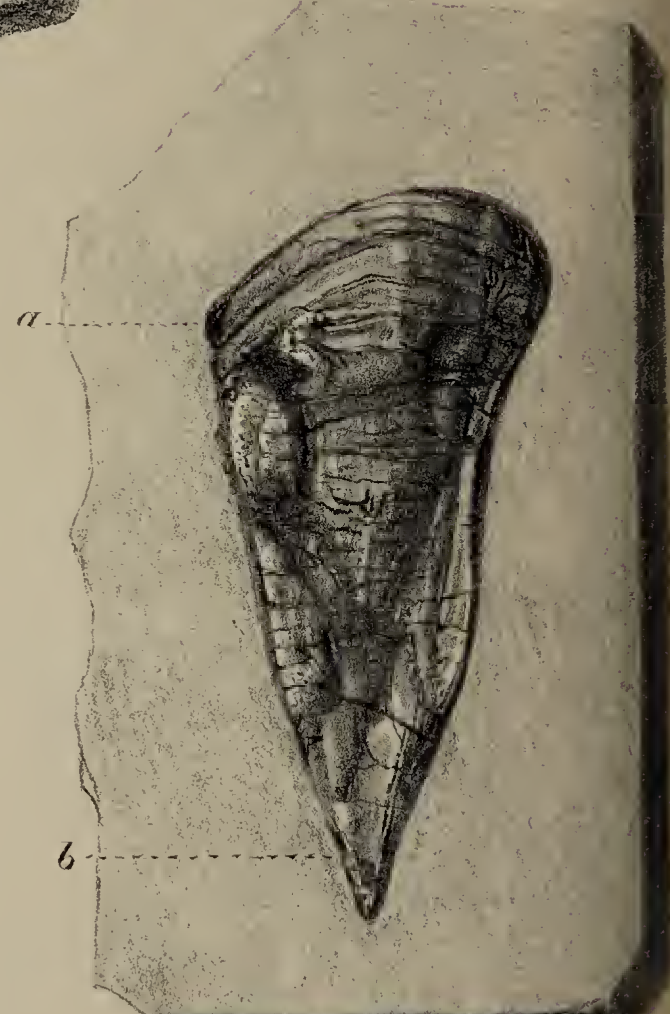

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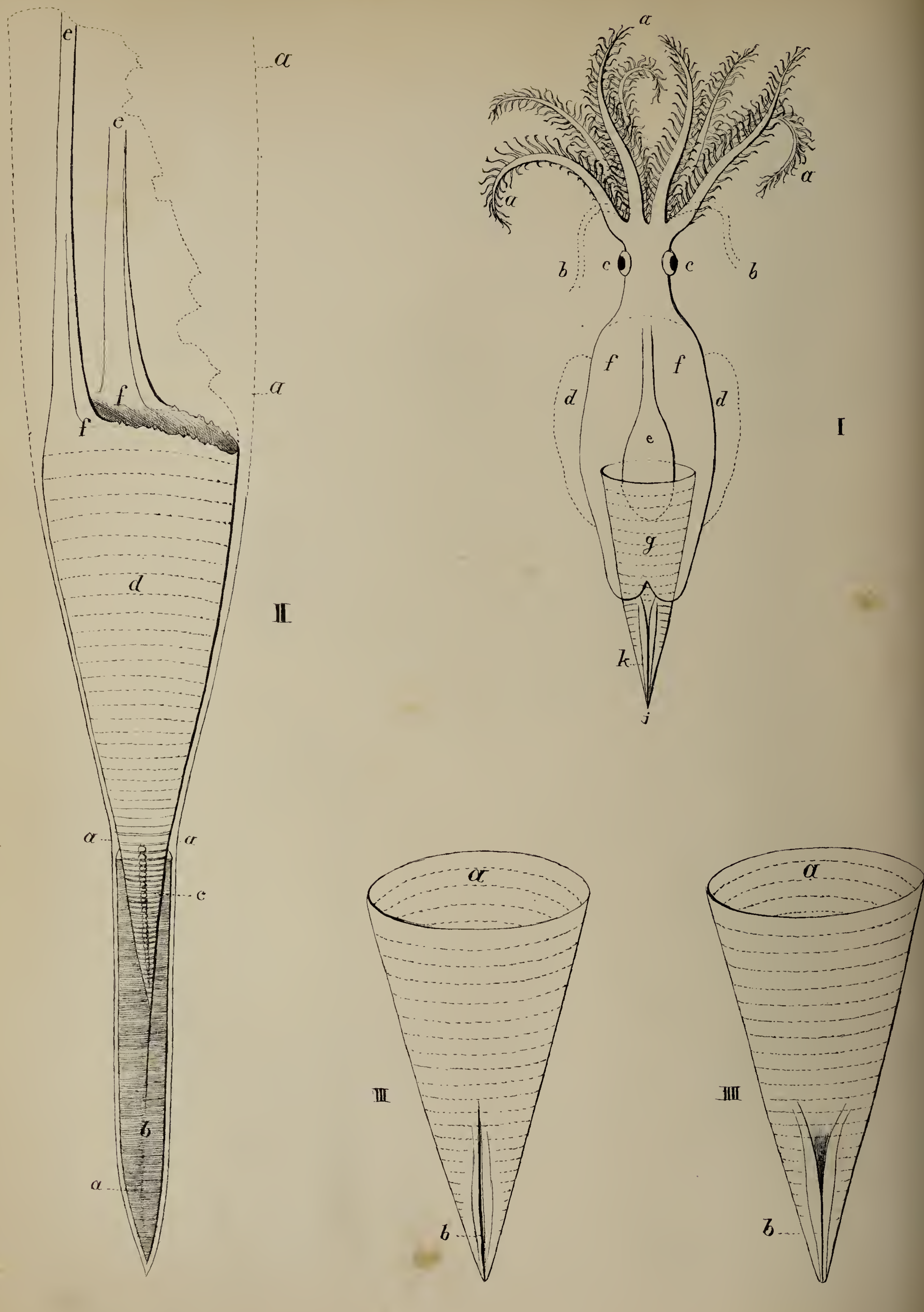

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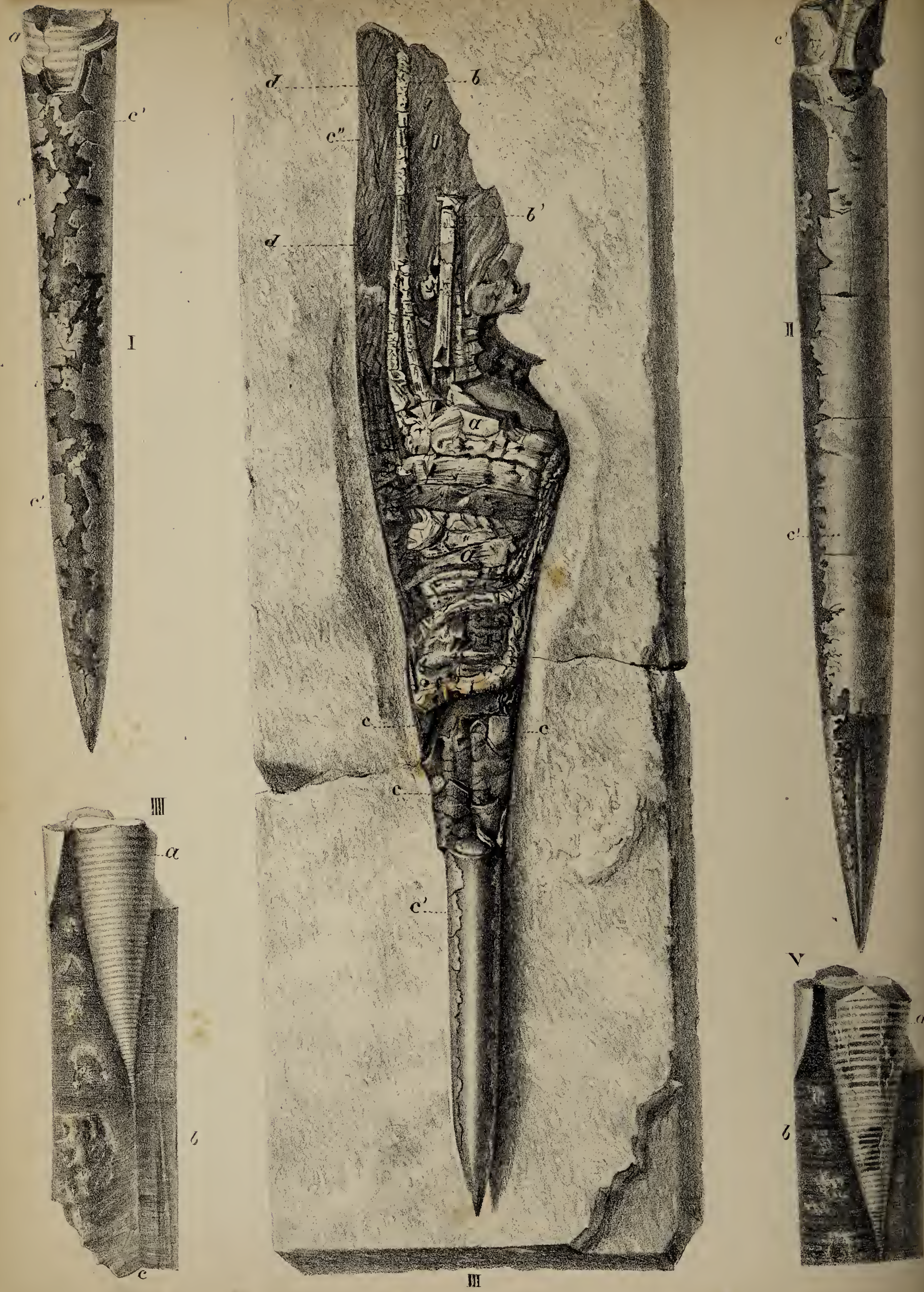

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PLATE XV.

Fig. I. The guard or osselet of a Belemnite (*B. attenuatus*) invested by the *periostricum,* or external shelly capsule, *c′*, *c′*, *c′*.

a. Part of the chambered phragmocone seen in the alveolus.

Fig. II. Another example of *B. attenuatus,* on which a large portion of the capsule remains, *c′*, *c′*.

Fig. III. Exhibits the most considerable part of the structure of the Belemnite hitherto discovered. The original is twenty-two inches in length.

a, a. Remains of the basal portion of the shelly phragmocone; the apical chambered extremity is concealed by the investing capsule and guard, *c, c.*

b, b. Two shelly processes proceeding from the margin of the peristome.

c, c′, c″. The capsule partially investing the guard (at *c′*), and extending upwards conceals at *c, c, c,* the siphonated part of the phragmocone, and expands into the horny receptacle (*c″*), within which the peristomal processes (*b, b′*) are contained.

A layer of dark brown animal matter extends from within the outline of the capsule at *c″*, under the processes of the peristome; and is probably the remains of the soft parts that occupied the interval between the receptacle and the phragmocone. The band on the left of the process *b,* presents a fibrous structure (*d, d*), and is marked with oblique lines, as if it were part of the muscular tunic or mantle.

Fig. IV. The alveolar part of the guard of a Belemnite split open longitudinally to show the apical chambered portion of the phragmocone (*a*), with the shell perfect, and imbedded in the alveolus. The phragmocone terminates in a very fine point, and is unsymmetrical; inclining considerably to one side, in consequence of the position of the axis of the guard (*c*). A comparison of this figure with figs. 2, 3, 5, Plate XIII., will at once enable the observer to perceive how essentially the phragmocone of the Belemnite differs from that of the Belemnoteuthis.

b. The radiated fractured surface of the spathose substance of the guard.

Fig. V. The same fossil, with the phragmocone partially divested of the outer shell, to display the edges of the transverse septa. I have never seen in the Oxford Clay, the apical part of the phragmocone of a Belemnite detached from the alveolus.

XIII. *On the Structure of the Jaws and Teeth of the Iguanodon.*

By Gideon Algernon Mantell, *Esq., LL.D., F.R.S., F.L.S., Vice-President of the Geological Society, &c.*

Received May 25,—Read May 25, 1848.

In the deltas and estuaries of rivers that are of great extent, and which flow through countries of varied geological structure, we naturally expect to find the remains of terrestrial vertebrated animals that have been transported by the currents from far-distant lands, in a more or less mutilated state; the skeletons broken up—the bones dissevered, fractured, and waterworn—the teeth detached from the jaws and dispersed—and all these separated parts promiscuously imbedded in the mud, silt, and sand of the delta, and intermingled with the debris of the flora of the country, and the durable remains of fishes, mollusks, and crustaceans, that inhabited the freshwater, or were denizens of the adjacent sea. Such is the condition in which the bones and teeth of oviparous quadrupeds are found in the Wealden formation of the south-east of England; and hence the difficulty of obtaining satisfactory evidence of the form and structure of the extinct reptiles whose relics are so abundant in some of these deposits.

To this cause may be ascribed the remarkable fact, that although several hundred teeth, belonging to seven or eight genera of Saurians, have been collected from these fluviatile strata, scarcely a portion of the cranium, and but a few fragments of the jaws, have been discovered. Every relic of this kind is consequently in the highest degree interesting, and it is therefore most gratifying to me to have it in my power to lay before the Royal Society a considerable portion of the lower jaw, with teeth, of an Iguanodon, recently obtained from a quarry near Cuckfield in Sussex; the locality in which, nearly thirty years since, I first discovered the teeth of this colossal herbivorous Lizard.

In the communication which I had the honour to address to this Society in 1841 *, a fragment of the lower jaw of a Saurian was described as that of a young Iguanodon, and the anatomical considerations which led me to offer that interpretation were fully detailed. But although from the form and mode of implantation of the fangs of the mature teeth, and the position of the germs of the successional ones, this inference appeared to be highly probable, yet as none of the crowns of the teeth remained, the peculiar dental characters of the Iguanodon were absent, and the presumed generic identity could not be unequivocally established; since it was possible

* Philosophical Transactions, Part II. p. 131.

that the fossil might belong to the Hylæosaurus, or to some other genus of reptiles whose bones occur in the Wealden deposits.

The specimen to which I now solicit attention consists of nearly the entire dentary and coronoid bones of the right side of the lower jaw of an adult animal, retaining two successional teeth in place, and the germ of a third, with the alveoli or sockets of seventeen or eighteen mature molars. This fossil is the first indisputable portion of a jaw of the Iguanodon hitherto brought to light; although nearly a quarter of a century has elapsed since the publication in the Philosophical Transactions* of my first memoir on the teeth of this extinct reptile.

From the striking analogy presented by the worn teeth of the Iguanodon to the condition of the abraded molars in some of the large herbivorous Mammalia, the discovery of the maxillary organs of this reptile was an object of the highest palæontological interest, in order to determine in what manner the mastication of vegetables was effected by an animal belonging to a class, in which no living species is provided with jaws so constructed as to be capable of a triturating or grinding motion; nor with cheeks to retain the food while such a process is performed†. And though from the absence of mature teeth in the sockets, and of the articular portion of the jaw, the proximal end being destroyed, the specimen before us does not afford a complete solution of the problem, yet it possesses characters sufficiently definite and intelligible to throw important light on the structure and functions of the dental organs of the Iguanodon; and it has also enabled me to ascertain the form of the upper jaw, from a portion of the left maxillary bone, collected many years since, and now in the British Museum, but whose peculiar characters I was previously unable satisfactorily to interpret.

Before entering upon the description of the highly interesting fossil which forms the principal subject of this memoir, I beg to express my most grateful acknowledgments to Captain Lambart Brickenden, of Warminglid, Sussex, by whom it was discovered, and skilfully extricated from the sandstone in which it was imbedded; and who, although I was personally unknown to him, in the true spirit of an ardent and liberal promoter of science, placed it at my disposal, as the original investigator of the fossil Saurians of the Wealden; a tribute of respect that I regard as a high reward for my humble efforts to advance those branches of natural knowledge, to which I have devoted the leisure moments of a life of professional toil.

The specimen when discovered was imbedded in a block of the fawn-coloured sandstone which occurs interstratified with beds of clay and limestone, throughout a considerable part of the Wealden districts of the south-east of England; fortunately this stone is not very compact, so that organic remains can be extricated from it by a skilful manipulator, with but little difficulty. This fossil, like most of the bones and teeth

* "Notice on the Iguanodon, a newly discovered Fossil Reptile, from the Sandstone of Tilgate Forest, in Sussex."—Philosophical Transactions for 1825.

† See Cuvier, Oss. Foss. tome v. p. 351.

found in the sandstone, is heavy, and of a rich umber colour, from impregnation with oxides of iron. It consists of the dentary, and part of the coronoid or complementary bone, of the right side, and is entire at the anterior part, but the posterior or opposite extremity is imperfect, probably to the extent of several inches. Its original relative position in the jaw will be understood by referring to my former communication*, in which the peculiar construction of the lower jaw in Saurian reptiles is described. In this place it may, however, be proper to remark, that of the six pieces on each side of which the inferior maxilla consists, that containing the teeth, and forming the anterior portion or symphysis, is termed the *dentary*; and the posterior part of this bone is united on the outside by suture to the *complementary* or *coronoid*, *angular*, and *surangular*; while on the mesial or inner side, it is covered below and behind the teeth by an expansion of the *opercular* bone.

The specimen is represented of the natural size in Plate XVI.; its dimensions are as follow:—

	inches.
Length from the front of the symphysis to the posterior extremity of the bone	21
Greatest width of the outer surface measured over the convexity, from the lower margin to the upper alveolar edge	$6\frac{3}{4}$
Greatest thickness at the posterior part	$2\frac{3}{4}$
Length of the alveolar parapet for twenty teeth	15
Breadth from the anterior termination of the alveolar space across to the inner margin	$4\frac{1}{4}$
Height of the alveolar parapet at the posterior part	2
Width of the alveolar space at the posterior part	$1\frac{1}{4}$
Width of the alveolar space at the anterior part	$\frac{3}{4}$
Length from the first anterior tooth to the symphysial extremity	5

Height of the successional tooth (Plate XVI. fig. 1, *b*) $1\frac{1}{2}$ inch; greatest width $\frac{7}{8}$.

The mesial or inner aspect of the fossil (Plate XVI. fig. 1) is flattened and smooth, and shows the successional teeth which remain in their original places (*a*, *b*), and the sockets (*f*, *f*, *f*,) for nineteen or twenty teeth; the inner alveolar plate having been destroyed, and the mature molars dislodged, before the bone was imbedded in the rock. The deep conical groove or furrow, so constantly present on the inner side of the dentary bone in reptiles, and which from its being covered by the splenial or opercular piece, it may be convenient to designate the *opercular furrow*, is here entirely exposed (fig. 1, *d*, *d*) in consequence of the removal and destruction of that maxillary element. It is very large, and prolonged anteriorly (*d*) to within six inches of the symphysis; the opercular piece must therefore have more nearly corresponded with that of the Varanians or Monitors than with the Iguanas, in which it is of a rhomboidal figure, and relatively of limited extent. The lower margin of the jaw is thick and convex at the posterior part, and gradually becomes thinner towards the front, where it expands horizontally into a broad scoop-like process, which is terminated anteriorly by an obtuse projection or tubercle (Plate XVI. fig. 1, *e*); it thins out mesially to form the symphysial suture (*s*) that connects it with the opposite ramus.

The upper margin is formed by the alveolar process, which has a thick external

* See Philosophical Transactions for 1841, Plate V. figs. 3. 7.

parapet, deeply furrowed on the inner side, as seen in this view, by the sockets for the mature teeth (*f, f*). Strongly-defined ridges occupy the interspaces, and rising above the sockets produce a sharp crenated upper border. The alveolar space is protected on the inner side by a moderately strong plate or wall, which must originally have nearly equalled the outer parapet in height, but is now in a great measure broken away: within this process the germs of the successional teeth were developed.

The mode of implantation of the teeth appears to have been intermediate between the pleurodont and thecodont types; for the teeth were not anchylosed to the alveolar wall as in the Iguanas, but free as in the Crocodiles; yet as the ridges that separate the dental sockets are smooth and rounded, it may be inferred that these were not rendered complete alveoli by transverse plates extending from the outer to the inner parapet, as is the case in the Megalosaurus*.

The dental sockets diminish in size, but somewhat irregularly, from the posterior to the anterior termination of the alveolar process; and the latter suffers a corresponding diminution in breadth, and terminates suddenly at the distance of five inches from the front. At this point the upper margin becomes attenuated and contracted in a vertical direction, and descending with a gentle curve, expands horizontally and mesially to unite at the symphysial suture (*s*) with the opposite ramus, the anterior part of the jaw being edentulous.

From the fortunate preservation of two successional teeth in their original position, the mode of dental development in the Iguanodon is clearly demonstrated. The coronal portion of the tooth was first formed, as seen in the germ, Plate XVI. fig. 1, *a*; and the entire crown was completed (fig. 1, *b*) before the secretion of the shank or fang commenced, as in the existing Saurians. The formative pulp was situated in a distinct depression or cavity, on the inner side of the root of the tooth it was destined to supplant: this is obvious by the positions of the teeth above described; and also by the remains of a third germ towards the posterior part (fig. 1, *c*).

Although the peculiar characters of the molars of the Iguanodon were described somewhat in detail in my former communications, and the present fossil confirms in every essential particular the inferences suggested by the detached teeth, as stated in my memoir of 1825, yet several new and important points relating to the development and functions of the dental organs, are elucidated by the new acquisition which Capt. Brickenden's researches have brought to light. The second tooth (Plate XVI. fig. 1, *b*) which occupies its natural position in the alveolar space, consists of the entire crown, having the serrated margin as perfect as in the recent state; and this is the first evidence I have obtained as to the mode in which the teeth were implanted. The flat enamelled front, characterized by its longitudinal ridges, is placed mesially, and parallel to, and within the inner alveolar wall; the smooth convex face filling up a depression in the outer parapet, in the interspace of two sockets of the mature molars. This position is the reverse of that in which the successional teeth in the Iguana are developed; for in that reptile the coronal germ occupies the same relative place as

* See Dr. Buckland's Bridgewater Essay, Plate 23.

in the mature state; namely, with the ridged face outwards, and the smooth side inwards, or towards the cavity of the mouth.

As the coronal portion of the tooth in the Iguanodon is not symmetrical, one lateral margin presenting a gentle curvature (Plate XVIII. fig. 4, *a*), and the other forming a broad angle at the base of the serrated border (Plate XVIII. fig. 4, *b*), the teeth belonging to one side of the lower jaw may readily be distinguished from those of the other; the lateral marginal angle (*b*) being always situated posteriorly. Guided by this character, Dr. Melville and myself examined the numerous teeth in the British Museum and in my own collection, and were enabled readily to determine to which ramus any tooth belonged. Thus, for example, the specimen represented, Plate XVIII. fig. 4, which is the very counterpart of that implanted in the jaw (Plate XVI. fig. 1, *b*), is a perfect successional tooth, consisting of the crown before the formation of the fang, and belonging to the right side (*a* denotes the anterior, and *b* the posterior angle). The specimen, Plate XVIII. fig. 5, appertained to the opposite or left lower side, as is shown by the situation of the posterior marginal angle (*b*).

The position of the lower teeth in relation to the alveolar process, appears to have been somewhat changed during the upward growth of the coronal portion consequent on the progressive development of the fang; and it seems probable that the face of the crown became inclined rather obliquely forwards and outwards, and that the mature teeth were arranged in an imbricated manner. This opinion is supported by the form of the alveoli in the outer parapet, and the corresponding oblique curvature in the fangs of the mature molars, as shown in Plate XVIII. fig. 5; but this inference does not admit of that absolute proof which the perfect adaptation of a full-grown tooth to one of the sockets would afford; for the alveoli are irregular, and none of the detached teeth in my possession will fit either of the sockets in the recently discovered dentary bone.

The situation of the germ in relation to the tooth it was destined to supplant, is invariably on the inside of the mouth; in the lower molars the excavation in the mature tooth occasioned by the upward growth of the germ, is consequently on the enamelled mesial face, as is shown in my original memoir*: in the upper tooth the germ was lodged in an excavation on the smooth convex aspect, as will subsequently be demonstrated (see Plate XVIII. fig. 2^{a}, *f*).

Xylograph, No. 1.

Lower tooth of the right side.

In the fossil represented in the annexed sketch, the cavity produced by the pressure of the germ is situated in the fang of the tooth in place; in other examples, however, the successional dental excavation is on the base of the enamelled crown; for in the Iguanodon the old teeth were retained till nearly the entire coronal portion was worn away, and the crown of the tooth, from the abrasion by use above, and the removal of the fang by absorption below, was reduced to a mere disk, before it was finally shed; as in the specimen (figured in Xylograph, No. 2, p. 188).

* Philosophical Transactions, 1825, Plate XIV. fig. 7 *a*.

Xylograph, No. 2.

Lower tooth reduced by abrasion of the crown and absorption of the shank.

As the surface of the crown when abraded by mastication invariably possesses two distinct facets (Xylograph, No. 4, and Plate XVIII. figs. 1^b and 2^b), it is obvious that the arrangement of the lower teeth in relation to the upper was intermediate, or sub-alternate, as is the case in the ruminants ; the further consideration of the dental characters of the Iguanodon will be resumed in a subsequent part of this memoir.

The *external aspect* of the specimen (Plate XVI. fig. 2, and Plate XVII. fig. 4) presents in its transverse diameter a gentle convexity, traversed by a slightly elevated longitudinal ridge, which lies parallel to, and immediately beneath, the row of vascular foramina, commonly met with in this part of the lower jaw in reptiles ; and towards the posterior extremity the side of the bone is somewhat compressed below the longitudinal eminence; according in this respect with the portion of a lower jaw of a much smaller reptile, described in a previous memoir*. The upper edge of the bone is formed by the outer alveolar parapet, which is deeply scalloped or crenated by the terminations of the sockets of the teeth ; the angular eminences indicate the intra-alveolar ridges. A reference to the figures (Plates XVI. and XVII.) will impart a more correct idea of the configuration of this part of the fossil than any verbal description. The whole surface of the bone is covered with minute punctuations and striæ.

The relative thickness and proportions of the different parts of the specimen, and its external contour, are shown in the reduced figure of the section exposed by the fractured posterior extremity, Plate XVII. fig. 3 ; and in this view is seen the dental canal (3) which contained the large blood-vessels and nerves that supplied the teeth and integuments of the lower jaw; its longest diameter at this part is six-tenths of an inch.

The numerous and large vascular foramina which afforded passage to the vessels and nerves from the dental canal to the external integuments, form a striking character in this aspect: they open obliquely forwards; nine are distributed at regular intervals in a line with the alveolar margin, from the posterior end of the bone to nearly opposite the successional tooth in place. A fracture in the middle of the outer surface (Plate XVII. fig. 4, *h*) at the distance of $4\frac{1}{2}$ inches from the posterior end, exposes the dental canal filled with sandstone: its diameter is here two-fifths of an inch.

At the anterior termination of the alveolar space, a slight protuberance (Plate XVII. fig. 4, *i*) marks the commencement of the upper margin of the symphysial region, which is defined by a sharp smooth ridge, that sweeps downwards and inwards to form the front of the jaw. A deep groove, beset with foramina (Plate XVII. fig. 4, *k*),

* Philosophical Transactions, 1841, Plate V. figs. 1, 8, 9.

constitutes a strong line of demarcation between the inner and outer boundary of this area; the latter is thick and convex, and terminates anteriorly, as already mentioned, in a mammillary protuberance or tubercle (Plate XVI. *e*, and Plate XVII. fig. 4, *e*). A series of foramina, eight in number, extends along the outer and inferior surface of the symphysis; the terminal one, which is three-fourths of an inch in its transverse diameter, is situated immediately under the mental tubercle (*e*) above described. The mesial or inner edge of the symphysis, which in connection with the ramus of the left side formed the median suture of the lower jaw, is thin and expanded; the articulating surfaces of the two dentary bones appearing to have overlapped each other (Plate XVI. fig. 1, *s*, and Plate XVII. fig. 1, *s*, *s*); but as the edge of the bone is somewhat broken, the line of junction is not quite determinable; but the two rami do not seem to have been united by anchylosis.

On the under surface of the symphysis there is a depressed oval area, bounded laterally and posteriorly by a slightly elevated ridge (Plate XVII. fig. 2, *l*, *l*); probably for the insertion of the protractor muscles of the tongue.

The coronoid bone (Plate XVI. fig. 1, *m*, and Plate XVII. fig. 4, *m*), of which only a portion remains, is much more expanded outwardly than in any recent Saurian.

With respect to the length of the jaw to which this specimen belonged, an approximative estimate only can be offered, since we have no means at present of determining the relative size of all the different pieces that entered into the construction of the maxillary organs of the Iguanodon. From the appearance of the fractured end, it seems probable that the dentary bone was prolonged backwards five or six inches before it united with the surangular and angular: upon this supposition its entire length must have been two feet, and the number of teeth about twenty. In the Iguana and most Lizards the dentary element is half the length of the jaw; and if this proportion be taken as the standard of comparison—and it appears to be the most probable one—the length of the jaw of this individual was four feet. An eminent palæontologist* has estimated the length of the head of the largest Iguanodon at only thirty inches; having taken as the basis of his calculation, the length of six dorsal vertebræ, which in the Iguana is equal to that of the lower jaw. But the specimen before us proves either that the same scale of proportions is not applicable to this colossal Saurian, or that much larger dorsal vertebræ than those from which the measurement was taken, are yet to be discovered; for several teeth in my possession exceed in magnitude the largest sockets of this dentary bone. Even if we take the

* Reports of the British Association for 1841. Article 'British Fossil Reptiles,' p. 143. "If there be any part of the skeleton of the Iguana which may with greater probability than the rest be supposed to have the proportions of the corresponding part of the Iguanodon, it is the lower jaw, by virtue of the analogy of the teeth and the substances they are adapted to prepare for digestion. Now the lower jaw gives the length of the head in the Iguana, and this equals the length of six dorsal vertebræ; so that as five inches rather exceeds the length of the largest Iguanodon vertebra yet obtained, with the intervertebral space superadded, on this calculation *the length of the head of the largest Iguanodon must have been two feet six inches.*"

abbreviated proportions of the short blunt-headed lizards as the scale—as for example the Chameleons—the length of the jaw of this Iguanodon must have exceeded three feet.

I will now describe the portion of a left upper maxilla in the British Museum, which corresponds so perfectly in its general characters with the lower jaw, as to leave not the slightest doubt of its having belonged to the Iguanodon, although no teeth remain to establish the identity.

Portion of the Upper Jaw, Plate XIX. figs. 1, 2*.—This specimen consists of the anterior part of the left maxillary bone, having on the under surface (fig. 1) the alveolar furrow with the bases of the sockets of ten teeth ; and on the upper (fig. 2), the deep channels of the infra-orbital vessels and nerves that supplied the teeth and integuments of the front of the jaw and face on the left side : this fossil is represented of the natural size. Dr. Melville, who has most kindly aided me by his profound anatomical knowledge throughout this investigation, and liberally devoted much time and attention in instituting the necessary comparisons between my specimens and those formerly collected by me, and now in the British Museum, with the jaws and teeth of recent reptiles, has favoured me with the following observations on this subject.

"This fragment of the left maxilla, which is eight inches five lines long, and two inches seven lines broad, formed the lower boundary of the nasal surface ; it is broken off where the vertical parapet rises to enclose the olfactory fossa. The corresponding part in the skull of an Iguana (*I. tuberculata*), measuring four inches two lines in length, is six lines long, or nearly one-eighth that of the cranium ; this ratio gives five feet four inches as the length of the skull of the Iguanodon to which the fossil belonged ; but as the brain and the organs of sense would have a less proportion to the whole bulk in these gigantic Saurians than in the small species of existing Lizards, we may infer a diminution in the absolute size of the head, corresponding with the abbreviation and contraction of the cranium ; and the length in the adult would probably average about four feet.

"The breadth of the fragment continues uniform ; in front it is rounded off externally (x), and exhibits the oblong terminal irregular surface for articulation with the intermaxillary bone by which it was overlapt. The large infra-orbital canal (fig. 2, a, a) opens at the junction of the posterior and middle third, and midway between its margins ; passing into a broad and deep sigmoid groove which curves inwards as it advances so as nearly to reach the inner edge in the centre of its course, where it gives off a retrograde furrow extending over the internal margin (fig. 2, b). In front it is deflected outwards, extending along the posterior border of the intermaxillary

* I discovered this fossil in 1838, in a quarry near Cuckfield, and it was labelled in my Catalogue "*fragment of the upper jaw of an Iguanodon.*" By the kind permission of Mr. König, the specimen has recently been cleared of the sandstone with which it was partially invested, so as to expose its peculiar characters as represented in Plate XIX.

surface; and rising to a higher level, becomes faint, and terminates one inch behind the anterior extremity; several small vascular foramina open where it ceases. A rough, irregularly triangular, and excavated tract, separates it from the outer edge of the bone; its apex is behind, about 1½ inch from the infra-orbital orifice; its base corresponds to the rounded anterior and external angle, where it passes into the low external border.

"The roof of the infra-orbital canal presents a deep concavity behind (fig. 2, *c*), with the smooth lateral surface of the groove rising into it externally; this is evidently part of the floor of an upwardly emergent canal about an inch in diameter; the thin partition continued backwards between it and the canal below is destroyed, and the posterior segment of the circumference removed; a comparison of the fossil with the skull of an Iguana will confirm this statement.

"The infra-orbital canal, which is eight lines wide behind and four lines high, bends inwards as it retrogrades from its anterior opening. The inner surface is only four lines from the nasal aspect of the fragment behind, so that after a course of a few inches, it would have emerged on the floor of the nasal cavity. The roof is incised obliquely outwards, and the inner portion of it extends forwards to the retrograde groove. The portion of the external surface of the alveolar process that remains, slopes inwards, and exhibits no traces of vascular foramina."

From the almost entire destruction of the alveolar walls of the furrow, deep transverse excavations (Plate XIX. fig. 1, *f*, *f*, *f*) are the only remains of the dental sockets. As the fangs of the teeth of the upper jaw, as will be shown in the sequel, were more curved than in the lower series, and their implantation presented a corresponding modification, as is the case in the dental organs of certain existing Monitors, the width of the alveolar space is greater than in the lower jaw.

Dental characters.—Although the peculiar form and structure by which the teeth of the Iguanodon are distinguished from those of all other animals have been described in my former communications, yet as the mode in which the teeth were implanted in the jaws was then unknown from actual observation, no attempt was made to ascertain the dextral or sinistral position of the detached specimens; nor to separate the lower from the upper series, and thus determine the dental arrangement by which the jaws of this colossal reptile were invested with the functions of those of the existing herbivorous Mammalia. To solve this highly interesting problem, it became necessary to institute a rigorous and minute comparison and examination of all the teeth of the Iguanodon to which we could obtain access; the results of the investigation are detailed by Dr. Melville in the following statement.

"*Teeth of the Lower Jaw*, Plate XVIII. figs. 4, 5, 6.—The lower tooth is curved, with the concavity outwards, or towards the external alveolar parapet; the upper and lower limbs, corresponding respectively to the wedge-shaped crown, and elongated taper fang, are not separated by a constriction or neck, but are flattened in opposite directions. In the upper moiety of the coronal segment, it is compressed

transversely with a convex outer, and a flat inner aspect, and gradually increases downwards in width and thickness, from the broadly-rounded excentric apex to its greatest longitudinal diameter. It continues to expand transversely while decreasing in breadth; subconcave planes also replace the serrated edges at which the surfaces meet above; it obtains its greatest thickness where the tooth is bent on itself to form the fang; the latter diminishes rapidly in both diameters, and the lateral facets are brought in contact below, and obliterate the inner surface; in fully-formed teeth the fang tapers to a point*.

"The inner coronal aspect has a rhomboidal outline, and is covered with a thin layer of enamel extending on the margins; it is flat in the antero-posterior diameter, and only slightly convex vertically. The upper serrated edges ascend converging; the anterior is the longest and most curved; it sweeps rapidly backwards above to the excentric apex, which thus presents a broad front shoulder. The inferior margins are striated or granulated, but destitute of serrations; the posterior is bent forward below to meet the straight lower front edge. The angle to which the hinder margin inclines is more prominent and acute than that formed by the anterior edge; and by this character the teeth belonging to the respective rami of the lower jaw may be distinguished.

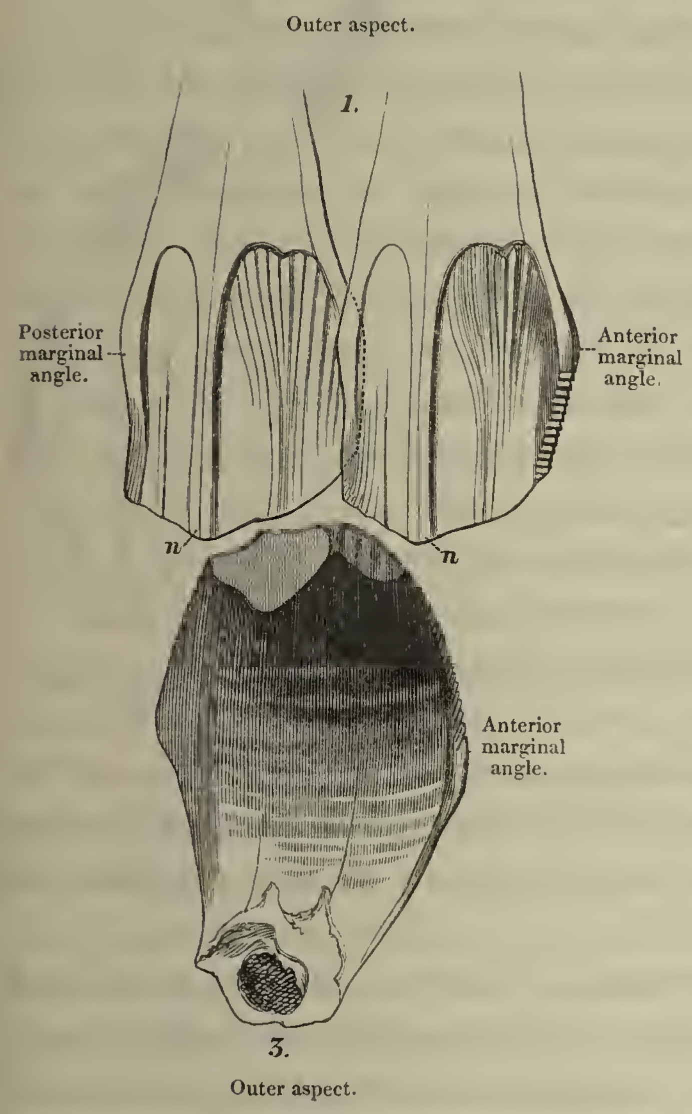
"The enamelled surface is divided into two unequal channelled areas by a primary longitudinal ridge (Plate XVIII. fig. 4, *n*, and fig. 5, *n*); commencing at the apex, it intersects the long diagonal, and terminates behind the lower angle, from which a broad secondary elevated tract ascends along the floor of the wide anterior groove, nearly obliterating it in front. A slight convexity, rapidly subsiding, passes upwards in front of the superior posterior edge.

"The relative width and depth of the longitudinal grooves, and the prominence of the intervening ridges, vary in different specimens; and the edges of the ridges are more or less replaced by planes; the ratio of the upper to the lower margins also differs. The serrations are produced by small convex mammillated ridges separated by slight intervals; the inner edges of the anterior apical ones are prolonged downwards; those on the posterior margin are abraded, apparently by absorption, during the upward growth of the germ. The inner convex surface of the fang is variously grooved and flattened, becoming ridge-like below; it is in apposition with the outer alveolar parapet. The lateral planes converge inwards, and are grooved longitudinally: the posterior plane presses forwards the inferior hinder coronal margin; they extend as high as the obtuse angle of the crown, and leave between them, as they diverge in their ascent, an unenamelled triangular space on the inner aspect. Expansions of the alveolar septa on each side are adapted to the lateral planes of the fang, and the inner parapet is deficient opposite the triangular tract above-mentioned, but is closed below, separating the alveolus from the cavity of reserve in the secondary dental groove.

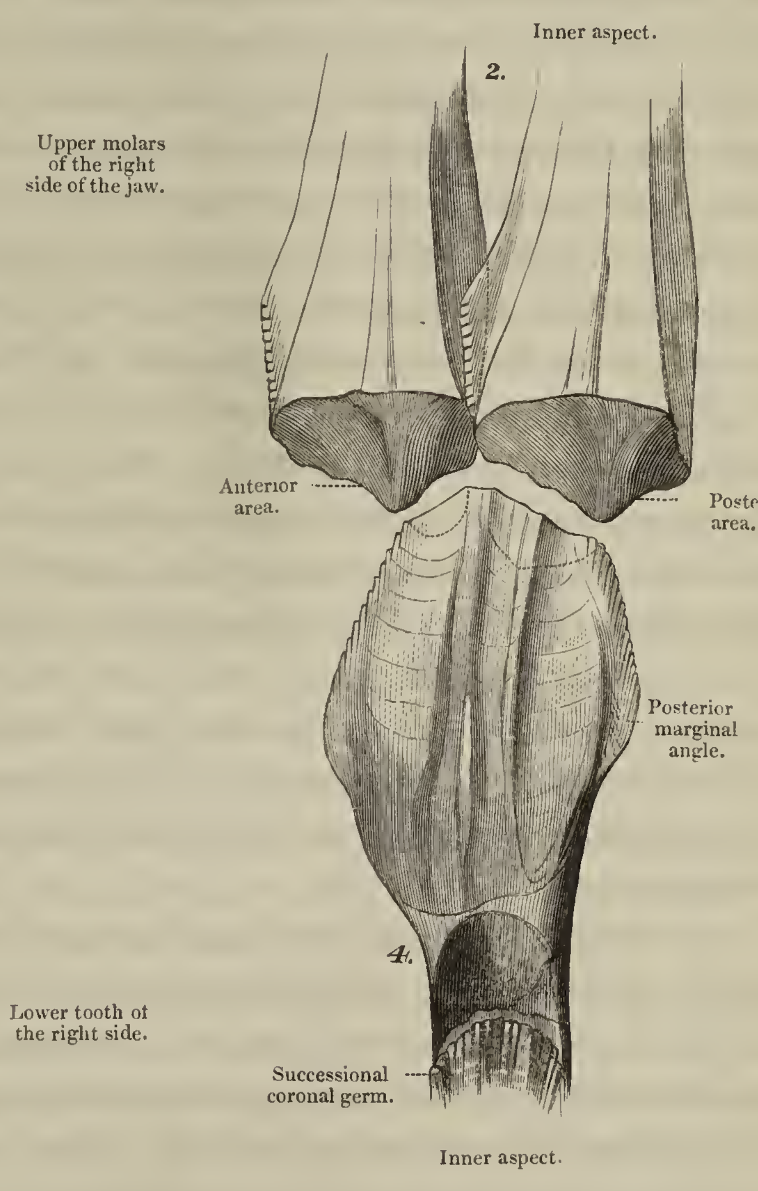
* Philosophical Transactions, 1841, Plate VII. figs. 1, 2.

"The teeth never become anchylosed to the sockets; the great transverse diameter of the dentary element of the jaw above appears to have allowed of the outward curvature of the elongating fang, while the inner surface was maintained nearly vertical. By the same provision the germ attained a considerable size before it pressed upon and excavated the root of the tooth, which it was destined ultimately to displace. The wedge-shaped crown and the anterior serrated recurved trenchant edge, must have rendered the teeth in this early stage very efficient instruments, in the absence of incisors, for cutting vegetable food.

Xylograph, No. 3.

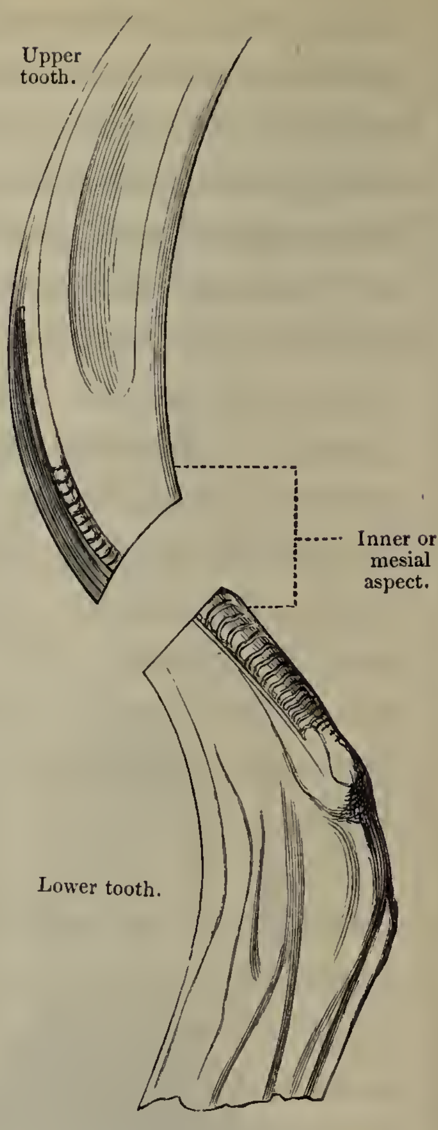


Xylograph, No. 4.



The arrangement of the upper and lower molars, and the situation of a lower successional germ, are shown in the above figures. In the wood-cut No. 3, two upper molars of the right side are represented on their external or enamelled aspect, and a corresponding lower molar beneath them; in No. 4, are seen the opposite or internal aspect, and the position of the successional germ in the fang of the lower tooth.

Xylograph, No. 5.



View in profile of an upper and lower molar, of the left side.

" *Teeth of the Upper Jaw*, Plate XVIII. figs. 1, 2.—After the determination of the form and position of the teeth of the lower jaw, yielded by the instructive specimen previously described, the next question to be determined was, whether the teeth in the upper maxilla had the same shape and curvature as those of the lower. Upon examining the extensive series in the British Museum, several teeth were found differing in shape from the now-ascertained type of the lower molars of the Iguanodon. These are, however, in all essential respects so analogous to the inferior teeth, that no reasonable doubt can exist of their having belonged to the upper jaw of this reptile. From the mutual adaptation of the grinding surfaces, and the situation of the excavation produced by the replacing germ—which in all analogous cases is in the mesial aspect of the fang—the inference was obvious that these teeth not only did belong to the upper series, but that they were curved in an opposite direction to those of the lower; namely, with the convexity external, and the concavity internal; the hollow for the successional germ being situated in the latter (Plate XVIII. fig. 3^a, *f*). Thus the upper and lower molars were related to each other nearly as in the Ruminants; the outer aspect below corresponding to the inner above; the triturating facet inclining from above downwards and outwards in the inferior series, and from below upwards and inwards in the superior; in the lower teeth the enamelled edge is within and the most elevated, while in the upper it is external and inferior. The annexed diagram (Xylograph, No. 5) of an upper and lower tooth, seen in profile, explains this arrangement.

"By this adjustment the hard unvascular dentine with its coating of enamel, played on the softer vaso-dentinal tract of the tooth opposed to it below; and a bevelled or chisel-like surface was maintained for triturating the food when drawn into the mouth by the large prehensile tongue, which is indicated by the procumbent and inferiorly excavated symphysis. The upper molar is also distinguished by the smaller antero-posterior diameter of the crown—by the great prominence of the primary ridge (*n*)—by the breadth of the vertically convex surface of the fang—by the width of the lateral facets—and by the contraction of the internal or vertically concave surface which becomes ridge-like below.

"The anterior and inferior serrated edge is rapidly recurved towards the blunt excentric apex, and forms above an obtuse prominent angle with the upper segment of that border, which is striated, and everted as it were by the encroachment of the anterior deeply concave lanceolate facet impressing the lower portion of the crown

and fang. The broad area between it and the primary ridge, exhibits numerous striæ of enamel converging as they descend to a secondary ridge; on the crest of which, one is prolonged before the outer edge of an anterior apical serration. The posterior coronal margin is nearly straight, plain above, and serrated below. The primary ridge forms a strong convex buttress subsiding towards each extremity; it is slightly inclined to the hinder edge, and nearly obliterates the smaller posterior area.

"From the inversion of the teeth in the upper jaw, we naturally expect to find some alteration in the configuration of the molars; while the position of the apex, of the primary ridge, and of the areas into which it subdivides the crown, will remain as in the lower series. On these grounds I am led to conclude that the smaller area always indicates the posterior part of the tooth, whether of the upper or lower jaw; but the marginal angle and the reflected edge are anterior in the upper molars, and posterior in the lower, these characters having relation to the manner in which the series of teeth are arranged. In the lower jaw the posterior margin of the molar overlaps, or rather projects internally beyond the anterior edge of the tooth behind, but in the upper it is situated externally to it (Xylographs, No. 3 and 4, page 193.); a similar arrangement takes place in the Ruminants. The mechanical advantages resulting from the opposite curvature of the teeth in the upper and lower jaw, are too obvious to require comment."

As it is very rarely that a specimen occurs in which the absorption of the fang, from the upward growth and pressure of a new tooth, has not taken place in a greater or lesser degree, it is probable that the formation of successional teeth was in constant progress at all periods of the animal's existence, as is the case in most of the Saurian reptiles.

The internal structure of the teeth is in striking accordance with the external form and mechanical arrangement of the dental organs; for the central body of dentine or tooth ivory is of a softer and coarser texture than in any other known Saurian, and closely resembles that observable in the colossal vegetable feeders of the Sloth tribe—the Mylodon and Megatherium*. But towards the periphery of the anterior part of the tooth the dentine is finer and harder; and not only the front of the crown, but also the inserted base, is covered by a thick layer of enamel; and from this peculiar arrangement of substances of different degrees of hardness, the tooth, in every stage, must have been admirably adapted for the trituration and comminution of vegetable food. A vertical slice of a tooth, showing the pulp-cavity, with the dentine traversed by vascular canals radiating from the centre, and running parallel with the calcigerous tubes, is represented as seen by transmitted light, and magnified eight diameters, in Plate XVIII. fig. 3.

The dental pulp becomes ossified in the old teeth, so that whatever the degree of abrasion, the exposed masticating coronal surface is solid; this is seen even in the

* "The intimate structure of the vascular dentine of the Bradypus resembles that of the inner half of the dentine of the Iguanodon."—*Odontography*, p. 329.

last stage, when the crown is reduced to a mere plate of dentine, as in the specimen previously figured (Xylograph, No. 2, *ante*, p. 188)*.

As the articular piece which contains the socket of the lower jaw for receiving the inferior head of the *os quadratum* is unfortunately wanting, the mechanism of the articulation can only be conjectured; for although several examples of the tympanic bone —which in reptiles as in birds connects the lower with the upper maxilla—are preserved in my former collection, neither of the specimens is sufficiently perfect to indicate the precise mechanism of the joint. One of the tympanic bones found in the same quarry with teeth and bones of the Iguanodon, and which I am led to consider as having belonged to that reptile, is 6 inches high and 5½ inches wide; it consists of a thick pillar, which is contracted at the sides in its vertical direction, and has two thin expanded lateral processes; it terminates both above and below in an elliptical and nearly flat surface; it is very cavernous from the large size of the tympanic cells. This bone differs considerably from the corresponding element in the Iguana, the peculiarity, doubtless, having relation to some modification in the mechanism of the lower jaw, by which the more complete comminution of vegetable substances was effected, than by maxillary organs constructed after the usual Saurian type.

Physiological inferences.—In instituting a comparison between the maxillary organs above-described, and those of the existing herbivorous lizards, with a view of obtaining some physiological deductions from the peculiar osteological characters of the fossil remains, we are at once struck with their remarkable deviation from all known types in the class of reptiles. In the *Amblyrhynchi*, the most exclusively vegetable feeders of the Saurian order, the alveolar process beset with teeth is continued round the front of the mouth; the junction of the two rami of the lower jaw at the symphysis presenting no edentulous interval whatever, and the lips are not more developed than in other reptiles: in the Iguanas, as shown in my former memoir, the same character exists. In the carnivorous Saurians the teeth are also continued to the symphysial suture on each side. The extinct colossal lizards offer no exception to this rule; in the acrodont Mosasaurus of the Chalk, and in the thecodont Megalosaurus of the Oolite and Wealden, the jaws are armed with teeth to the anterior extremity. In short, the edentulous, expanded, scoop-shaped, procumbent symphysis of the lower jaw of the Iguanodon has no parallel among either existing or fossil reptiles; and we seek in vain for maxillary organs at all analogous, except among the herbivorous Mammalia. The nearest approach is to be found in certain *Edentata* —as for example in the *Cholœpus didactylus* or Two-toed Sloth, in which the anterior part of the lower jaw is edentulous and much-prolonged: the correspondence is still closer in the gigantic extinct *Mylodons*; in which the symphysis resembles the blade of a spade used by turf-diggers, and has no traces of incisive sockets; and were not this part of the jaw elevated vertically in front, and the rami confluent, it would pre-

* For a detailed account of the microscopical structure of the teeth of the Iguanodon, see "Odontography, or a Treatise on the Comparative Anatomy of the Teeth," by Professor OWEN, p. 246–253.

sent the very counterpart of that of the Iguanodon. This striking resemblance will be obvious to any one who will compare the illustrations subjoined to this memoir, with the magnificent specimen of the *Mylodon robustus* in the Hunterian Museum*.

The great size and number of the vascular foramina distributed along the outer side of the dentary bone and beneath the border of the symphysis (Plate XVI. fig. 2, and Plate XVII. fig. 4), and the magnitude of the anterior outlets which gave exit to the vessels and nerves that supplied the front of the mouth, indicate the great development of the integuments and soft parts with which the lower jaw was invested†.

The sharp ridge bordering the deep groove of the symphysis, in which there are also several foramina, evidently gave attachment to the muscles and integuments of the under lip; and there are strong reasons for supposing that the latter was greatly produced, and capable of being protruded and retracted, so as to constitute in conjunction with a large fleshy prehensile tongue, a powerful instrument for seizing and cropping the leaves and branches, which, from the construction of the teeth, we may infer constituted the chief food of the Iguanodon.

Thus we find the mechanism of the maxillary organs of the Wealden herbivorous Saurian, as elucidated by recent discoveries, in perfect harmony with the remarkable dental characters which rendered the first known teeth so enigmatical to the palæontologist, and another interesting proof is obtained of the constancy of those laws by which the correlation of organic structures is governed.

In the Iguanodon we have a solution of the problem how the integrity of the type of organization peculiar to the class of cold-blooded Vertebrata was maintained, and yet adapted, by slight and simple modifications, to fulfil the conditions required by the economy of a gigantic terrestrial reptile, destined to obtain support exclusively from vegetable substances; in like manner as the extinct colossal herbivorous Edentata which flourished in South America, ages after the Country of the Iguanodon and its inhabitants had been swept from the face of the earth.

Thus in the unlimited production of successional teeth at every period of the animal's existence, in the mode of implantation of the teeth, and in the composite structure of the lower jaw—each ramus consisting of six distinct elements—the Saurian type of organization is unequivocally manifest; while the intimate structure of the dental organs corresponds with that of the Sloths, and the subalternate arrangement and reversed position of the upper and lower series of teeth, with that of the Ruminants. And again, the edentulous and prolonged symphysis, and the great develop-

* In the *Mylodon Darwinii* the rami of the lower jaw anterior to the teeth are contracted vertically, and converge to a longer and narrower symphysis, which is inclined forwards at a more open angle with the horizontal ramus, than in the *Mylodon robustus*; and therefore still more nearly approaches that of the Iguanodon. See Professor Owen on the Mylodon.

† The external vascular foramina are remarkably augmented in the dentary bone of the Iguanodon as compared with other reptiles. In the colossal Mosasaurus, the dentary piece of a jaw $4\frac{1}{2}$ feet long, has but ten or twelve relatively small foramina; in the Iguana but five or six; in the Monitors six; in the Crocodiles they are numerous, but small and irregular. In the Megalosaurus there are four foramina, and several small ones.

ment of the lower lip and the integuments of the jaws, as indicated by the size and number of the vascular foramina, present a striking analogy to the Edentata, with which indeed other parts of the osteology of the Iguanodon bear a remarkable resemblance; as for example, the sacrum formed by the anchylosis of five vertebræ, the expanded vertebral portion of the ribs, &c.; while the massive femur with its medullary cavity, well-marked trochanters and condyles, and the short and strong metatarsal and phalangeal bones, remind us of the gigantic recent pachyderms.

In fine, we have in the Iguanodon the type of the terrestrial herbivora, which in that remote epoch of the earth's physical history, termed by geologists "*The Age of Reptiles*," occupied the same relative station in the scale of being, and fulfilled the same general purposes in the economy of nature, as the Mastodons, Mammoths, and Mylodons of the tertiary periods, and the large pachyderms of modern times.

Although many important characters of the osteology of the Iguanodon are still unknown, we may, I conceive, from the data hitherto obtained, safely infer that this gigantic herbivorous reptile was equal in bulk to the Elephant, and as massive in its proportions; for living exclusively on vegetables there must have been a large development of the abdominal region. Its limbs must have been of proportionate size to sustain and move so enormous a carcass. The hinder extremities in all probability presented the unwieldy contour of those of the Hippopotamus and Rhinoceros, and were supported by strong short feet, protected by broad horny ungual phalanges. The fore-legs appear to have been less bulky, and adapted for seizing and pulling down plants and branches of trees. The teeth and jaws demonstrate its power of mastication and the nature of its food; and the remains of coniferous trees, arborescent ferns and cycadeous plants, with which its relics are commonly associated, indicate the character of the flora adapted for its sustenance.

I forbear entering at this time upon other considerations relating to the fossil reptiles of the Wealden, in the hope of being able to lay before the Royal Society, on some future occasion, an account of several other interesting Saurian remains from the strata of Tilgate Forest; but I beg to subjoin the following supplementary remarks on my last memoir.

Regnosaurus Northamptoni, Phil. Trans. 1841, Plate V. figs. 1, 8.—The portion of the lower jaw of a Saurian formerly described as that of a young Iguanodon*, is proved by the specimen discovered by Captain BRICKENDEN, to be generically or subgenerically distinct from the *I. Mantelli*, although unquestionably belonging to the same remarkable family of herbivorous reptiles; and the internal structure of the teeth in this specimen, so far as I can judge from a microscopical examination of a very small fragment from the upper part of the fang, corresponds more nearly with that of the Iguanodon than with the very fine and dense tooth-ivory of the teeth of

* Philosophical Transactions, 1841, p. 131.

the Hylæosaurus. I therefore propose to distinguish this fossil Saurian by a name indicative of the district from which, in common with so many relics of the same class, it was obtained—*Regnosaurus** ; with the specific appellation of *Northamptoni*, as a tribute of respect to the eminent nobleman whose approaching retirement from the Presidency of the Royal Society is so much to be regretted.

19 *Chester Square, Pimlico,*
May 24, 1848.

Description of the Plates.

PLATE XVI.

A considerable portion of the right ramus of the lower jaw of an adult Iguanodon from the Wealden sandstone of Tilgate Forest, discovered by Capt. Lambart Brickenden, F.G.S. The figures are of the size of the original.

Fig. I. The inner or mesial aspect.

a. The coronal germ of the first anterior tooth, in its alveolus.

b. The crown of a successional tooth in its natural situation.

c. Portion of the base of a successional tooth. The above are visible in consequence of the removal of the inner parapet of the alveolar process.

d, d. The deep elongated *opercular furrow,* originally covered by the splenial or opercular bone.

e. An obtuse tubercle or projection forming the anterior termination of the symphysial portion of the jaw.

f, f, f. Denote some of the alveoli or sockets of the mature molar teeth.

m. The coronoid or complementary bone.

s. The articulating surface forming the symphysial suture.

Fig. II. An oblique external view of the same specimen. In this figure the coronoid process (*m* of fig. 1) is not delineated, and the fractured posterior end of the fossil is represented to show the situation of the great dental canal (*g*), and the thickness of the parietes of the bone ; the internal course of this canal is exposed by the fracture (*h*).

PLATE XVII.

Fig. I. Outline of the anterior part of the lower jaw of the Iguanodon, seen from above ; reduced one-half the natural size in linear dimensions.

s, s. The symphysial suture.

e, e. The mental tubercle.

* *Sussex Saurian* ; the County of Sussex was anciently inhabited by the *Regni*.

Fig. II. Outline of the under or inferior aspect of the same.

l. l. Area for the attachment of the protractor muscles of the tongue.

e, e. The mental tubercle.

Fig. III. Section presented by the posterior fractured end of the specimen (see Plate XVI. fig. 2); reduced half linear.

1, 1. The outer wall or parapet of the alveolar process.

2. Remains of the inner or mesial alveolar parapet.

3. Posterior section of the great dental canal; the lesser foramen indicates the channel for the vessels sent off from the main trunk to supply the dental germs.

4. The *opercular furrow.*

Fig. IV. The external aspect of the specimen represented in Plate XVI., reduced one-half in linear dimensions. In this view the numerous vascular foramina leading from the dental canal are distinctly shown; they extend in a line parallel with the alveolar margin, and are continued to the anterior extremity of the symphysis; one large foramen being situated immediately under the mental tubercle (*e*).

e. The mental tubercle forming the anterior extremity of the symphysis.

h. A fracture on the side of the bone, by which the course of the great dental canal is exposed to view.

i. An eminence at the commencement of the anterior edentulous portion of the jaw.

k. A deep groove beneath the margin of the symphysis.

m. The coronoid process.

PLATE XVIII.

Teeth of the Iguanodon.

Fig. 1. A molar tooth belonging to the left side of the upper jaw, having the crown worn by use: from Tilgate Forest.

1. The outer or external aspect, showing the ridged and enamelled surface of the crown.

1^a. Lateral view of the tooth.

1^b. Surface of the crown worn smooth by mastication, and presenting two distinct facets (1 and 2), produced by the attrition of the corresponding lower molars.

f. The fang.

n. Marks the primary ridge.

Fig. 2. Upper molar of the left side, in which the crown of the tooth is much abraded

and the fang in a great measure absorbed, from the downward growth of a successional tooth.

2. The enamelled and ridged external aspect of the crown.

2^a. The smooth convex inner aspect of the same.

2^b. The abraded coronal surface with its double facets (1 and 2).

a. Anterior margin.

b. Posterior marginal angle.

f. Remains of the fang, showing the cavity produced by a successional germ.

n. Primary ridge.

Fig. 3. Vertical section of the crown of an unused tooth, seen by transmitted light, magnified eight diameters.

e. Enamel.

d. Vascular dentine.

p. Pulp-cavity filled with mineral matter.

Fig. 4. A successional tooth of the right side of the lower jaw, with the serrated coronal margin entire, from Brook Point, Isle of Wight. This specimen is identical with the tooth in place (Plate XVI. fig. 1), but must have belonged to a larger individual.

4. The mesial or inner aspect.

a. The anterior margin.

b. The posterior marginal angle.

n. The primary ridge of the enamelled front of the crown.

4^a. The external smooth convex aspect of the same.

p. Pulp-cavity.

Fig. 5. A tooth belonging to the left side of the lower jaw; the apex of the crown partially worn away. This specimen, together with figs. 1, and 4, with numerous vertebræ, a femur $3\frac{1}{2}$ feet long*, and other bones, were obtained from the Weald Clay, forming the cliff near Brook Point, Isle of Wight; the whole probably belonged to the same individual, which must have been an aged reptile of very colossal proportions†.

5. The smooth, convex, outer aspect.

5^a. Oblique view, showing the posterior side and marginal angle, and the ridged enamelled inner surface.

a. The anterior margin.

b. The posterior marginal angle.

f. The fractured extremity of the fang exposing the pulp-cavity (*p*).

Fig. 6. Molar of the left side of the lower jaw: in this example the whole of the

* I have presented this bone to the Hunterian Museum.

† See my Geology of the Isle of Wight, p. 315.

serrated part of the crown is worn away, and the fang absorbed by the pressure of a successional tooth.

6. The smooth convex outer aspect; the worn surface of the crown has two distinct facets, as in the abraded coronal planes of figs. 3 and 4.

6^a. The inner or mesial aspect of the same.

PLATE XIX.

A portion of the anterior part of the left upper maxillary bone of an adult Iguanodon, from Tilgate Forest: represented of the size of the original*.

Fig. I. The inferior or alveolar aspect, showing the remains of the sockets of ten molars.

f, *f*. Dental alveoli.

x. The intermaxillary articulating surface.

Fig. II. The upper or nasal surface, with deep channels for the infra-orbital vessels and nerves.

a, *a*. The infra-orbital canal.

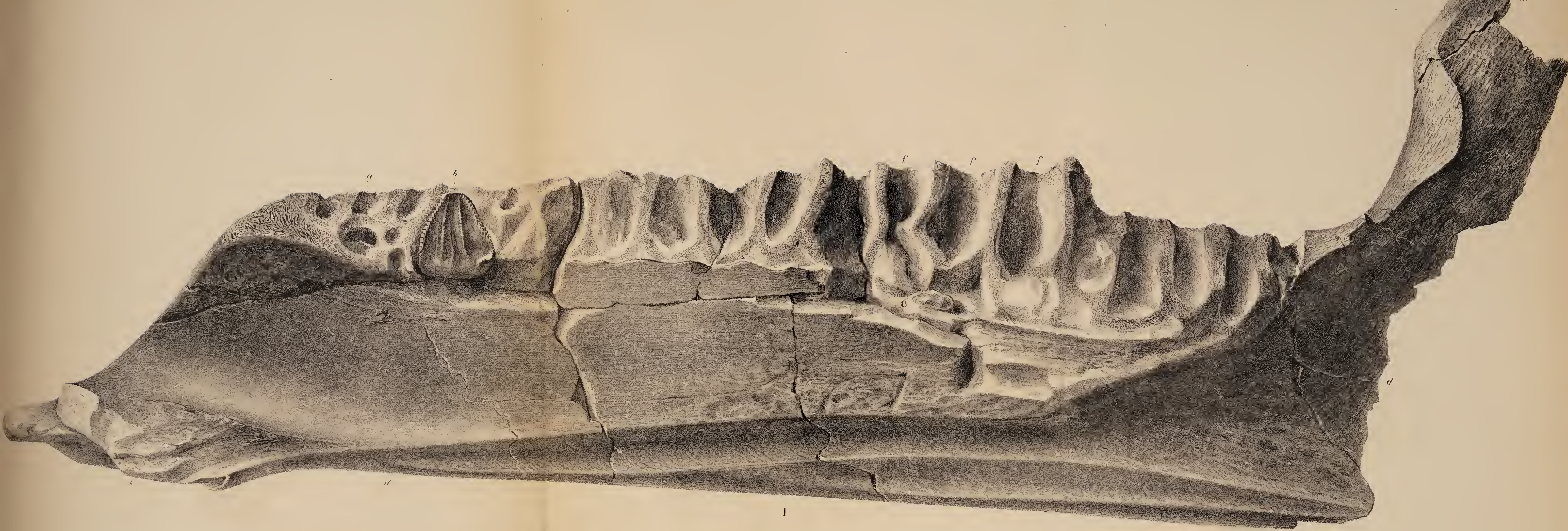
b. Retrograde furrow of the same.

c. A deep concavity in the roof of the infra-orbital canal (see description, *ante*, p. 191).

x. The anterior marginal surface that articulated with the intermaxillary bone.

* This specimen is in the British Museum.

⁂ The nearly perfect tooth figured in Philosophical Transactions, 1841, Plate VI. figs. 1, 2, 3, is an upper molar of a very young Iguanodon. Fig. 4 of the same plate is the coronal germ of an upper tooth.



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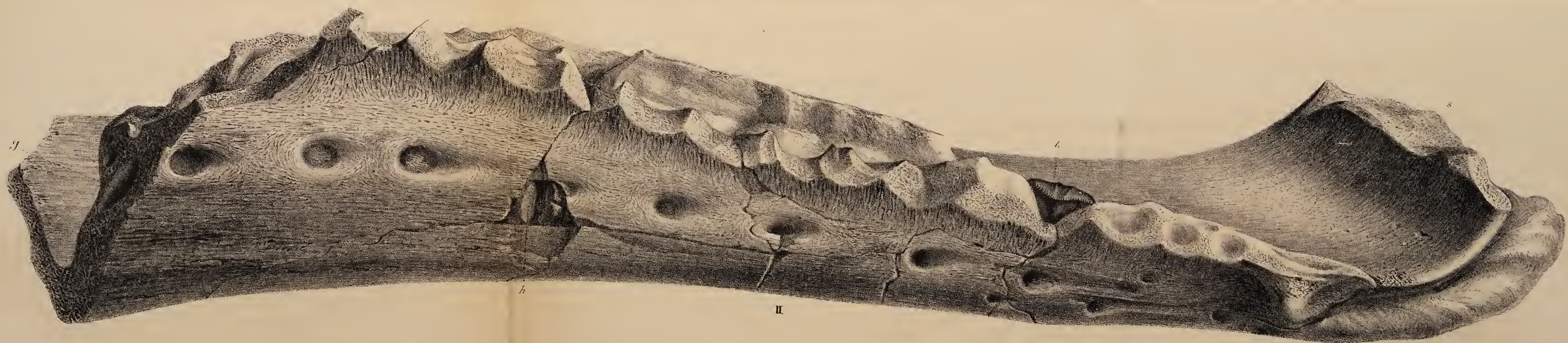

II

Dinkel del et lith

The anterior part of the right side of the Lower Jaw
of the *Iguanodon Mantelli*; from Tilgate Forest.

Fig. 1. Inner or mesial aspect. Fig. 2 Oblique external view.

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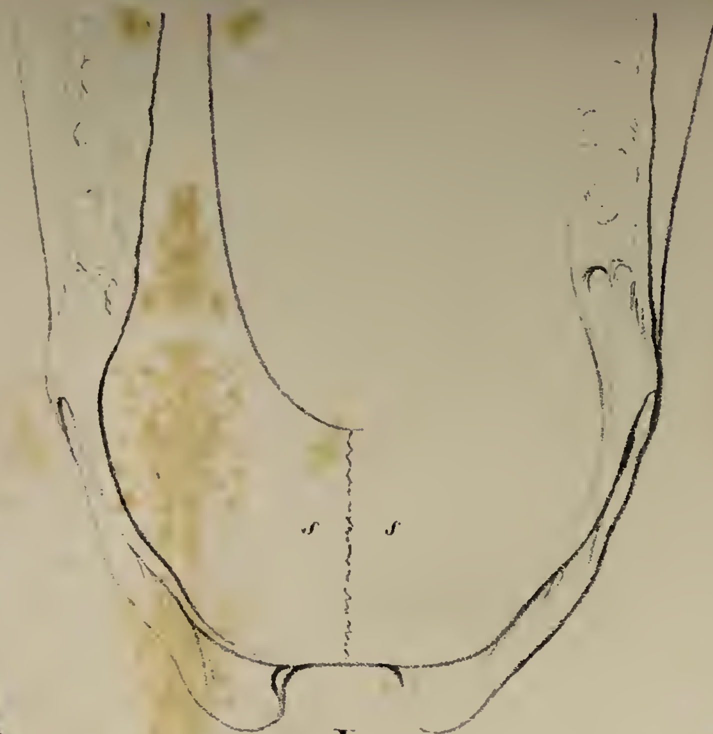

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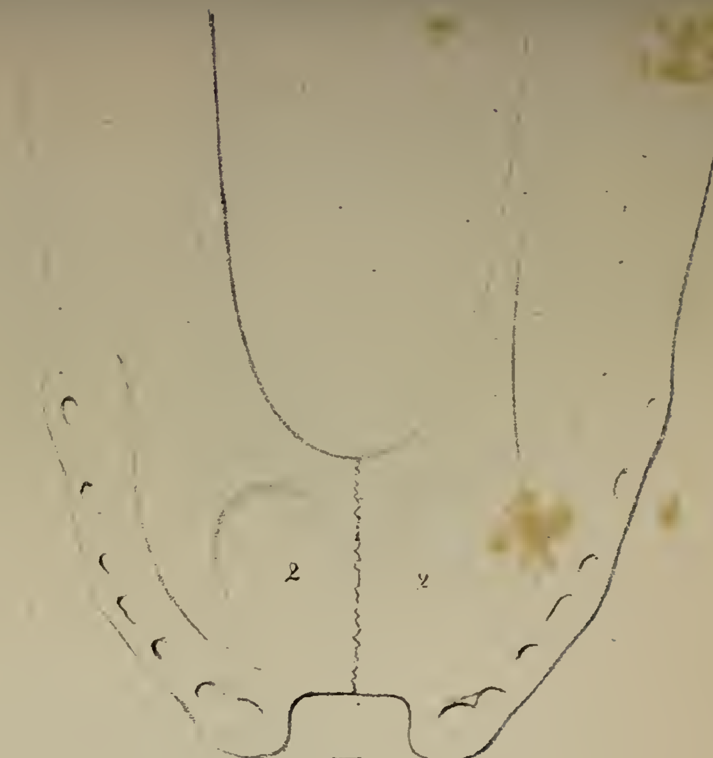

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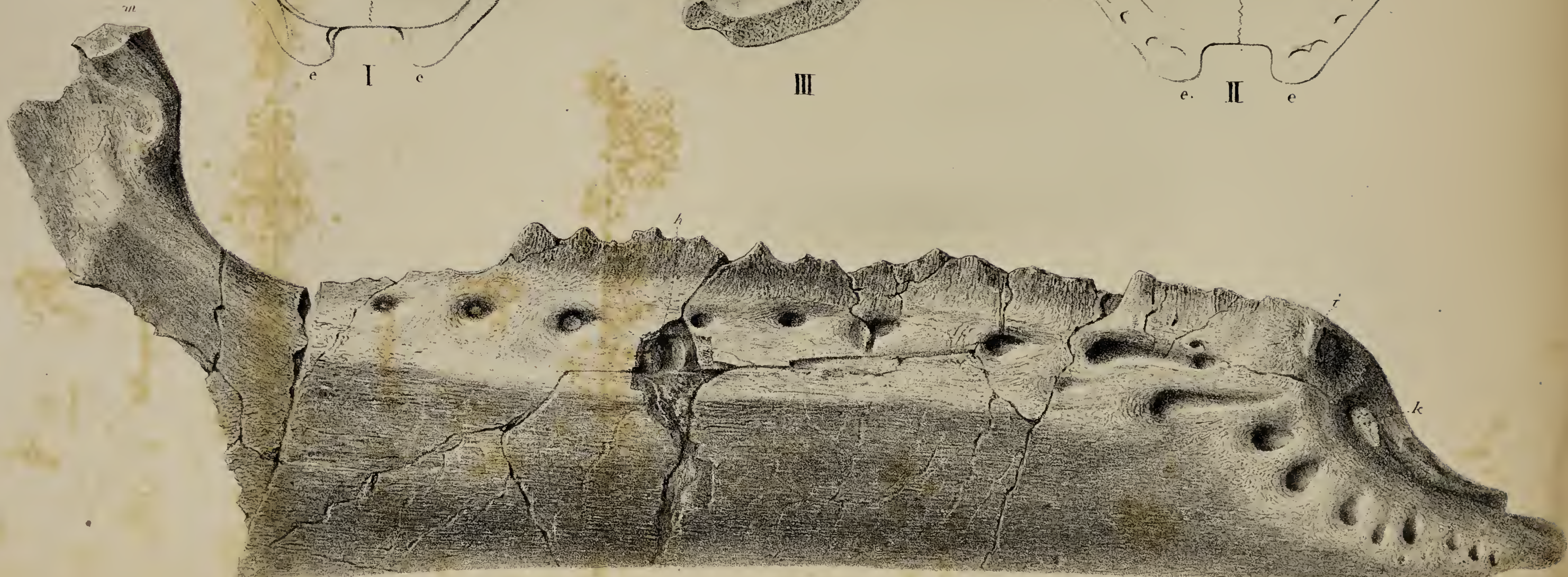

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Dinkel del. et lith

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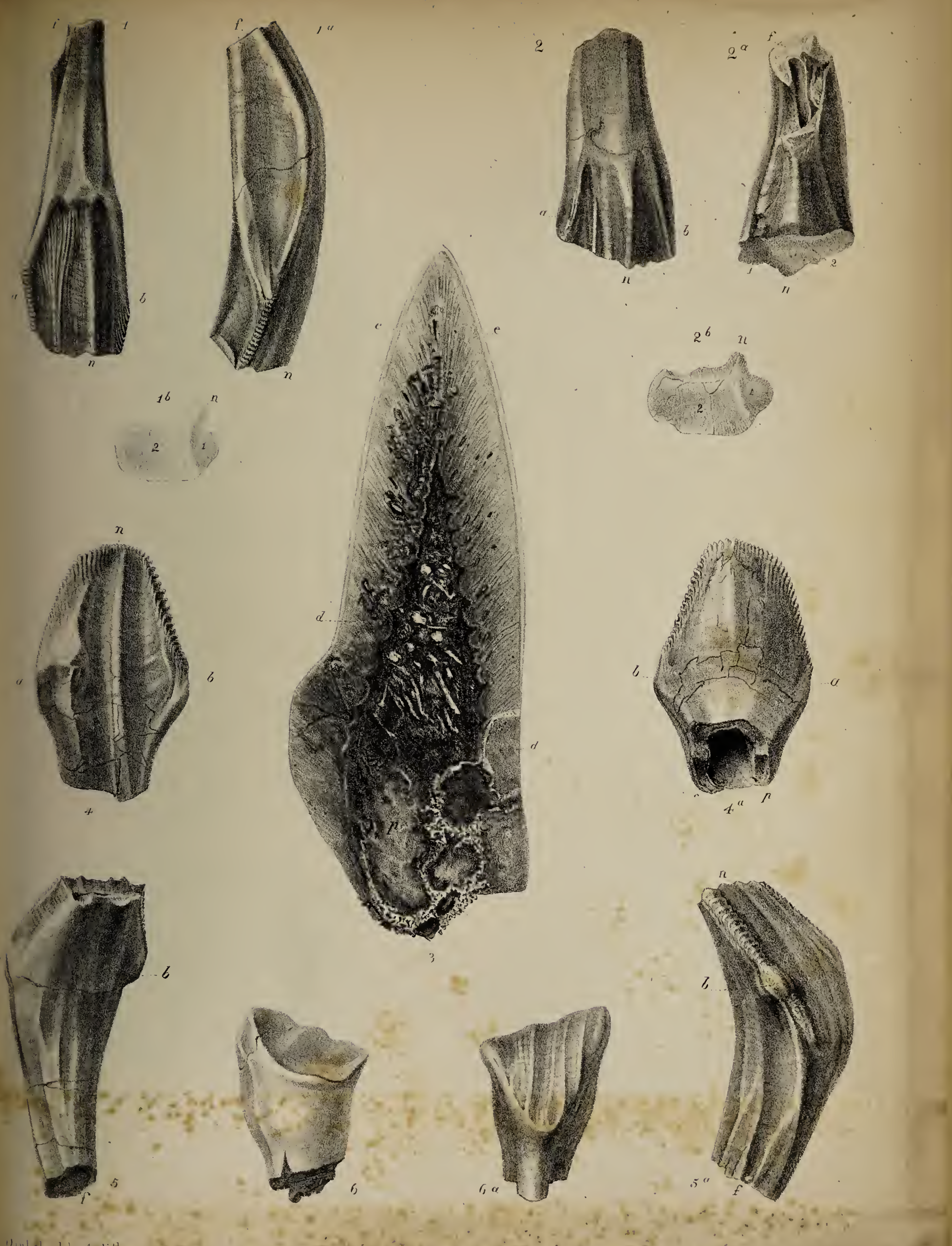

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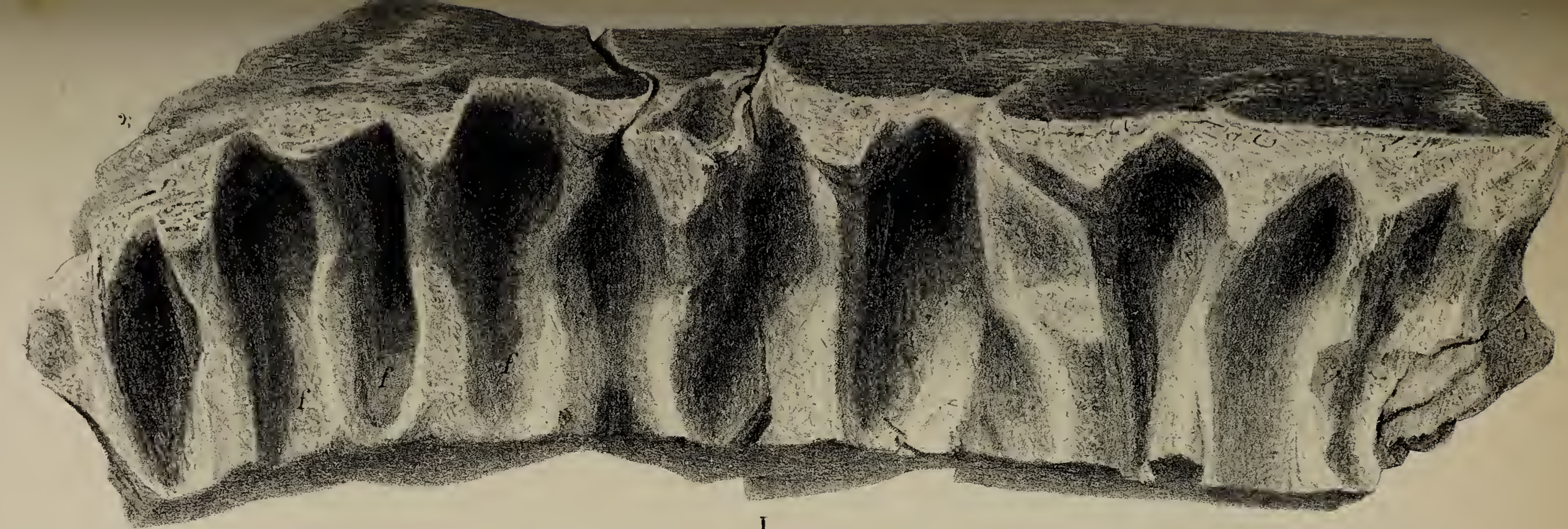


I

II

Dinkel del. et lith.

Printed by J. Basire.

Portion of the Upper Jaw of the Iguanodon.

XIV. *Determinations of the Magnetic Inclination and Force in the British Provinces of Nova Scotia and New Brunswick, in the Summer of* 1847.

By Professor GEORGE W. KEELY, *of Waterville College, Maine, United States. Communicated by Lieut.-Colonel* SABINE, *For. Sec. R.S.*

Received March 1,—Read April 6, 1848.

IN an excursion in the provinces of Nova Scotia and New Brunswick in August and September 1847, I took with me some magnetical instruments, with which I made a few observations for determining the magnetic intensity. The observations were of two kinds; those for the relative total force, made with a pair of LLOYD needles, which I shall designate L(1) and L(2), and an inclination circle, seven inches in diameter, with two verniers reading to single minutes, constructed by BARROW, successor to ROBINSON; and those for the absolute horizontal force, made with a unifilar magnetometer by JONES. The positions of the needles in the inclination circle are determined by two reading microscopes with micrometer scales. The unifilar has a theodolite base and circle of six inches, divided on silver, and reading to twenty seconds. Both instruments are described in Captain RIDDELL'S "Supplement" to his "Magnetical Instructions." Observations to determine the temperature coefficients of the LLOYD needles were made in a small building fastened with nothing but copper, and containing a copper stove. For the weights sent by the maker with these needles, which were inconvenient, I substituted two platinum weights, which have never been removed from the holes in which they were placed before my observations commenced. Using t, t', v, v', θ, θ', φ, φ' for the temperatures, the angles of deflection, the inclinations, and the relative forces, respectively, at low and high temperatures, the following Table exhibits the observations and results by the well-known formula $\varphi=\frac{\cos v}{\sin(\theta+v)}$. I find the results the same whether the mean values of t, v, and θ, &c. are used to obtain φ and φ', or a mean of the daily results is taken.

Table I.

Observations for the Temperature Coefficient of L(2). Waterville Mean Time.

Low temperatures.					High temperatures.				
1847.	t.	v.	θ.	φ.	1847.	t'.	v'.	θ'.	φ'.
d h					d h				
Feb. 10	26°·1	26° 26′ $33\frac{1}{8}$″	75° 55′ $57\frac{1}{4}$″	0·9166800	Feb. 10	72°·1	26° 59′ $46\frac{1}{4}$″	75° 55′ $57\frac{1}{4}$″	0·9142133
18 20·4	11·4	26 16 $33\frac{3}{4}$	76 02 14	0·9177840	20 00·5	70·0	27 11 41	75 56 $45\frac{5}{8}$	0·9133754
19 20·3	14·8	26 18 $13\frac{3}{4}$	75 56 $45\frac{5}{8}$	0·9173431	21 23·7	64·5	27 15 $33\frac{1}{8}$	75 56 $17\frac{3}{8}$	0·9130579
21 20·5	17·6	26 39 35	75 56 $17\frac{3}{8}$	0·9157335	22 23·7	60·5	27 09 $20\frac{5}{8}$	76 00 $58\frac{1}{2}$	0·9138112
22 20·5	15·6	26 24 $20\frac{5}{8}$	76 00 $58\frac{7}{16}$	0·9171375	23 23·7	84·2	27 50 $06\frac{7}{8}$	75 55 $26\frac{3}{4}$	0·9104202
23 20·7	34·1	26 59 $42\frac{1}{2}$	75 55 $26\frac{3}{4}$	0·9141865	26 00·3	80·6	27 38 $39\frac{3}{8}$	75 56 $16\frac{7}{8}$	0·9113319
25 21·3	33·5	26 52 $51\frac{7}{8}$	75 56 $16\frac{7}{8}$	0·9147465	27 00	73·3	27 38 $46\frac{7}{8}$	75 57 $38\frac{7}{8}$	0·9114102
26 21·3	25·6	26 59 $49\frac{3}{8}$	75 57 $38\frac{7}{8}$	0·9143125	28 23·7	69·7	27 27 $59\frac{3}{8}$	76 00 $01\frac{1}{2}$	0·9123667
28 20·5	35·4	27 00 $01\frac{1}{4}$	76 00 $01\frac{1}{2}$	0·9144437	Mar. 1 23·5	67·7	27 26 $27\frac{1}{2}$	75 57 $30\frac{3}{4}$	0·9123212
Mar. 1 20·5	31·0	26 56 56	75 57 $30\frac{3}{4}$	0·9145187	Feb. 18 23·5	62·3	27 04 $12\frac{1}{2}$	76 02 14	0·9142698
Means.........	24·51	26 41 $27\frac{3}{4}$	75 57 $54\frac{3}{4}$	0·9156886	Means.........	70·49	27 22 $15\frac{3}{8}$	75 57 $54\frac{3}{4}$	0·9126578
Calculated value from means of t, v, and θ ...... 0·9156908					Calculated value from means of t', v', and θ'...... 0·9126596				

Observations for the Temperature Coefficient of L(1). Waterville Mean Time.

1847.	t.	v.	θ.	φ.	1847.	t'.	v'.	θ'.	φ'.
Feb. 7 20·5	29·9	35 55 $26\frac{7}{8}$	75 54 $21\frac{1}{2}$	0·8723502	Feb. 8 03	75·1	36 11 $41\frac{1}{4}$	75 54 $21\frac{1}{2}$	0·8710132
8 20	31·9	36 00 $08\frac{1}{8}$	75 57 55	0·8723284	9 02	86·9	36 23 $47\frac{1}{2}$	75 57 55	0·8703828
9 20	34·2	35 59 $36\frac{1}{4}$	75 56 $05\frac{1}{2}$	0·8721856	10 01	78·3	36 15 $45\frac{5}{8}$	75 56 $05\frac{1}{2}$	0·8708558
10 20	32·6	35 52 $53\frac{1}{8}$	75 58 $51\frac{1}{2}$	0·8730186	10 23	78·8	36 16 50	75 58 $51\frac{1}{2}$	0·8710536
11 20·5	24·0	35 51 $37\frac{1}{2}$	76 00 $06\frac{1}{4}$	0·8732486	12 01	74·8	36 15 $36\frac{1}{4}$	76 00 $06\frac{1}{4}$	0·8712842
12 20	25·9	35 53 $28\frac{3}{4}$	75 58 $32\frac{3}{4}$	0·8729382	13 00	74·3	36 13 $11\frac{7}{8}$	75 58 $32\frac{3}{4}$	0·8713204
14 20·5	16·7	35 44 $27\frac{1}{2}$	75 58 $34\frac{1}{2}$	0·8736786	14 23	63·0	36 13 $38\frac{3}{4}$	75 58 $34\frac{1}{2}$	0·8712866
15 20·5	7·0	35 44 $33\frac{3}{4}$	75 58 $24\frac{3}{4}$	0 8736534	15 23	71·0	36 18 $04\frac{3}{8}$	75 58 $24\frac{3}{4}$	0·8709048
16 21	19·0	35 54 $58\frac{3}{4}$	75 58 $04\frac{1}{2}$	0·8727674	16 23·5	75·7	36 24 $26\frac{1}{4}$	75 58 $04\frac{1}{2}$	0·8703460
18 20·7	15·0	35 57 $57\frac{1}{2}$	76 02 14	0·8729488	19 00·7	67·8	36 29 $45\frac{5}{8}$	76 02 14	0·8703426
19 20·7	20·4	35 55 $41\frac{7}{8}$	75 56 $45\frac{5}{8}$	0·8725744	20 01	81·0	36 28 25	75 56 $45\frac{5}{8}$	0·8698804
Means.........	23·33	35 53 $42\frac{4}{5}$	75 58 $10\frac{1}{2}$	0·8728811	Means.........	75·15	36 19 12	75 58 $10\frac{1}{2}$	0·8707882
Calculated value of φ from means of t, v, and θ ...·8728810					Calculated value of φ' from means of t', v', and θ'...0·8707881				

whence by the formula $q=\frac{\varphi-\varphi'}{\varphi'(t-t')}$ we have for L(1) the coefficient ·0000463; and for L(2) the coefficient ·0000722.

The values of θ in the above observations were obtained by the two dipping-needles accompanying the Lloyd needles, and which I shall call A(1) and A(2). The observations were made in the usual way with poles direct and reversed, in each case the marked limb of the circle facing both east and west; each single reading being a mean of from four to eight, successively obtained by lifting the needle from its agate supports by the lifting frame. The mean of the whole is, for A(1), twenty complete observations, 75° 57′ 03″; for A(2), nineteen complete observations, 75° 58′ 50″; whence the mean dip for Waterville (lat. 44° 33′ N., long. 293° 23′), Feb. 17, 1847, is 75° 57′ $56\frac{1}{2}$″. Between June 14th and June 30th I made at the same place, with each of the needles, six complete observations, on as many days, and when, from the less variations of temperature, the adjustments were more under control; the results were for A(1), 75° 56′ 27″; for A(2), 75° 59′ 31″; mean dip for June 22nd, 75° 57′ 59″. In all my observations with these dipping-needles, up to the time of my excursion, I had observed far greater irregularities in A(2) than in A(1); in this last set, for instance,

the greatest difference between any partial result by A(1) and the mean dip was 01′·7, while for A(2) it was 03′·2; the difference for A(1) indeed was, in two-thirds of the results, less than 1′. Though this might not affect the means of a great number of observations, it is evident that if the observer is limited to one observation with each of these needles, he is far more likely, setting aside the effects of carelessness, to obtain a truly comparable value of θ by needle A(1) alone, than by a mean of the two. I have therefore confined my observations for θ, on my tour, with one exception, to A(1). The direction of the meridian has always been obtained with A(1), by considering the meridian as 90° from a mean of the positions of the vertical limb of the circle, when the needle resting on its supports had an inclination of 90° with its marked face alternately north and south. Moreover, in all my observations with the dipping or force needles, whenever the instrument was newly placed, the adjustments for correct position of the microscopes, the axis, the level, and the planes of support, were duly made if necessary. Table II. gives the details of the observations made in the provinces for v and θ with the logs of φ, by each needle reduced to 50° Fahr.

Table II.

No. for reference.	Place.	Date. 1847.	Latitude to nearest minute.	Longitude to nearest minute.	For inclination.				Observations with Lloyd needles.			Logarithms of φ.
					Needle.	Poles direct.	Poles reversed.	θ.	Needle.	Temp. Fahr.	v.	
			° ′	° ′		° ′	° ′	° ′		°	° ′	Means used.
1.	Halifax, N.S.	Aug. 20.	44 39	296 23	A(1)	75 37	75 37·1	75 37	L(1)	77·3	39 32·1	$\bar{1}$·9310218
	Halifax, N.S.	21.			A(1)	75 37·6	75 38·3	75 37·9	L(1)	77·5	39 32·5	
	Halifax, N.S.	21.							L(2)	76·9	32 15·1	$\bar{1}$·9495554
2.	Hiltz's, N.S.	23.	44 57	295 9	A(1)	75 37	75 37	75 37	L(1)	74·0	39 36·5	$\bar{1}$·9307381
3.	Windsor, N.S.	24.	45 10	295 44	A(1)	75 40·6	75 42·2	75 41·4	L(1)	76·8	39 38	$\bar{1}$·9309837
4.	Kentville, N.S.	25.	45 12	295 14	A(1)	75 45·1	75 46·3	75 45·7	L(1)	52·1	39 09·6	$\bar{1}$·9320107
5.	Bridgetown, N.S.	26.	44 51	294 22	A(1)	75 40·9	75 41·9	75 41·4	L(1)	75·3	39 05·2	$\bar{1}$·9323155
6.	Annapolis, N.S.	27.	44 45	294 04	A(1)	75 41·7	75 41·2	75 41·5	L(1)	68·5	38 58·8	$\bar{1}$·9325703
	Annapolis, N.S.	27.							L(2)	70·3	31 41·2	$\bar{1}$·9508199
7.	St. John, N.B.	Sept. 1.	45 14	293 57	A(1)	75 55·7	75 55·7	75 55·7	L (1)	59·9	38 22·2	$\bar{1}$·9348103
8.	Fredericton, N.B.	2.			A(1)	76 59·2	76 58·5	76 58·9	L(1)	72·0	39 19·7	$\bar{1}$·9364098
9.	Woodstock, N.B.	4.	46 09	292 25	A(1)	77 12	77 10·2	77 11·1	L(1)	73·2	39 14·8	$\bar{1}$·9373967
	Woodstock, N.B.							76 59·6				$\bar{1}$·9377287
	Woodstock, N.B.	11.							L(2)	63·2	30 56·1	$\bar{1}$·9553924
10.	Riviere des Chutes	6.	46 36	292 16	A(1)	77 11·1	77 11·7	77 11·4	L(1)	69·0	38 54·3	$\bar{1}$·9381584
	Riviere des Chutes	11.							L(2)	45·3	30 37·8	$\bar{1}$·9559425
11.	Grand Falls of St. John	7.	47 03	292 15	A(1)	77 28·1	77 31	77 29·5	L(1)	70·7	39 29·1	$\bar{1}$·9379492
	Grand Falls of St. John	10.				77 27·5	77 31·6	77 29·5	L(2)	59·0	31 09	$\bar{1}$·9561125
12.	Grand River	8.	47 11	292 03	A(1)	77 36·4	77 34·8	77 35·6	L(1)	49·2	38 51·4	$\bar{1}$·9392578
	Grand River	8.							L(2)	61·4	31 01·6	$\bar{1}$·9565685
13.	Madawaska	9.	47 22	291 41	A(1)	77 44·9	77 44·8	77 44·8	L(1)	65·1	39 09·3	$\bar{1}$·9395932
	Madawaska	9.			A(1)				L(2)	77·6	31 08·7	$\bar{1}$·9573159
	Madawaska	9.			A(2)	77 49·3	77 42	77 45·6				

Remarks on Table II.

1. This station was on the east side of Citadel Hill, in an enclosure surrounded with a picket fence, and as much as forty or fifty rods from the N.E. outer line of the works. There were no iron stores in the works, and only two or three guns mounted, which were on the S.W. side. To ascertain if there was local attraction, I took the dip again in the plain on the west side of the hill, at least 100 rods from the works, and found it did not vary by one minute.

2. On the hill, back of Hiltz's Tavern, half-way from Halifax to Windsor.

3. McBride's Garden, one-fifth of a mile N. of the Catholic Chapel.

4. Garden, back of Terrey's Hotel.

5. Forty rods S.E. of Quirk's Tavern.

6. An open field back of Hall's Tavern, and about forty rods S. of Catholic Chapel.

7. On the sea-side E. of the Barracks.

8. River-side in front of the Province House.

9. In an open field, a few rods, say thirty, N. of the Woodstock Hotel. On examining my notes at home, I found that a somewhat thick pencil-mark, in noting the azimuths for meridian, had caused the observations for θ and v on the 4th to be made just 10° out of it: calling those angles θ' and v', θ' was reduced to the meridian by the formula $\cot\theta = \cot\theta' - \cos 10°$; I then obtained the value ϕ' from θ' and v', as if rightly observed; and then obtained ϕ from the equation $\phi = \phi' \frac{\sin\theta'}{\sin\theta}$, which, I believe, will be found a correct process.

10. In the road near Woolverton's.

11. Near the Barracks; needles unsteady on the 7th.

12. On the lower cape, at the junction with the St. John. Observations with L(1) disturbed.

13. On the lower cape, at the junction of the Madawaska with the St. John.

In regard to the latitudes and longitudes, I am indebted to Major Graham for those of Nos. 9, 10, 11, 12 and 13. Messrs. Crauford and Agnew, chronometer raters at Halifax and St. John respectively, furnished me with those for Nos. 1 and 7; the rest were obtained from the best authorities of books or maps that I could procure.

Table III. exhibits the change in force of needles L(1) and L(2); the results are reduced to a temperature of 50°.

Table III.

1847.	No. of observations.	ϕ for L(1).	Difference.	1847.	No. of observations.	ϕ for L(2).	Difference.
February 13.	22	·8718039	·0010902	February 23.	21	·9140079	·0069450
June ......17.	10	·8707137	·0000015	June ......30.	10	·9070629	·0002127
August...16.	5	·8707122		August...16.	5	·9068502	

From the above Table it is obvious that both needles lost considerable force between February and June, and that after that time L(2) continued to lose, while L(1) retained its force. I have therefore reduced all observations made with L(2) to the 30th of June, by considering the loss as proportional to the time, the coefficient of reduction being $(1+{\cdot}000005.d)$, d being the number of days after the 30th of June. Column (3.) in Table IV. contains the logarithms corrected in this manner for L(2), and those of L(1) as originally found. The numbers ·8707137 and ·9070629 are the values of ϕ *at Waterville* on the 30th of June by L(1) and L(2) respectively; if the factor 1·04175 which connects them is compared with the corresponding factors in column (4.) (omitting that for Grand River for a reason already stated), these factors will all be found to exceed it by a small quantity: waiving discussion, at present, on the cause of this difference, I shall assign half the difference between the mean of these factors and 1·04175, or 0·00036 as a correction common to both, positive for L(1) and negative for L(2), which, while it leaves the mean determinations where both needles were used unaltered, will secure a more just comparison for stating where but one needle was used. I believe the difference, however, has hitherto been considered wholly within the errors of observation. Column (5.) contains the corrected logarithms, that is, the values of ϕ for L(1) increased in the ratio of 1 to 1·00036, and those of ϕ for L(2) reduced in the same ratio. Column (7.) contains the total forces for both needles, considering ·8707137, or the number expressing the relative force by L(1) on the 30th of June at Waterville, as the unit of force; and column (8.) contains the means.

Table IV.

(1.) Station.	(2.) Needle.	(3.) Logarithms corrected for change of force in L(2).	(4.) Factors connecting the numbers corresponding to the logarithms (in col. 3) for L(1) and L(2).	(5.) Logarithms corrected for factor connecting L(1) and L(2).	(6.) Logarithms reduced to Waterville as unit.	(7.) Forces to Waterville unit.	(8.) Means.
Halifax ……………	L(1)	$\bar{1}$·9310218	1·04387	$\bar{1}$·9311781	$\bar{1}$·9913028	·98017	·98081
Halifax ……………	L(2)	$\bar{1}$·9496683		$\bar{1}$·9495120	$\bar{1}$·9918746	·98146	
Hiltz's……………….	L(1)	$\bar{1}$·9307381		$\bar{1}$·9308944	$\bar{1}$·9910191	·97953	·97953
Windsor ……………	L(1)	$\bar{1}$·9309837		$\bar{1}$·9311400	$\bar{1}$·9912647	·98009	·98009
Kentville …………	L(1)	$\bar{1}$·9320107		$\bar{1}$·9321670	$\bar{1}$·9922917	·98241	·98241
Bridgetown ………	L(1)	$\bar{1}$·9323155		$\bar{1}$·9324718	$\bar{1}$·9925965	·98310	·98310
Annapolis …………	L(1)	$\bar{1}$·9325703	1·04322	$\bar{1}$·9327266	$\bar{1}$·9928513	·98367	·98401
Annapolis …………	L(2)	$\bar{1}$·9509458		$\bar{1}$·9507895	$\bar{1}$·9931521	·98436	
St. John ……………	L(1)	$\bar{1}$·9348103		$\bar{1}$·9349666	$\bar{1}$·9950913	·98876	·98876
Fredericton ………	L(1)	$\bar{1}$·9364098		$\bar{1}$·9365661	$\bar{1}$·9966908	·99241	·99241
Woodstock…………	L(1)	$\bar{1}$·9377287	1·04189	$\bar{1}$·9378850	$\bar{1}$·9980097	·99543	·99514
Woodstock…………	L(2)	$\bar{1}$·9555509		$\bar{1}$·9553946	$\bar{1}$·9977572	·99485	
Riviere des Chutes…	L(1)	$\bar{1}$·9381584	1·04218	$\bar{1}$·9383147	$\bar{1}$·9984394	·99641	·99626
Riviere des Chutes…	L(2)	$\bar{1}$·9561010		$\bar{1}$·9559447	$\bar{1}$·9983073	·99611	
Grand Falls ………	L(1)	$\bar{1}$·9379492	1·04308	$\bar{1}$·9381055	$\bar{1}$·9982302	·99593	·99621
Grand Falls ………	L(2)	$\bar{1}$·9562688		$\bar{1}$·9561125	$\bar{1}$·9984751	·99649	
Grand River ………	L(1)	$\bar{1}$·9392578	1·04102	$\bar{1}$·9394141	$\bar{1}$·9995388	·99894	·99823
Grand River ………	L(2)	$\bar{1}$·9567205		$\bar{1}$·9565642	$\bar{1}$·9989268	·99753	
Madawaska ………	L(1)	$\bar{1}$·9395932	1·04202	$\bar{1}$·9397495	$\bar{1}$·9998742	·99971	·99948
Madawaska ………	L(2)	$\bar{1}$·9574700		$\bar{1}$·9573137	$\bar{1}$·9996763	·99925	
Waterville …………	L(1)	…………	…………	$\bar{1}$·9398753	10·	1·	1·0000
Waterville …………	L(2)	…………	…………	$\bar{1}$·9576374	10·	1·	
Logarithmic factors used in the above changes.	L(1)	…………	…………	+·0001563	+·0601247		
	L(2)	…………	…………	−·0001563	+·0423626		

Observations with the Unifilar.

At Halifax, Kentville, Annapolis and Fredericton, I made observations with the unifilar; the magnet vibrated, and used also as a deflector, was marked H(10) and was a cylinder 2·953 inches in length; the deflected magnet was marked I(14), a cylinder 2·414 inches in length.

The magnet H(10) being suspended,—by turning the telescope through arcs of three or four degrees, and applying a correction for torsion, observed at the same times,—I obtained on the 27th of March for a scale division 10′·0999, and on the 29th 10′·0963; the value used is 10′·1.

Table V. exhibits the results of observations to determine the temperature coeffi-

cient of magnet H(10); and Table VI. shows the values of P from the formula $P=-\frac{r^2r'^5\sin u'-r'^2r^5\sin u}{r'^5\sin u'-r^5\sin u}$, using the distances 1 foot and 1·4 foot.

TABLE V.

April 29th and 30th.	·000160 ·000169 ·000158 ·000168
Mean......	·000164

TABLE VI.

Date.	Values of P.
July 28.	−00042
July 29.	−00091
Sept. 25.	−00157
Nov. 27.	−00162
Dec. 8.	−00102
Mean ...	−00111

LAMONT's method was adopted of obtaining the moment of inertia of cylinder H(10) by means of a brass ring accompanying the instrument, whose dimensions were,—

	ft.	
External diameter . .	0·2153	Whence $\log k'=·5489695$.
Internal diameter . .	0·1667	
Weight*		

The cylinder H(10) had on it, at the ends, two small brass rings, the contour of which I shortened by filing, so as to bear conveniently the metal ring in vibration. Twenty-one sets of vibrations without the ring, and eight sets with it, made between the 2nd and 6th of October, varying, in number of vibrations, from 340 to 466 each, and corrected for temperature, arc, torsion and rate of chronometer, all observed at the same times, gave for $\log \pi^2k$ at a mean temperature of 63°·3, 1·3320923. Observing, however, that the cylinder suspended seemed very slightly depressed at the N. end, I shifted the little ring there to the S. end, putting it in contact with the other ring, and at such a distance from the end of the cylinder as I had calculated would give the same moment of inertia; and then on the 8th and 9th of October made more observations for π^2k, the particulars of which are in Table VII.

TABLE VII.

Date. Civil reckoning.	Whole time.	Number of vibrations.	Time of one vibration.	Commencing arc.	Final arc.	Temperature.	Torsion for 90° in scale divisions.	Rate chron.	k.
d h m				Sc. div.	Sc. div.	°		″	
Oct. 8 9 33 A.M.	1448·3572	330	4·38896	3·0	·4	50·5	·8725	−6·5	2·1743 by 1st set.
8 10 21 A.M.	2091·8929	294	7·11528	3·0	·9	54·5	1·1800	−6·5	
8 01 18 P.M.	1500·8215	342	4·38836	2·9	·7	65·4	·76	−6·5	2·1787 by 2nd set.
8 02 32 P.M.	2103·	296	7·10473	2·8	·1	65·75	1·065	−6·5	
8 03 17 P.M.	1490·7857	340	4·38466	3·0	·6	64·0	·69	−6·5	
9 01 00 P.M.	1501·	342	4·38889	2·9	·7	59·5	·55	−6·5	2·1766 by 3rd set.
9 01 55 P.M.	2217·2643	312	7·10662	2·9	·1	59·3	·945	−6·5	
9 02 53 P.M.	1499·0357	342	4·38314	2·9	·4	56·0	·6875	−6·5	
9 03 47	2228·8857	314	7·09836	2·7	·9	56·1	1·0925	−6·5	2·1764 by 4th set.
9 04 33	1523·4785	348	4·37781	3·0	·6	50·5	·795	−6·5	
									2·1765 mean value.

* Weight not stated in the MSS.—E. S.

These observations gave for the mean value of log k, $\cdot 3377587$, at a mean temperature of $58^\circ \cdot 2$, and hence log $\pi^2 k = 1 \cdot 3320584$. If this value of $\pi^2 k$ is reduced to the temperature of $63^\circ \cdot 3$, by the formula $1 + 2e(t' - t)$, it gives $1 \cdot 3320885$, differing from the former observed value only $\cdot 0000038$. Where in the table three observations are included in one set, the mean of the true corrected values of T for the first and third was taken and combined with T′, so as in some degree to eliminate changes of force.

Table VIII. contains the uncorrected particulars of my observations of vibrations and deflections at Halifax, Annapolis and Fredericton, and those of vibration only at Kentville; with the nearest observations, before and after my tour, at Waterville.

Table IX. contains m, X, and the total force; a mean for the value of m at Kentville having been taken from the values at Halifax and Annapolis.

Table X. contains the total forces at the four stations obtained by the unifilar reduced to Waterville by the observations of the LLOYD needle L(1) at the same stations, in order to test the accuracy of the observations; an additional column shows the same comparison for the only two stations where the two LLOYD needles and unifilar were all used; in other words, having obtained the total force at, say, Halifax by the unifilar, and also the relative force by the LLOYD needles, and knowing also the relative force at Waterville by the LLOYD needles, and therefore the *ratio* of the force at Halifax to the force at Waterville by the LLOYD needles, I multiply the aforesaid total force at Halifax got by the unifilar by this *ratio*, and obtain the total unifilar force at Waterville, and so with the rest.

TABLE VIII.

Station.	Date.	Magnet.	u.	Temperature.	u'.	Temperature.	Time of one vibration.	Number of vibrations.	Temperature.	Commencing arc.	Final arc.	Rate of chron.	Torsion for 90° in sc. divisions.
			° ′	°	° ′	°			°	Sc. div.	Sc. div.	″	
Waterville ...	July 29.	H(10)	12 21·3	63·8	4 28·5	64·1	4·35021	548	66·4	6·5	1·1	......	·8925
Waterville ...	30.	H(10)	12 21·6	67·6	....	......	4·35277	460	72·0	5·25	0·75	......	1·009
Halifax	Aug. 21.	H(10)	....	......	4 26	64·9	4·34901	338	71·5	5·0	1·0	−8·5	·69
Kentville......	24.	H(10)	....	......	....	......	4·36005	238	61·9	5·0	1·0	−8·5	·6018
Annapolis ...	27.	H(10)	....	......	4 26·2	65·0	4·35008	344	68·5	5·5	0·8	−8·0	·52
Fredericton...	Sept. 2.	H(10)	....	......	4 49	71·2	4·53733	344	72·5	4·9	0·7	−7·0	·7225
Waterville ...	25.	H(10)	12 10·9	61	4 24·9	60·4	4·37520	356	56·0	8·4	1·4	−6·2	1·01

In this Table u and u' are given to the nearest tenth of a minute, but in the calculations the value in seconds was employed.

Table IX.

Station.	Date.	Corrected angles of deflection.	Time of one vibration.	m.	X.	Total force.
Waterville	July 29.	12° 23′ 02″·5	4·34798	·34928	3·2536	13·4174
Waterville	29.	4 29 04·8		·34926	3·2538	13·4181
Waterville	30.	12 23 43·9	4·34901	·34937	3·2515	13·4086
Halifax	Aug. 21.	4 26 41·2	4·34457	·34799	3·2710	13·1757
Kentville............	24.	....	4·35914	*	3·2493	13·2112
Annapolis	27.	4 26 54	4·34777	·34787	3·2672	13·2198
Fredericton.........	Sept. 2.	4 49 58·7	4·53242	·34780	3·0071	13·3491
Waterville	25.	12 12 15·6	4·27832	·34437	3·2544	13·4207
Waterville	25.	4 25 19·8		·34441	3·2540	13·4192

Table X.

Halifax reduced to Waterville	13·442	13·435
Kentville reduced to Waterville	13·448	
Annapolis reduced to Waterville	13·439	13·437
Fredericton reduced to Waterville	13·451	
	13·445	13·436

XV. *On a new Case of the Interference of Light.*

By the Rev. Baden Powell, *M.A., F.R.S., F.R.A.S., F.G.S., Savilian Professor of Geometry in the University of Oxford.*

Received April 6,—Read May 18, 1848.

(1.) In the state of advance at which the theory of light has now arrived, a single case of interference directly explicable by the ordinary principles of undulations, even though occurring under new conditions, could hardly be deemed of sufficient importance to form the subject of a separate communication to the Royal Society.

But in the present instance though the case presented, in its more *general* features, is easily accounted for on the acknowledged doctrine of interference and retardation, yet it offers many particulars in its *details*, in reference to which such explanation is, at least, not equally obvious: while some points require for their elucidation investigations of a more extended character.

Again, it is found to be a case which by no means stands isolated, but offers analogies with other classes of phenomena which have excited considerable interest and discussion, especially with regard to what has been termed, perhaps improperly, a "polarity" in the prismatic rays,—a new instance of which is here exhibited. In these respects, then, I trust the subject may not appear unworthy of the notice of the Royal Society.

Having arrived at the primary results in July 1847, I communicated them to my friend Mr. G. G. Stokes of Pembroke College, Cambridge, who has gone extensively into the whole theory of these and other allied cases, and has afforded me much valuable aid in the investigation. In the present paper I propose only to describe my own experiments, with the general application of the undulatory theory to the explanation of them, supported by some numerical comparisons.

(2.) The main experiment is as follows:—in a *hollow glass prism*, or rather *trough*, containing some highly refractive and dispersive liquid, such as oil of sassafras, anise, or cassia, *a plate of glass is inserted* with its lower edge parallel to the edge of the prism, and so that its plane nearly bisects the angle of the prism, while it extends only through the upper half of the liquid, leaving the lower, or thinner part, clear (see fig. 1). Light being admitted through a narrow horizontal slit in the usual manner, *the spectrum thus formed is seen crossed by a number of dark bands parallel to the slit or edge of the prism.*

(3.) I have tried various combinations of oils and other media with plates of different thicknesses, both of glass and of other transparent substances. In these different instances some remarkable distinctions are exhibited. In some cases the bands are sensibly equidistant, in others increasing in number and fineness towards one

Fig. 1.

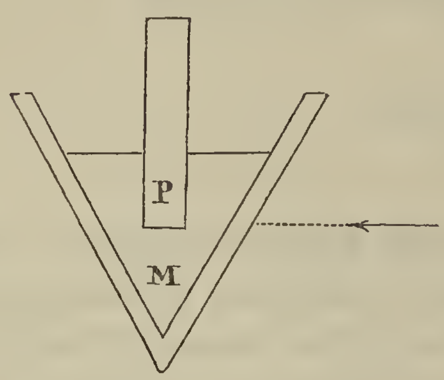


Fig. 2. Position 1.

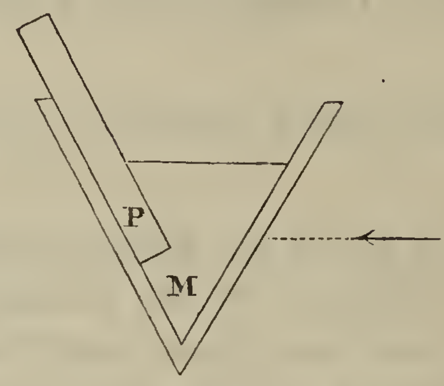
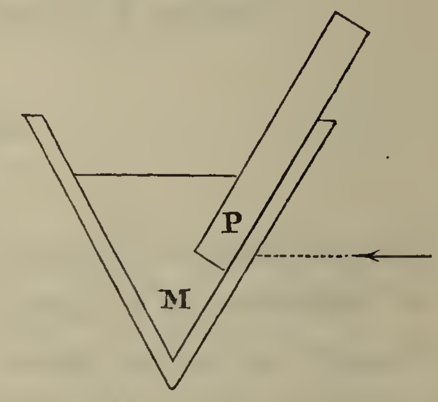


Fig. 3. Position 2.



Arrangement I.

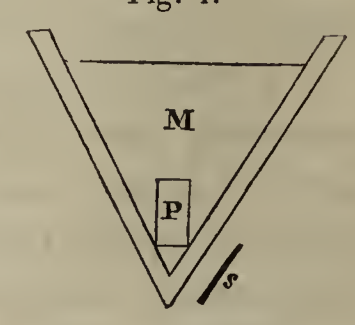
end of the spectrum: in most cases extending throughout, but in some deficient at one part.

(4.) With the same oil or medium, if the plate exceed a certain thickness, the bands become too numerous and fine to be seen: if less than a certain other limit, they become too few, broad and faint: and for some intermediate thickness they appear most vivid and distinct.

(5.) With plates and media of different refractive and dispersive powers as well as different thicknesses, changes in the number and vividness of the bands take place, as well as in the limits of their visibility, in a manner evidently dependent on the thickness and relative refractions jointly; though not in any such obvious relation as can be stated by a simple experimental law.

(6.) It is not *necessary* that the plate should have precisely the vertical position just described; if *inclined* either way, even to being in contact with either side of the prism, the bands are still seen; but they undergo a slight shifting downwards as the plate is inclined, and are perhaps less vivid (see figs. 2, 3).

Fig. 4.



Arrangement II.

(7.) Some combinations of a medium and a plate, such as glass with oils of turpentine or angelica, or water, &c., *give no bands with this arrangement.* But Mr. Stokes pointed out from theory, that in these cases bands might be expected to appear with a *reverse* arrangement; that is, by placing a narrow slip of glass, &c. to intercept the *thinner* part of the prism, leaving the upper or thicker part clear, and of course cutting off any portion of light, which might otherwise pass below the plate (see fig. 4, where *s* is a small screen for intercepting the light below).

On using such an arrangement for the cases just mentioned, I accordingly found bands produced.

(8.) In any case to see the bands well, especially towards the violet end of the spectrum, a strong light is requisite: that of the sun direct, shaded by a blue glass, or at least that of the bright part of the sky near the sun, was usually thrown in by the shutter apparatus; and though a small telescope was usually employed, yet the bands, in cases where they are vivid and broad, may be seen by the naked eye; and for such cases lamp-light may be used, but it will not suffice for the more delicate.

(9.) When *crystallized* substances are employed as *plates* other peculiar phenomena are presented.

A plate of calc-spar (formed by the natural cleavage) with oil of cassia gives two distinct sets of bands; the one finer and narrower than the other, which about the middle of the spectrum may be seen distinctly superimposed one on the other.

On applying a Nicol-prism each set disappears alternately, leaving the other visible at each quarter of a revolution of the analyser; showing them due to the two oppositely polarized pencils. It is easily ascertained that the *finer* bands belong to the *extraordinary*, the *broader* to the *ordinary ray*.

(10.) A plate of quartz, cut perpendicular to the axis, with oil of sassafras, gives very distinct bands, which may be seen to be in fact composed of two sets superimposed and nearly coinciding, since when the plate is inclined (as in position 1, see fig. 2) at intervals throughout the spectrum the near coincidence of the dark spaces of one set, with the bright of the other, occasions an extreme faintness in the bands. In this case the two pencils deviate sufficiently from the axis to approximate to plane polarized light, and thus the Nicol-prism causes the indistinctness to disappear at each quarter of a revolution by stopping one of them alternately.

(11.) For an *explanation* of the *general* phenomena of the formation of bands under the conditions specified, the simple interference-theory suffices.

Of the homogeneous pencil going to form any one ray of the spectrum, that half which passes through the thicker part of the prism is more retarded than that through the thinner; but uniformly, and in proportion to the difference of refraction, throughout the spectrum.

The plate of glass, however, having different indices for the several primary rays from those of the medium, interrupts this uniformity, and causes the one part of the pencil to be always retarded in an increasing ratio with respect to the other, throughout the spectrum; and as this difference of retardation amounts successively to an odd or an even multiple of a half wave-length, the rays will be in discordance or accordance, or give a dark or bright band accordingly.

(12.) But to account for the different conditions which determine the number and

character of the bands, and the limits of thickness, as well as which of the two arrangements before described, will produce them; it is necessary to consider more precisely the relative refractions of the plate and medium, or the amount and direction of the retardation ; and it is found that on such considerations we obtain an expression which includes these conditions ;—the difference of *sign* corresponding to the *two arrangements*: while it assigns the *number* of bands which will be formed between any two given rays, or throughout the whole spectrum, with a given thickness of the plate.

(13.) In the comparison of theory and observation, the broad facts,—that bands will be produced in the respective cases by the one or the other arrangement,—as well as their general character,—show an entire agreement with theory.

The more precise comparison of the *number* of bands formed throughout the spectrum, or within certain definite spaces of it, though in some cases unavoidably imperfect from the difficulty of distinguishing the bands, yet upon the whole gives accordances as good as perhaps can be expected.

(14.) When plates of doubly-refracting crystals are employed, the calculation for the extraordinary ray in particular directions in the crystal, becomes more complex, involving the laws of crystalline refraction.

In all cases a small error in the index causes a comparatively large difference in the calculated number of bands. But, from whatever cause, in these last instances the application of theory is, as yet, less satisfactory than in others ; at least in the instance of calc-spar.

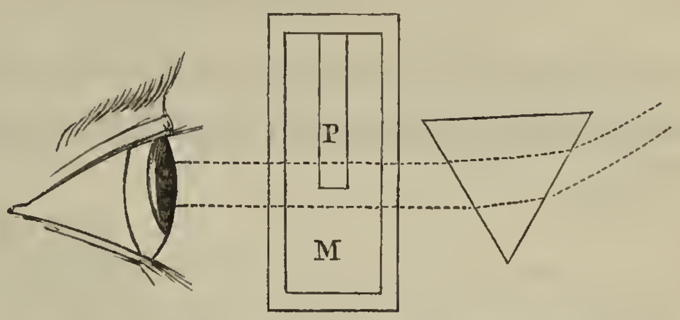
(15.) The first observation of the general phenomenon reminded me of the precisely *analogous* result obtained by Baron von Wrede, though in a manner so totally different*; in which two portions of light, unequally retarded the one by reflexion from the *first*, the other from the *second* surface of a plate of mica, impinge jointly on a prism, whence results a spectrum crossed with bands; while the author's profound analysis (founded on the hypothesis of the internal reflexions of a ray among the molecules of a medium) opens an extended analogy between these cases and those of the absorption of definite rays by media in general.

(16.) There is also obviously a general analogy between these phenomena and those observed by Mr. Fox Talbot and Sir D. Brewster, on partially intercepting the spectrum by a plate of mica covering half the pupil of the eye ;—especially in the circumstance, that here also the plate must be applied towards *one side* of the prism, which corresponds to what has been described as a species of *polarity*. In those experiments the retardation is the difference of the retardations of the plate of mica and of a plate of air which would be contained between the surfaces of the mica produced: in mine it is the difference of the retardation of the glass and of an equal thickness of the liquid medium of the prism.

* Taylor's Foreign Scientific Memoirs, vol. i. part 3, p. 487.

(17.) Hence Mr. STOKES illustrated the case by supposing the glass plate inserted in a vessel with parallel sides filled with the medium ; when if the spectrum formed by any prism were viewed through this combination, with the plate towards the thick part of the prism, the effect would be the same ; and if the prism were of the same substance, the cases would be identical ; the liquid prism in my experiment serving the double office of the prism and the flat vessel. (See fig. 5, where P is the plate and M the medium.)

Fig. 5.



(18.) Another remark here offers itself:— If it should be considered that the theory is sufficiently established to give confidence in deductions from it, it may be applied to the *inverse problem* of finding the refractive indices of a plate, those of the medium, and the number of bands, being known. This may be important for many substances which occur in the form of plates, but cannot be cut into prisms.

(19.) There remain also other features of the case to be accounted for, which, if slight in appearance, are yet not unimportant in a theoretical point of view ; such as relate to the changes in the vividness of the bands, especially as affected by the thickness or inclination of the plate, by enlarging or contracting the aperture, or breadth of the prism,—and some other points,—to which the simple interference-theory cannot apply.

In all experiments of this kind it is now generally understood that there must be, theoretically at least, even if it be practically insignificant, another species of action concerned, dependent on *the diffraction of the lens*, whether that of the eye, or of the object-glass of the telescope, producing that "diffusion," as it has been termed, in the optical image, which, if of sensible amount, may influence the phenomena.

The method of investigating this species of action in general is equally well understood, though in some parts the theory has been found susceptible of improvement. In the present instance such an investigation is necessary to include some of the peculiar modifications of the phenomena observed ; and this constitutes the subject of Mr. STOKES's researches, which he will give in a separate form.

The remainder of this paper is devoted to the details of the observations and the theoretical investigations.

Theoretical Investigation.

(20.) The intensity of the incident light being unity, and the direct vibration for any point in the spectrum being

$$\sin \frac{2\pi}{\lambda}(vt-x) ;$$

that for the part retarded by r will be

$$\sin \frac{2\pi}{\lambda}(vt-x-r);$$

or,
$$\sin \frac{2\pi}{\lambda}(vt-x) \cos \frac{2\pi}{\lambda} r - \cos \frac{2\pi}{\lambda}(vt-x) \sin \frac{2\pi}{\lambda} r.$$

Consequently the intensity I for any point of the whole resulting wave will be the sum of the squares of the coefficients of $(vt-x)$, or we shall find on squaring and reducing,

$$\mathrm{I}=2\left(1+\cos \frac{2\pi r}{\lambda}\right);$$

or if we assume $p=\frac{4r}{\lambda}$, it becomes

$$\mathrm{I}=2\left(1+\cos \frac{\pi}{2} p\right).$$

(21.) And if we suppose p to be originally an even number and to increase by unity for successive rays of the spectrum, we shall have the corresponding values,

p	$\cos \frac{\pi}{2} p=-1$, and therefore	$\mathrm{I}=0$
$p+1$	$=\ \ 0$	$\mathrm{I}=2$
$p+2$	$=+1$	$\mathrm{I}=4$
$p+3$	$=\ \ 0$	$\mathrm{I}=2$
$p+4$	$=-1$	$\mathrm{I}=0$
&c.	&c.	&c.

Thus for any two values p_1, p_2, if $p_1-p_2=4$, they correspond to a change from one dark band to another, and consequently, if for two rays $p_1-p_2=4n$, n will be the number of bands in the space comprised between those two rays: and if they be the extremes of the spectrum, n will be the whole number of bands. Its value may be assigned by considering the nature of the retardation, or obtaining an expression for r, as follows :—

(22.) On inserting the plate P of thickness τ whose index is μ_p, as above, into the medium whose index is μ_m, the retardation of the light which passes through the plate being the difference of the retardations of the plate and of an equal thickness of the liquid, will be expressed by

$$r=(\mu_p-\mu_m)\tau.$$

But since $\frac{p}{4}=\frac{r}{\lambda}$ (20.), we shall have

$$\frac{p}{4}=\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau.$$

And for any two rays whose indices are μ_1, μ_2, and wave-lengths λ_1, λ_2, we have

$$\frac{p_1-p_2}{4}=n=\left[\left(\frac{\mu_p-\mu_m}{\lambda}\right)_1-\left(\frac{\mu_p-\mu_m}{\lambda}\right)_2\right]\tau.$$

This formula may apply to the whole length of the spectrum, taking the two ex-

treme rays. And the thickness of the plate being known, we may find the number of bands by computing the coefficient in terms of μ and λ which are known.

(23.) Writing this coefficient $=q$ or $n=q\tau$, since here n is supposed a whole positive number, and q may be either $+$ or $-$ according to the values of μ and λ, we have,

$$\frac{n}{q}=\pm\tau.$$

Hence if, taking successive rays, we have always through the spectrum from the red to the violet,

$$\left(\frac{\mu_p-\mu_m}{\lambda}\right)_1<\left(\frac{\mu_p-\mu_m}{\lambda}\right)_2,$$

or the value of q negative, the effect of the plate will be such that the arrangement I. will give bands : if q be positive, the effect of the plate will be such that arrangement II. will give bands.

If for any combination of a plate and a medium $\left(\frac{\mu_p-\mu_m}{\lambda}\right)$ should have a *maximum* or *minimum* value at any ray, the difference would *change signs*, and bands be formed towards that end of the spectrum where it was $-$ with arrangement I.; and towards that end where it was $+$ with arrangement II.

(24.) The general principles of the "diffraction-theory," as applicable to the present case, are precisely the same as in Mr. Airy's paper*. But it will not be necessary here to go into the subject any further, since Mr. Stokes has greatly generalized and improved this theory so as to lead to other important results, the whole of which are discussed in his paper, in the present part of the Philosophical Transactions.

Observations.

(25.)

Arrangement.	Plate.	Medium.	Number of bands.				
	Glass.	Oil of Sassafras.	B to D.	D to F.	F to G.	G to H.	Total.
I.	inch. $\tau=$·5 ·34 ·17 ·08 ·04 ·015	 No bands visible. Very fine and close. Fine. Clear. Broad and clear. Very broad and faint.	6	14	21	24	65
II.		No bands.					
(26.)							
		Oil of Cassia.					
I.	·08 ·04 ·015	Too fine to count. Fine.	15	29	32	Faint. 40 ?	116
II.		No bands.					

* Philosophical Transactions, 1841, Part I.

(27.)

Arrangement.	Plate.	Medium.	Number of bands.					
	Glass.	Oil of Cummin.	B to D.	D to E.	E to F.	F to G.	G to H.	Total.
I.	inch. $\tau = \cdot 04$	Broad.	0	0	5	11	{ Faint. 15 ? }	31
II.		No bands.						

(28.)

		Oil of Turpentine.					
I.		No bands.					
II.	·08 ·04	Fine. { Clear in the red; broader towards blue. }	14	20	15	10	59

(29.)

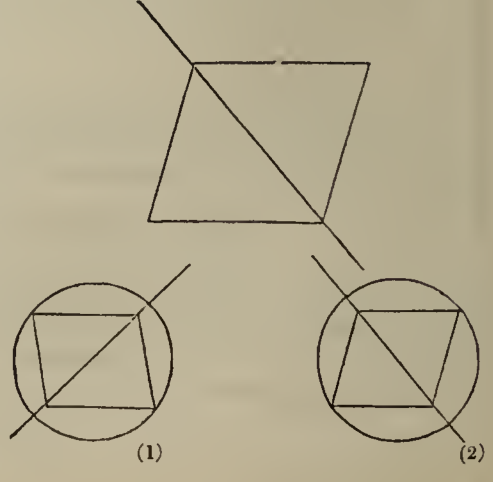
		Water.					
I.		No bands.					
II.	·04 ·015	Too fine to count. { Fine and faint; difficult to count. }	19	30 ?	22 ?	15 ?	86

(30.)

Arrangement.	Plate.	Medium.	Number of bands.			
	Calc-spar (by natural cleavage).	Oil of Cassia.	B to D.	D to F.	F to G.	G to H.
I.	inch. $\tau = \cdot 04$ Position vertical.	Two sets of bands superposed. With Nicol prism. Broad set.	0	Broad and faint. 14	30 ?	40 ?
		With Nicol prism. Fine set.	20	36 ?	Too fine to count.	
	Position 1. Position 2.	Both sets broader. Both sets finer.				
	$\tau = \cdot 08$	Both sets too fine to count.				

(31.) To determine which set of bands belongs to the ordinary and which to the extraordinary pencil, we may proceed as follows:—Placing a rhomb of calc-spar with a small aperture behind it, so as to give two images, in the same relative position as the plate (see fig. 6) when the section of the Nicol-prism has its short diagonal perpendicular to that of the rhomb (1.), E disappears; when parallel (2.) O disappears. In the spectrum, in the former case the finer bands disappear, in the latter the broader. The fine bands therefore belong to E, the broad to O.

Fig. 6.



Or the same result might perhaps be deduced more directly from considering that as O is polarized parallel, and E perpendicular to the principal section, and that the *short* diagonal of the Nicol-prism is *horizontal* when light polarized by reflexion from an *horizontal* surface is *transmitted*,—or that the *short* diagonal is perpendicular to the plane of polarization of the *transmitted* ray. Then since when the short diagonal is perpendicular to the principal section of the calc-spar, the *fine* bands are stopped and the broad transmitted, it follows that the broad bands are polarized *parallel* to the principal section, or belong to O.

(32.)

Arrangement.	Plate.	Medium.	Number of bands.			
	Quartz cut perpendicular to axis.	Oil of Sassafras.	B to D.	D to F.	F to G.	G to H.
I.	inch. $\tau = \cdot 15$ Position vertical.	Bands fine and clear, broader towards red end	8	43	Too fine to count; estimated at 70 ?	80 ?
	Position 1 ...... Position 2	Bands rather finer.				

With the plate inclined in position 1, eight intervals of indistinctness (extending over four or five bands each) occur from H to about E; from E to D the bands altogether become very faint: perhaps two such intervals may be discerned: from D to B no bands appear. On applying a Nicol-prism, the intervals of indistinctness disappear at each quarter of a revolution.

Calculation.

(33.) In all the following calculations for n by the formula (22.), the values employed for the reciprocals of the wave-lengths of primary rays are as follows:—

Ray.	λ (decimals of 1 French inch).	$\frac{1}{\lambda}$.
B.	·00002541	39354
D.	·00002175	45977
E.	·00001945	51413
F.	·00001794	55741
G.	·00001587	63011
H.	·00001464	68306

The values of the term $\left(\frac{\mu_p - \mu_m}{\lambda}\right)\tau$ are interesting as expressing the absolute interval of route in wave-lengths of the two interfering rays, and thus, when compared with observation, conveying an idea of the extent to which the regularity of the undulations is kept up.

The indices here used are those contained in my "Report on Refractive Indices"*. For glass I have assumed the indices of FRAUNHOFER's crown glass, No. 9.

* British Association Report, 1839.

Ray.	μ_p.	μ_m.	$\mu_p-\mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff.$=n$.	Remarks.
(34.) Glass and Oil of Sassafras, $\tau=\cdot04$ inch.							
B.	1·526	1·526	0	0	0		
D.	1·529	1·532	—·003	— 138	— 5·5	— 5	
F.	1·536	1·545	—·009	— 501	—20	—15	Hence in this case arrangement I. will give bands.
G.	1·542	1·557	—·015	— 945	—37·8	—18	
H.	1·546	1·569	—·023	—1571	—62·8	—25	
						—65	
(35.) Glass and Oil of Cassia (Report, No. ii.), $\tau=\cdot015$ inch.							
B.	1·526	1·594	—·068	— 2676	— 40		
D.	1·529	1·607	—·078	— 3586	— 54	— 14	
F.	1·536	1·636	—·100	— 5574	— 84	— 30	Here arrangement I. will give bands.
G.	1·542	1·667	—·125	— 7867	—118	— 34	
H.	1·546	1·702	—·156	—10656	—160	— 42	
						—120	
(36.) Glass and Oil of Cummin, $\tau=\cdot04$ inch.							
B.	1·526	1·502	+·024	+ 944	+37·7		
D.	1·529	1·507	+·022	+1011	+40·4	+ 2	
E.	1·533	1·513	+·020	+1028	+40·8	+ 1	Here arrangement II. would give bands from B. to E, and arrangement I. from E to H.
F.	1·536	1·520	+·016	+ 891	+35	— 6	
G.	1·542	1·533	+·009	+ 567	+22·6	—12	
H.	1·546	1·543	+·003	+ 205	+ 8	—14	
						—32	

(37.) We may remark that the indices for oil of cummin are all open to some uncertainty*, and it is easily found that a change in the indices of D and E of ·001 only would give *no bands* between B and E with either arrangement; for we should have on this supposition—

Ray.	μ_p.	μ_m.	$\mu_p-\mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff. $=n$.	Remarks.
B.	......	......	......	......	+37·7		
D.	......	1·508	+·021	+564	+38·5	0	No bands.
E.	......	1·514	+·019	+976	+38·8	0	
(38.) Glass and Oil of Turpentine, $\tau=\cdot04$ inch.							
B.	1·526	1·470	+·056	+2203	+ 88		
D.	1·529	1·474	+·055	+2528	+101	+13	
F.	1·536	1·482	+·054	+3009	+120	+19	Hence arrangement II. will give bands.
G.	1·542	1·488	+·054	+3402	+136	+16	
H.	1·546	1·494	+·053	+3619	+145	+ 9	
						+57	

* See Report, British Association, 1839, p. 12.

(39.) Glass and Water, $\tau = \cdot 015$ inch.

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p - \mu_m}{\lambda}$.	$\left(\frac{\mu_p - \mu_m}{\lambda}\right)\tau$.	Diff. $= n$.	Remarks.
B.	1·526	1·331	+195	+ 7672	+115		
D.	1·529	1·333	+196	+ 9009	+135	+20	
F.	1·536	1·337	+199	+11092	+166	+31	Hence arrangement II. will give bands.
G.	1·542	1·341	+201	+12663	+190	+24	
H.	1·546	1·344	+202	+13796	+207	+17	
						+92	

(40.) Calc-spar and Oil of Cassia, ordinary ray. RUDBERG's indices.

Ray.	$\mu_{p\,O}$.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p - \mu_m}{\lambda}$.	$\left(\frac{\mu_p - \mu_m}{\lambda}\right)\cdot\tau$ $\tau = \cdot 04$.	Diff.	Remarks.
B.	1·6531	1·5945	+·0586	+2306	+92		Hence with arrangement I. there will be no bands from B to D, but bands from D to H.
D.	1·6585	1·6073	+·0512	+2353	+94	+ 2	
F.	1·6680	1·6358	+·0322	+1794	+71	− 23	
G.	1·6762	1·6671	+·0091	+ 573	+23	− 48	
H.	1·6833	1·7025	−·0192	+1311	−52	− 75	
						146	

(41.) For the *extraordinary ray*, in a plate bounded by the planes of cleavage, Mr. STOKES has calculated the results as follows:—

"The incidence is supposed to be perpendicular; that is, strictly, the rays B, D, &c. are supposed to be received in succession at a perpendicular incidence: but the results may be applied with very little error to the case in which rays of mean refrangibility are incident perpendicularly.

"The dihedral angle of the rhombohedron of calc-spar is 105° 5′*.

"If i be the inclination of the axis to the normal of the plate, we get by a spherical triangle,

$$\sin i = \cos 52^\circ\ 32'\cdot 5 \operatorname{cosec} 60^\circ$$

whence
$$i = 44^\circ\ 36'\cdot 6.$$

Now† we have

$$v = \sqrt{a^2 \cos^2 i + c^2 \sin^2 i} = a \cos i \sec \theta$$

where
$$\tan \theta = \frac{c}{a} \tan i = \frac{\mu'_o}{\mu'_E} \tan i,$$

and if
$$\mu' = \frac{a}{v}\mu'_o \quad \mu' = \mu'_o \sec i \cos \theta.$$

Also
$$\log \tan i = 9\cdot 99409, \quad \text{and} \log \sec i = 10\cdot 14758.$$

Hence
$$\log \tan \theta = \log \mu'_o - \log \mu'_E + 9\cdot 99409$$
$$\log \mu' = \log \mu'_o + \log \cos \theta + 10\cdot 14758 - 20.$$

* PHILLIPS's Mineralogy.

† AIRY's Tract, Art. 151.

"In this manner we obtain the following table:—

(42.) Calc-spar and Oil of Cassia. Extraordinary ray.

Ray.	Calc-spar μ'_O.	Calc-spar μ'_E.	θ.	μ'.	Oil of Cassia, μ.	$\mu'-\mu$.	$\frac{\mu'-\mu}{\lambda}$.	$\frac{\mu'-\mu}{\lambda}\times\cdot 0$	Diff.
B.	1·6531	1·4839	47° 42'·0	1·5628	1·5945	—·0317	—1248	— 49·9	
D.	1·6585	1·4863	44·8	1·5665	1·6073	—·0408	—1876	— 75·0	— 25·1
F.	1·6680	1·4907	49·5	1·5731	1·6358	—·0627	—3495	—139·8	— 64·8
G.	1·6762	1·4945	53·6	1·5787	1·6671	—·0884	—5570	—222·8	— 83·0
H.	1·6833	1·4978	57·0	1·5837	1·7025	—·1188	—8115	—324·6	—101·8
									275

"The direction considered being about 45° from the axis, μ' ought to be nearly equal to the mean of μ'_O μ'_E. Now the values of μ' found above, fall short of the mean of μ'_O and μ'_E by the following quantities:—

For B	. . . . . .	·0057
D	. . . . . .	·0059
F	. . . . . .	·0062
G	. . . . . .	·0066
H	. . . . . .	·0068

"The smallness and regularity of these numbers is a test of the correctness of the arithmetic."

(43.) *Quartz and Oil of Sassafras.*

The angle of the prism being 60° and the ray F at the minimum deviation, and ϕ, ϕ' being the angles of incidence and refraction for the first surface, we have for either pencil,

$$\phi'_F=30°, \quad \sin\phi_F=\mu_F \sin\phi'_F,$$

which gives ϕ_F, and therefore ϕ for the other rays;

also $$\sin\phi'=\frac{\sin\phi}{\mu} \text{ gives } \phi' \text{ for the other rays.}$$

If i be the angle of incidence *on the plate* then in position (1) (see figs. 2, 3) $i=60°-\phi'$, and in position (2) $i=\phi'$.

Then (as in AIRY's Tract, Art. 151) if i' be the angle of refraction referring to the *normal of the wave*, we have

$$\tan i'=\frac{a\sin i}{\sqrt{v^2-c^2\sin^2 i}}$$

where $$\frac{a}{c}=\frac{\mu_E}{\mu_O}$$

and $$v' = \sqrt{a^2 \cos^2 i' + c^2 \sin^2 i'},$$

and if V be the velocity in air, so that $\frac{V}{v} = \mu$, and we take $\frac{v'}{v} = \frac{\mu}{\varkappa}$, then $\frac{V}{v'} = \varkappa$;

and M being the retardation expressed by the number of waves' lengths, we get

$$M = \frac{\tau}{\lambda \cos i'} \left(\frac{V}{v'} - \mu \cos (i - i') \right).$$

Also i' is found to differ but little from 30° (about 30′ for the extreme rays) and $\cos (i - i') = \cdot 9999$, which may be put $= 1$. Thus

$$M = \frac{\tau}{\lambda} \sec i' \ (\varkappa - \mu).$$

For the ordinary ray we have simply

$$\sin i' = \frac{\mu}{\mu_0} \sin i$$

and $$M = \frac{\tau}{\lambda} \sec i' (\mu_0 - \mu).$$

(44.) In this way the following results are obtained, $\tau = \cdot 15$ inch.

Ray.	Quartz μ_{p0}.	Oil of Sassafras, μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p - \mu_m}{\lambda}$.	$M' = \left(\frac{\mu_p - \mu_m}{\lambda}\right) \tau$.	Position vertical diff. $= n$.	$M = M' \sec i'$.	Position 1. Diff. $= n'$.
B.	1·5409	1·5257	+·0152	+598	+ 89·7		+103·4	
D.	1·5441	1·5321	+·0120	+552	+ 82·8	− 7	+ 95·7	− 7
E.	1·5471	1·5387	+·0084	+432	+ 64	− 18	+ 73·9	− 22
F.	1·5496	1·5448	+·0048	+267	+ 40	− 24	+ 46	− 28
G.	1·5542	1·5575	−·0033	−208	− 31	− 71	− 35·8	− 82
H.	1·5582	1·5693	−·0111	−758	−113	− 82	−131	95
						202		234

(45.) For the extraordinary ray Mr. Stokes has made a calculation, of which the following are the principal steps and results. From the expression above,

$$v' = a \cos i' \sqrt{1 + \frac{c^2}{a^2} \tan^2 i'};$$

and assuming $$\tan \theta = \frac{c}{a} \tan i' = \frac{\mu_0}{\mu_E} \tan i',$$

whence $$v' = a \cos i' \sec \theta,$$

$$\varkappa = \frac{V}{v'} = \mu_0 \sec i' \cos \theta.$$

(46.) To obtain $\varkappa$, assuming i' approximately as differing little from i, which is near enough for this purpose, the following values result :—

Ray.	μ_O.	μ_E.	$\varkappa$. Pos. (1).	$\varkappa$. Pos. (2).	μ. Sassafras.	$\sin\varphi=\mu\sin\varphi'$. φ'.	Pos. 1. $i=60-\varphi'$.	Pos. 2. $i=\varphi$.
B.	1·5409	1·5499	1·5430	1·5432	1·52575	30° 25′ ″	29° 35′ ″	30° 25′ ″
D.	1·5442	1·5533	1·5464	1·5465	1·53215	30 16 30	29 43 30	30 16 30
E.	1·5471	1·5563	1·5494	1·5494	1·53870	30 30 80	29 52 0	30 30 80
F.	1·5496	1·5589	1·5519	1·5519	1·54485	30 0 0	30 0 0	30 0 0
G.	1·5542	1·5636	1·5566	1·5565	1·55750	29 44 0	30 16 0	29 44 0
H.	1·5582	1·5677	1·5607	1·5605	1·56935	29 29 0	30 31 0	29 29 0

(47.) Then with more accurate values of i' we obtain, successively,

Ray.	$\sin i'=\frac{\mu}{\varkappa}\sin i$. i'. Position 1.	i'. Position 2.	$\varkappa-\mu$. Pos. 1.	$\varkappa-\mu$. Pos. 2.	$\frac{\varkappa-\mu}{\lambda\cos i'}$. Pos. 1.	Pos. 2.	$\left(\frac{\varkappa-\mu}{\lambda\cos i'}\right)\tau=$M. Pos. 1.	Pos. 2.	Diff.$=n'$. Pos. 1.	Pos. 2.
B.	29° 13′ 0″	30° 2′ 0″	+·0173	+·0175	+779	+795	+117	+119		
D.	29 25 30	29 58 0	+·0143	+·0144	+754	+762	+113	+114	− 4	− 5
E.	29 38 30	29 54	+·0107	+·0107	+633	+634	+ 95	+ 95	−18	−19
F.	29 51	29 51	+·0071	+·0071	+451	+451	+ 68	+ 68	−27	−27
G.	30 17	29 45	−·0009	−·0010	− 65	− 73	− 10	− 11	−78	−79
H.	30 42	29 40	−·0086	−·0088	−683	−694	−102	−104	−92	−93
									219	223

Hence arrangement I. gives bands.

(48.) For the ordinary ray with the plate vertical, the number of bands agrees well with observation as far as it goes.

It is obvious in general, that if two sets of bands differing in number be superposed, there will be a number of coincidences equal to their difference, separated by spaces of partial obliteration, which if the bands be of sensible breadth, will extend over several bands.

On comparing the above values of p for O and for E in position 1, we find—

$$\left.\begin{array}{lr} \text{From B to E} \quad . \quad . \quad . & 29-\ \ 22=\ \ 7 \\ \text{From E to H} \quad . \quad . \quad . & 205-197=\ \ 8 \\ \text{From B to H} \quad . \quad . \quad . \quad . \quad . \quad . & =15 \end{array}\right\}=\text{number of intervals of indistinctness.}$$

From E to H this agrees with observation, and may do so from B to E, occasioning a total disappearance of bands.

Postscript.—At the time the foregoing paper was communicated to the Royal Society, I had not seen Mr. STOKES's paper; nor in writing his, had he seen mine at length; hence it will be found that there are some repetitions in the latter, of points mentioned in mine, but usually put in so much clearer a light that the reader will not regret the repetition.

XVI. *On the Theory of certain Bands seen in the Spectrum.*

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Communicated by the Rev. Baden Powell, *M.A., F.R.S., &c.*

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SOME months ago Professor Powell communicated to me an account of a new case of interference which he had discovered in the course of some experiments on a fluid prism, requesting at the same time my consideration of the theory. As the phenomenon is fully described in Professor Powell's memoir, and is briefly noticed in art. 1 of this paper, it is unnecessary here to allude to it. It struck me that the theory of the phenomenon was almost identical with that of the bands seen when a spectrum is viewed by an eye, half the pupil of which is covered by a plate of glass or mica. The latter phenomenon has formed the subject of numerous experiments by Sir David Brewster, who has discovered a very remarkable polarity, or apparent polarity, in the bands. The theory of these bands has been considered by the Astronomer Royal in two memoirs "On the Theoretical Explanation of an apparent new Polarity of Light," printed in the Philosophical Transactions for 1840 (Part II.) and 1841 (Part I.). In the latter of these Mr. Airy has considered the case in which the spectrum is viewed in focus, which is the most interesting case, as being that in which the bands are best seen, and which is likewise far simpler than the case in which the spectrum is viewed out of focus. Indeed, from the mode of approximation adopted, the former memoir can hardly be considered to belong to the bands which formed the subject of Sir David Brewster's experiments, although the memoir no doubt contains the theory of a possible system of bands. On going over the theory of the bands seen when the spectrum is viewed in focus, after the receipt of Professor Powell's letter, I was led to perceive that the intensity of the light could be expressed in finite terms. This saves the trouble of Mr. Airy's quadratures, and allows the results to be discussed with great facility. The law, too, of the variation of the intensity with the thickness of the plate is very remarkable, on account of its discontinuity. These reasons have induced me to lay my investigation before the Royal Society, even though the remarkable polarity of the bands has been already explained by the Astronomer Royal. The observation of these bands seems likely to become of great importance in the determination of the refractive indices, and more especially the laws of dispersion, of minerals and other substances which cannot be formed into prisms which would exhibit the fixed lines of the spectrum.

Section I.—*Explanation of the formation of the bands on the imperfect theory of Interferences. Mode of calculating the number of bands seen in a given part of the spectrum.*

1. The phenomenon of which it is the principal object of the following paper to investigate the theory, is briefly as follows. Light introduced into a room through a horizontal slit is allowed to pass through a hollow glass prism containing fluid, with its refracting edge horizontal, and the spectrum is viewed through a small telescope with its object-glass close to the prism. On inserting into the fluid a transparent plate with its lower edge horizontal, the spectrum is seen traversed from end to end by very numerous dark bands, which are parallel to the fixed lines. Under favorable circumstances the dark bands are intensely black; but in certain cases, to be considered presently, no bands whatsoever are seen. When the plate is cut from a doubly refracting crystal, there are in general two systems of bands seen together; and when the light is analysed each system disappears in turn at every quarter revolution of the analyser.

2. It is not difficult to see that the theory of these bands must be almost identical with that of the bands described by Sir David Brewster in the Report of the Seventh Meeting of the British Association, and elsewhere, and explained by Mr. Airy in the first Part of the Philosophical Transactions for 1841. To make this apparent, conceive an eye to view a spectrum through a small glass vessel with parallel faces filled with fluid. The vessel would not alter the appearance of the spectrum. Now conceive a transparent plate bounded by parallel surfaces inserted into the fluid, the plane of the plate being perpendicular to the axis of the eye, and its edge parallel to the fixed lines of the spectrum, and opposite to the centre of the pupil. Then we should have bands of the same nature as those described by Sir David Brewster, the only difference being that in the present case the retardation on which the existence of the bands depends is the difference of the retardations due to the plate itself, and to a plate of equal thickness of the fluid, instead of the absolute retardation of the plate, or more strictly, the difference of retardations of the solid plate and of a plate of equal thickness of air, contained between the produced parts of the bounding planes of the solid plate. In Professor Powell's experiment the fluid fills the double office of the fluid in the glass vessel and of the prism producing the spectrum in the imaginary experiment just described.

It might be expected that the remarkable polarity discovered by Sir David Brewster in the bands which he has described, would also be exhibited with Professor Powell's apparatus. This anticipation is confirmed by experiment. With the arrangement of the apparatus already mentioned, it was found that with certain pairs of media, one being the fluid and the other the retarding plate, no bands were visible. These media were made to exhibit bands by using fluid enough to cover the plate to a certain depth, and stopping by a screen the light which would otherwise have passed through the thin end of the prism underneath the plate.

3. Although the explanation of the polarity of the bands depends on diffraction, it may be well to account for their formation on the imperfect theory of interferences, in which it is supposed that light consists of rays which follow the courses assigned to them by geometrical optics. It will thus readily appear that the number of bands formed with a given plate and fluid, and in a given part of the spectrum, has nothing to do with the form or magnitude of the aperture, whatever it be, which limits the pencil that ultimately falls on the retina. Moreover, it seems desirable to exhibit in its simplest shape the mode of calculating the number of bands seen in any given case, more especially as these calculations seem likely to be of importance in the determination of refractive indices.

4. Before the insertion of the plate, the wave of light belonging to a particular colour, and to a particular point of the slit, or at least a certain portion of it limited by the boundaries of the fluid, after being refracted at the two surfaces of the prism, enters the object-glass with an unbroken front. The front is here called unbroken, because the modification which the wave suffers at its edges is not contemplated. According to geometrical optics, the light after entering the object-glass is brought to a point near the principal focus, spherical aberration being neglected; according to the undulatory theory, it forms a small, but slightly diffused image of the point from which it came. The succession of these images due to the several points of the slit forms the image of the slit for the colour considered, and the succession of coloured images forms the spectrum, the waves for the different colours covering almost exactly the same portion of the object-glass, but differing from one another in direction.

Apart from all theory, it is certain that the image of a point or line of homogeneous light seen with a small aperture is diffused. As the aperture is gradually widened the extent of diffusion decreases continuously, and at last becomes insensible. The perfect continuity, however, of the phenomenon shows that the true and complete explanation, whatever it may be, of the narrow image seen with a broad aperture, ought also to explain the diffused image seen with a narrow aperture. The undulatory theory explains perfectly both the one and the other, and even predicts the distribution of the illumination in the image seen with an aperture of given form, which is what no other theory has ever attempted.

As an instance of the effect of diffusion in an image, may be mentioned the observed fact that the definition of a telescope is impaired by contracting the aperture. With a moderate aperture, however, the diffusion is so slight as not to prevent fine objects, such as the fixed lines of the spectrum, from being well seen.

For the present, however, let us suppose the light entering the telescope to consist of rays which are brought accurately to a focus, but which nevertheless interfere. When the plate is inserted into the fluid the front of a wave entering the object-glass will no longer be unbroken, but will present as it were a *fault*, in consequence of the retardation produced by the plate. Let R be this retardation measured by

actual length in air, ϱ the retardation measured by phase, M the retardation measured by the number of waves' lengths, so that

$$\varrho=\frac{2\pi}{\lambda}\mathrm{R},\ \mathrm{M}=\frac{1}{\lambda}\mathrm{R};$$

then when M is an odd multiple of $\frac{1}{2}$, the vibrations produced by the two streams, when brought to the same focus, will oppose each other, and there will be a minimum of illumination; but when M is an even multiple of $\frac{1}{2}$ the two streams will combine, and the illumination will be a maximum. Now M changes in passing from one colour to another in consequence of the variations both of R and of λ; and since the different colours occupy different angular positions in the field of view, the spectrum will be seen traversed by dark and bright bands. It is nearly thus that Mr. Talbot has explained the bands seen when a spectrum is viewed through a hole in a card which is half-covered with a plate of glass or mica, with its edge parallel to the fixed lines of the spectrum. Mr. Talbot however does not appear to have noticed the polarity of the bands.

Let h, k be the breadths of the interfering streams; then we may take

$$h\sin\frac{2\pi}{\lambda}vt,\quad k\sin\left(\frac{2\pi}{\lambda}vt-\varrho\right)$$

to represent the vibrations produced at the focus by the two streams respectively, which gives for the intensity I,

$$\mathrm{I}=(h+k\cos\varrho)^2+(k\sin\varrho)^2=h^2+k^2+2hk\cos\varrho, \quad \dots\dots \quad (1.)$$

which varies between the limits $(h-k)^2$ and $(h+k)^2$.

5. Although the preceding explanation is imperfect, for the reason already mentioned, and does not account for the polarity, it is evident that if bands are formed at all in this way, the number seen in a given part of the spectrum will be determined correctly by the imperfect theory; for everything will recur, so far as interference is concerned, when M is decreased or increased by 1, and not before. This points out an easy mode of determining the number of bands seen in a given part of the spectrum. For the sake of avoiding a multiplicity of cases, let an acceleration be reckoned as a negative retardation, and suppose R positive when the stream which passes nearer to the edge of the prism is retarded relatively to the other. From the known refractive indices of the plate and fluid, and from the circumstances of the experiment, calculate the values of R for each of the fixed lines B, C . . . . H of the spectrum, or for any of them that may be selected, and thence the values of M, by dividing by the known values of λ. Set down the results with their proper signs opposite to the letters B, C . . . denoting the rays to which they respectively refer, and then form a table of differences by subtracting the value of M for B from the value for C, the value for C from the value for D, and so on. Let N be the number found in the table of differences corresponding to any interval, as for example from F to G; then the numerical value of N, that is to say, N or $-$N, according as N is positive

or negative, gives the number of bands seen between F and G. For anything that appears from the imperfect theory of the bands given in the preceding article, it would seem that the sign of N was of no consequence. It will presently be seen, however, that the sign is of great importance: it will be found in fact that the sign + indicates that the second arrangement mentioned in art. 2 must be employed; that is to say, the plate must be made to intercept light from the thin end of the prism, while the sign — indicates that the first arrangement is required. It is hardly necessary to remark that, if N should be fractional, we must, instead of the number of bands, speak of the number of band-intervals and the fraction of an interval.

Although the number of bands depends on nothing but the values of N, the values of M are not without physical interest. For M expresses, as we have seen, the number of waves' lengths whereby one of the interfering streams is before or behind the other. Mr. Airy speaks of the formation of rings with the light of a spirit-lamp when the retardation of one of the interfering streams is as much as fifty or sixty waves' lengths. But in some of Professor Powell's experiments, bands were seen which must have been produced by retardations of several hundred waves' lengths. This exalts our ideas of the regularity which must be attributed to the undulations.

6. It appears then that the calculation of the number of bands is reduced to that of the retardation R. As the calculation of R is frequently required in physical optics, it will not be necessary to enter into much detail on this point. The mode of performing the calculation, according to the circumstances of the experiment, will best be explained by a few examples.

Suppose the retarding plate to belong to an ordinary medium, and to be placed so as to intercept light from the thin end of the prism, and to have its plane equally inclined to the faces of the prism. Suppose the prism turned till one of the fixed lines, as F, is seen at a minimum deviation; then the colours about F are incident perpendicularly on the plate; and all the colours may without material error be supposed to be incident perpendicularly, since the directions of the different colours are only separated by the dispersion accompanying the first refraction into the fluid, and near the normal a small change in the angle of incidence produces only a very small change in the retardation. The dispersion accompanying the first refraction into the fluid has been spoken of as if the light were refracted from air directly into the fluid, which is allowable, since the glass sides of the hollow prism, being bounded by parallel surfaces, may be dispensed with in the explanation. Let T be the thickness of the plate, μ the refractive index of the fluid, μ' that of the plate; then

$$R=(\mu'-\mu)T. \quad \ldots\ldots\ldots\ldots \quad (2.)$$

If the plate had been placed so as to intercept light from the thick end of the prism, we should have had $-R=(\mu'-\mu)T$, which would have agreed with (2.) if we had supposed T negative. For the future T will be reckoned positive when the plate intercepts light from the thin end of the prism, and negative when it intercepts light

from the thick end, so that the same formulæ will apply to both of the arrangements mentioned in art. 2.

If we put $\mu=1$, the formula (2.) will apply to the experiment in which a plate of glass or mica is held so as to cover half the pupil of the eye when viewing a spectrum formed in any manner, the plate being held perpendicularly to the axis of the eye. The effect of the small obliquity of incidence of some of the colours is supposed to be neglected.

The number of bands which would be determined by means of the formula (2.) would not be absolutely exact, unless we suppose the observation taken by receiving each fixed line in succession at a perpendicular incidence. This may be effected in the following manner. Suppose that we want to count the number of bands between F and G, move the plate by turning it round a horizontal axis till the bands about F are seen stationary; then begin to count from F, and before stopping at G incline the plate a little till the bands about G are seen stationary, estimating the fractions of an interval at F and G, if the bands are not too close. The result will be strictly the number given by the formula (2.). The difference, however, between this result and that which would be obtained by keeping the plate fixed would be barely sensible. If the latter mode of observation should be thought easier or more accurate, the exact formula which would replace (2.) would be easily obtained.

7. Suppose now the nearer face of the retarding plate made to rest on the nearer inner face of the hollow prism, and suppose one of the fixed lines, as F, to be viewed at a minimum deviation. Let ϕ, ϕ' be the angles of incidence and refraction at the first surface of the fluid, i, i' those at the surface of the plate, 2ε the angle of the prism. Since the deviation of F is a minimum, the angle of refraction ϕ'_{F} for F is equal to ε, and the angle of incidence ϕ is given by $\sin\phi=\mu_{\mathrm{F}}\sin\phi'_{\mathrm{F}}$, and ϕ is the angle of incidence for all the colours, the incident light being supposed white. The angle of refraction ϕ' for any fixed line is given by the equation $\sin\phi'=\frac{1}{\mu}\sin\phi=\frac{\mu_{\mathrm{F}}}{\mu}\sin\varepsilon$; then $i=2\varepsilon-\phi'$, and i' is known from the equation

$$\mu'\sin i'=\mu\sin i. \quad \ldots\ldots\ldots \quad (3.)$$

The retardation is given by either of the formulæ

$$\mathrm{R}=\mu'\mathrm{T}\frac{\sin(i-i')}{\sin i}, \quad \ldots\ldots\ldots \quad (4.)$$

$$\mathrm{R}=\mathrm{T}(\mu'\cos i'-\mu\cos i). \quad \ldots\ldots\ldots \quad (5.)$$

These formulæ might be deduced from that given in AIRY's Tract, modified so as to suit the case in which the plate is immersed in a fluid; but either of them may be immediately proved independently by referring everything to the wave's front and not the ray.

By multiplying and dividing the second side of (5.) by $\cos i$, and employing (3.), we get

$$\mathrm{R}=\mathrm{T}\sec i\,.(\mu'-\mu)-\mathrm{T}\mu'\sec i\,\mathrm{versin}\,(i-i'). \quad \ldots\ldots \quad (6.)$$

When the refractive indices of the plate and fluid are nearly equal, the last term in this equation may be considered insensible, so that it is not necessary to calculate i' at all.

8. The formulæ (2.), (4.), (5.), (6.) are of course applicable to the ordinary ray of a plate cut from a uniaxal crystal. If the plate be cut in a direction parallel to the axis, and if moreover the lower edge be parallel to the axis, so that the axis is parallel to the refracting edge of the prism, the formulæ will apply to both rays. If μ_o, μ_e be the principal indices of refraction referring to the ordinary and extraordinary rays respectively, μ' in the case last supposed must be replaced by μ_o for the bands polarized in a plane perpendicular to the plane of incidence, and by μ_e for the bands polarized in the plane of incidence. In the case of a plate cut from a biaxal crystal in such a direction that one of the principal axes, or axes of elasticity, is parallel to the refracting edge, the same formulæ will apply to that system of bands which is polarized in the plane of incidence.

If the plate be cut from a biaxal crystal in a direction perpendicular to one of the principal axes, and be held in the vertical position, the formula (2.) will apply to both systems of bands, if the small effect of the obliquity be neglected. The formula would be exact if the observations were taken by receiving each fixed line in succession at a perpendicular incidence.

If the plate be cut from a uniaxal crystal in a direction perpendicular to the axis, and be held obliquely, we have for the extraordinary bands, which are polarized in a plane perpendicular to the plane of incidence,

$$R=T\left(\frac{\mu_o}{\mu_e}\sqrt{\mu_e^2-\mu^2\sin^2 i}-\mu\cos i\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

which is the same as the formula in AIRY's Tract, only modified so as to suit the case in which the plate is immersed in fluid, and expressed in terms of refractive indices instead of velocities. If we take a subsidiary angle j, determined by the equation

$$\sin j=\frac{\mu}{\mu_e}\sin i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

the formula (7.) becomes

$$R=T(\mu_o\cos j-\mu\cos i), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

which is of the same form as (5.), and may be adapted to logarithmic calculation if required by assuming $\frac{\mu_o}{\mu}=\tan\theta$. The preceding formula will apply to the extraordinary bands formed by a plate cut from a biaxal crystal in the manner described in the last paragraph, and held obliquely, the extraordinary bands being understood to mean those which are polarized in a plane perpendicular to the plane of incidence. In this application we must take for μ_e, μ_o those two of the three principal indices of refraction which are symmetrically related to the axis normal to the plate, and to the axis parallel to the plate, and lying in the plane of incidence, respectively; while

in applying the formula (4.), (5.) or (6.) to the other system of bands, the third principal index must be substituted for μ'.

It is hardly necessary to consider the formula which would apply to the general case, which would be rather complicated.

9. If a plate cut from a uniaxal crystal in a direction perpendicular to the axis be placed in the fluid in an inclined position, and be then gradually made to approach the vertical position, the breadths of the bands belonging to the two systems will become more and more nearly equal, and the two systems will at last coalesce. This statement indeed is not absolutely exact, because the whole spectrum cannot be viewed at once by light which passes along the axis of the crystal, on account of the dispersion accompanying the first refraction, but it is very nearly exact. With quartz it is true there would be two systems of bands seen even in the vertical position, on account of the peculiar optical properties of that substance; but the breadths of the bands belonging to the two systems would be so nearly equal, that it would require a plate of about one-fifth of an inch thickness to give a difference of one in the number of bands seen in the whole spectrum in the case of the two systems respectively. If the plate should be thick enough to exhibit both systems, the light would of course have to be circularly analysed to show one system by itself.

Section II.—*Investigation of the intensity of the light on the complete theory of undulations, including the explanation of the apparent polarity of the bands.*

10. The explanation of the formation of the bands on the imperfect theory of interferences considered in the preceding section is essentially defective in this respect, that it supposes an annihilation of light when two interfering streams are in opposition; whereas it is a most important principle that *light is never lost by interference.* This statement may require a little explanation, without which it might seem to contradict received ideas. It is usual in fact to speak of light as *destroyed* by interference. Although this is true, in the sense intended, the expression is perhaps not very happily chosen. Suppose a portion of light coming from a luminous point, and passing through a moderately small aperture, to be allowed to fall on a screen. We know that there would be no sensible illumination on the screen except almost immediately in front of the aperture. Conceive now the aperture divided into a great number of small elements, and suppose the same quantity of light as before to pass through each element, the only difference being that now the vibrations in the portions passing through the several elements are supposed to have no relation to each other. The light would now be diffused over a comparatively large portion of the screen, so that a point P which was formerly in darkness might now be strongly illuminated. The disturbance at P is in both cases the aggregate of the disturbances due to the several elements of the aperture; but in the first case the aggregate is insensible on account of interference. It is only in this sense that light is *destroyed* by interference, for the total illumination on the screen is the same in the two cases;

the effect of interference has been, not to annihilate any light, but only to alter the "distribution of the illumination," so that the light, instead of being diffused over the screen, is concentrated in front of the aperture.

Now in the case of the bands considered in Section I., if we suppose the plate extremely thin, the bands will be very broad ; and the displacement of illumination due to the retardation being small compared with the breadth of a band, it is evident, without calculation, that at most only faint bands can be formed. This particular example is sufficient to show the inadequacy of the imperfect theory, and the necessity of an exact investigation.

11. Suppose first that a point of homogeneous light is viewed through a telescope. Suppose the object-glass limited by a screen in which there is formed a rectangular aperture of length $2\,l$. Suppose a portion of the incident light retarded, by passing through a plate bounded by parallel surfaces, and having its edge parallel to the length of the aperture. Suppose the unretarded stream to occupy a breadth h of the aperture at one side, the retarded stream to occupy a breadth k at the other, while an interval of breadth $2\,g$ exists between the streams. In the apparatus mentioned in Section I., the object-glass is not limited by a screen, but the interfering streams of light are limited by the dimensions of the fluid prism, which comes to the same thing. The object of supposing an interval to exist between the interfering streams, is to examine the effect of the gap which exists between the streams when the retarding plate is inclined. In the investigation the effect of diffraction before the light reaches the object-glass of the telescope is neglected.

Let O be the image of the luminous point, as determined by geometrical optics, f the focal length of the object-glass, or rather the distance of O from the object-glass, which will be a little greater than the focal length when the luminous point is not very distant. Let C be a point in the object-glass, situated in the middle of the interval between the two streams, and let the intensity be required at a point M, near O, situated in a plane passing through O and perpendicular to OC. The intensity at any point of this plane will of course be sensibly the same as if the plane were drawn perpendicular to the axis of the telescope instead of being perpendicular to OC. Take OC for the axis of z, the axes of x and y being situated in the plane just mentioned, and that of y being parallel to the length of the aperture. Let p, q be the coordinates of M ; x, y, z those of a point P in the front of a wave which has just passed through the object-glass, and which forms part of a sphere with O for its centre. Let c be the coefficient of vibration at the distance of the object-glass ; then we may take

$$\frac{c}{\lambda}\cdot\frac{1}{\mathrm{PM}}\sin\frac{2\pi}{\lambda}(vt-\mathrm{PM})dxdy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a.)$$

to represent the disturbance at M due to the element $dxdy$ of the aperture at P, P being supposed to be situated in the unretarded stream, which will be supposed to lie at the negative side of the axis of x. In the expression $(a.)$ it is assumed that the

proper multiplier of $\frac{c}{\text{PM}}$ is $\frac{1}{\lambda}$. This may be shown to be a necessary consequence of the principle mentioned in the preceding article, that light is never lost by interference; and this principle follows directly from the principle of *vis viva*. In proving that λ^{-1} is the proper multiplier, it is not in the least necessary to enter into the consideration of the law of the variation of intensity in a secondary wave, as the angular distance from the normal to the primary wave varies; the result depends merely on the assumption that in the immediate neighbourhood of the normal the intensity may be regarded as sensibly constant.

In the expression (*a.*) we have

$$\text{PM}=\sqrt{\{z^2+(x-p)^2+(y-q)^2\}}=\sqrt{\{f^2+p^2+q^2-2px-2qy\}}=f-\frac{1}{f}(px+qy),\text{nearly,}$$

if we write f for $\sqrt{f^2+p^2+q^2}$. It will be sufficient to replace $\frac{1}{\text{PM}}$ outside the circular function by $\frac{1}{f}$. We may omit the constant f under the circular function, which comes to the same thing as changing the origin of t. We thus get for the disturbance at M due to the unretarded stream,

$$\frac{c}{\lambda f}\int_{-(g+h)}^{-g}\int_{-l}^{l}\sin\frac{2\pi}{\lambda}\left\{vt+\frac{1}{f}(px+qy)\right\}dxdy,$$

or on performing the integrations and reducing,

$$\frac{2chl}{\lambda f}\cdot\frac{\lambda f}{2\pi ql}\sin\frac{2\pi ql}{\lambda f}\cdot\frac{\lambda f}{\pi ph}\sin\frac{\pi ph}{\lambda f}\cdot\sin\frac{2\pi}{\lambda}\left(vt-\frac{pg}{f}-\frac{ph}{2f}\right). \quad . \quad . \quad . \quad . \quad (b.)$$

For the retarded stream, the only difference is that we must subtract R from vt, and that the limits of x are g and $g+k$. We thus get for the disturbance at M due to this stream,

$$\frac{2ckl}{\lambda f}\cdot\frac{\lambda f}{2\pi ql}\sin\frac{2\pi ql}{\lambda f}\cdot\frac{\lambda f}{\pi pk}\sin\frac{\pi pk}{\lambda f}\cdot\sin\frac{2\pi}{\lambda}\left(vt-\text{R}+\frac{pg}{f}+\frac{pk}{2f}\right). \quad . \quad . \quad . \quad . \quad (c.)$$

If we put for shortness τ for the quantity under the last circular function in (*b.*), the expressions (*b.*), (*c.*) may be put under the forms $u\sin\tau$, $v\sin(\tau-\alpha)$, respectively; and if I be the intensity, I will be measured by the sum of the squares of the coefficients of $\sin\tau$ and $\cos\tau$ in the expression

$$u\sin\tau+v\sin(\tau-\alpha),$$

so that

$$\text{I}=u^2+v^2+2uv\cos\alpha,$$

which becomes, on putting for u, v and α, their values, and putting

$$\left\{\frac{\lambda f}{2\pi ql}\sin\frac{2\pi ql}{\lambda f}\right\}^2=\text{Q}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.)$$

$$\text{I}=\text{Q}.\frac{4c^2l^2}{\pi^2p^2}\left\{\left(\sin\frac{\pi ph}{\lambda f}\right)^2+\left(\sin\frac{\pi pk}{\lambda f}\right)^2+2\sin\frac{\pi ph}{\lambda f}\cdot\sin\frac{\pi pk}{\lambda f}\cos\left[\varrho-\frac{\pi p}{\lambda f}(4g+h+k)\right]\right\}. \quad (11.)$$

12. Suppose now that instead of a point we have a line of homogeneous light, the

line being parallel to the axis of y. The luminous line is supposed to be a narrow slit, through which light enters in all directions, and which is viewed in focus. Consequently each element of the line must be regarded as an independent source of light. Hence the illumination on the object-glass due to a portion of the line which subtends the small angle β at the distance of the object-glass varies as β, and may be represented by $A\beta$. Let the former origin O be referred to a new origin O$'$ situated in the plane xy, and in the image of the line; and let η, q' be the ordinates of O, M referred to O$'$, so that $q=q'-\eta$. In order that the luminous point considered in the last article may represent an element of the luminous line considered in the present, we must replace c^2 by $Ad\beta$ or $\frac{A}{f}d\eta$; and in order to get the aggregate illumination due to the whole line, we must integrate from a large negative to a large positive value of η, the largeness being estimated by comparison with $\frac{\lambda f}{l}$. Now the angle $\frac{2\pi ql}{\lambda f}$ changes by π when q changes by $\frac{\lambda f}{2l}$, which is therefore the breadth, in the direction of y, of one of the diffraction bands which would be seen with a luminous point. Since l is supposed not to be extremely small, but on the contrary moderately large, the whole system of diffraction bands would occupy but a very small portion of the field of view in the direction of y, so that we may without sensible error suppose the limits of η to be $-\infty$ and $+\infty$. We have then

$$\int_{-\infty}^{\infty} Q d\eta = \int_{-\infty}^{\infty} \left\{ \frac{\lambda f}{2\pi l(q'-\eta)} \sin \frac{2\pi l(q'-\eta)}{\lambda f} \right\}^2 d\eta = \frac{\lambda f}{2\pi l} \int_{-\infty}^{\infty} \left(\frac{\sin\theta}{\theta} \right)^2 d\theta,$$

by taking the quantity under the circular function in place of η for the independent variable. Now it is known that the value of the last integral is π, as will also presently appear, and therefore we have for the intensity I at any point,

$$I = \frac{2A\lambda l}{\pi^2 p^2} \left\{ \left(\sin \frac{\pi ph}{\lambda f} \right)^2 + \left(\sin \frac{\pi pk}{\lambda f} \right)^2 + 2 \sin \frac{\pi ph}{\lambda f} \cdot \sin \frac{\pi pk}{\lambda f} \cdot \cos \left[\varrho - \frac{\pi p}{\lambda f}(4g+h+k) \right] \right\}, \quad (12.)$$

which is independent of q', as of course it ought to be.

13. Suppose now that instead of a line of homogeneous light we have a line of white light, the component parts of which have been separated, whether by refraction or by diffraction is immaterial, so that the different colours occupy different angular positions in the field of view. Let $B\beta\psi$ be the illumination on the object-glass due to a length of the line which subtends the small angle β, and to a portion of the spectrum which subtends the small angle ψ at the centre of the object-glass. In the axis of x take a new origin O$''$, and let ξ, p' be the abscissæ of O$'$, M reckoned from O$''$, so that $p=p'-\xi$. In order that (12.) may express the intensity at M due to an elementary portion of the spectrum, we must replace A by $Bd\psi$, or $\frac{B}{f}d\xi$; and in order to find the aggregate illumination at M, we must integrate so as to include all values of ξ which are sufficiently near to p' to contribute sensibly to the illumination

at M. It would not have been correct to integrate using the displacement instead of the intensity, because the different colours cannot interfere. Suppose the angular extent, in the direction of x, of the system of diffraction bands which would be seen with homogeneous light, or at least the angular extent of the brighter part of the system, to be small compared with that of the spectrum. Then we may neglect the variations of B and of λ in the integration, considering only those of ξ and ϱ, and we may suppose the changes of ϱ proportional to those of ξ; and we may moreover suppose the limits of ξ to be $-\infty$ and $+\infty$. Let ϱ' be the value of ϱ, and $-\varpi$ that of $\frac{d\varrho}{d\xi}$, when $\xi=p'$, so that we may put $\varrho=\varrho'+\varpi(p'-\xi)$; and take p instead of ξ for the independent variable. Then putting for shortness

$$\frac{\pi h}{\lambda f}=h_{\prime}, \qquad \frac{\pi k}{\lambda f}=k_{\prime}, \qquad \varpi-\frac{\pi}{\lambda f}(4g+h+k)=g_{\prime}, \quad . \quad . \quad . \quad . \quad . \quad (13.)$$

we have for the intensity,

$$\mathrm{I}=\frac{2\mathrm{B}\lambda l}{\pi^2 f}\int_{-\infty}^{\infty}\{\sin^2 h_{\prime}p+\sin^2 k_{\prime}p+2\sin h_{\prime}p\,.\,\sin k_{\prime}p\,.\,\cos(\varrho'-g_{\prime}p)\}\frac{dp}{p^2}.$$

Now $\int_{-\infty}^{\infty}\sin^2 h_{\prime}p\frac{dp}{p^2}=h_{\prime}\int_{-\infty}^{\infty}\sin^2\theta\frac{d\theta}{\theta^2}=\pi h_{\prime}$. Similarly, $\int_{-\infty}^{\infty}\sin^2 k_{\prime}p\,.\,\frac{dp}{p^2}=\pi k_{\prime}$.

Moreover, if we replace

$$\cos(\varrho'-g_{\prime}p) \text{ by } \cos\varrho'\,.\,\cos g_{\prime}p+\sin\varrho'\,.\,\sin g_{\prime}p,$$

the integral containing $\sin\varrho'$ will disappear, because the positive and negative elements will destroy each other, and we have only to find w, where

$$w=\int_{-\infty}^{\infty}\sin h_{\prime}p\,.\,\sin k_{\prime}p\,.\,\cos g_{\prime}p\,.\,\frac{dp}{p^2}.$$

Now we get by differentiating under the integral sign,

$$\frac{dw}{dg_{\prime}}=-\int_{-\infty}^{\infty}\sin h_{\prime}p\,.\,\sin k_{\prime}p\,.\sin g_{\prime}p\,.\,\frac{dp}{p}$$

$$=\frac{1}{4}\int_{-\infty}^{\infty}\Big\{\sin(g_{\prime}+h_{\prime}+k_{\prime})p+\sin(g_{\prime}-h_{\prime}-k_{\prime})p-\sin(g_{\prime}+h_{\prime}-k_{\prime})p-\sin(g_{\prime}+k_{\prime}-h_{\prime})p\Big\}\frac{dp}{p}.$$

But it is well known that

$$\int_{-\infty}^{\infty}\frac{\sin sp}{p}\,dp=\pi, \text{ or } =-\pi,$$

according as s is positive or negative. If then we use $\mathrm{F}(s)$ to denote a discontinuous function of s which is equal to $+1$ or -1 according as s is positive or negative, we get

$$\frac{dw}{dg_{\prime}}=\frac{\pi}{4}\Big\{\mathrm{F}(g_{\prime}+h_{\prime}+k_{\prime})+\mathrm{F}(g_{\prime}-h_{\prime}-k_{\prime})-\mathrm{F}(g_{\prime}+h_{\prime}-k_{\prime})-\mathrm{F}(g_{\prime}+k_{\prime}-h_{\prime})\Big\}.$$

This equation gives

$$\frac{dw}{dg_{\prime}}=0, \text{ from } g_{\prime}=-\infty \text{ to } g_{\prime}=-(h_{\prime}+k_{\prime})$$

$=\frac{\pi}{2}$, from $g_{\prime}=-(h_{\prime}+k_{\prime})$ to $g_{\prime}=-(h_{\prime}\backsim k_{\prime})$

$=0$, from $g_{\prime}=-(h_{\prime}\backsim k_{\prime})$ to $g_{\prime}=+(h_{\prime}\backsim k_{\prime})$

$=-\frac{\pi}{2}$, from $g_{\prime}=h_{\prime}\backsim k_{\prime}$ to $g_{\prime}=h_{\prime}+k_{\prime}$

$=0$, from $g_{\prime}=h_{\prime}+k_{\prime}$ to $g_{\prime}=\infty$.

Now w vanishes when $g_{\prime}$ is infinite, on account of the fluctuation of the factor $\cos g_{\prime}p$ under the integral sign, whence we get by integrating the value of $\frac{dw}{dg_{\prime}}$ given above, and correcting the integral so as to vanish for $g_{\prime}=-\infty$,

$w=0$, from $g_{\prime}=-\infty$ to $g_{\prime}=-(h_{\prime}+k_{\prime})$;

$w=\frac{\pi}{2}(h_{\prime}+k_{\prime}+g_{\prime})$, from $g_{\prime}=-(h_{\prime}+k_{\prime})$ to $g_{\prime}=-(h_{\prime}\backsim k_{\prime})$;

$w=\pi k_{\prime}$ or $=\pi h_{\prime}$, (according as $h_{\prime}>k_{\prime}$ or $h_{\prime}<k_{\prime}$,) from $g=-(h_{\prime}\backsim k_{\prime})$ to $g_{\prime}=+(h_{\prime}\backsim k_{\prime})$;

$w=\frac{\pi}{2}(h_{\prime}+k_{\prime}-g_{\prime})$, from $g_{\prime}=h^{\prime}\backsim k_{\prime}$ to $g_{\prime}=h_{\prime}+k_{\prime}$;

$w=0$, from $g_{\prime}=h_{\prime}+k_{\prime}$ to $g_{\prime}=\infty$.

Substituting in the expression for the intensity, and putting $g_{\prime}=\frac{\pi g^{\prime}}{\lambda f}$, so that

$$g^{\prime}=\frac{\varpi\lambda f}{\pi}-4g-h-k, \quad \dots \dots \dots \quad (14.)$$

we get

$$\mathrm{I}=\frac{2\mathrm{B}l}{f^2}(h+k), \quad \dots \dots \dots \quad (15.)$$

when the numerical value of $g^{\prime}$ exceeds $h+k$;

$$\mathrm{I}=\frac{2\mathrm{B}l}{f^2}\left\{h+k+(h+k-\sqrt{g^{\prime 2}})\cos g^{\prime}\right\}, \quad \dots \dots \dots \quad (16.)$$

when the numerical value of $g^{\prime}$ lies between $h+k$ and $h\backsim k$;

$$\mathrm{I}=\frac{2\mathrm{B}l}{f^2}(h+k+2h\cos g^{\prime}), \text{ or } =\frac{2\mathrm{B}l}{f^2}(h+k+2k\cos g^{\prime}), \quad \dots \dots \quad (17.)$$

according as h or k is the smaller of the two, when the numerical value of $g^{\prime}$ is less than $h\backsim k$.

The discontinuity of the law of intensity is very remarkable.

By supposing $g_{\prime}=0$, $k_{\prime}=h_{\prime}$ in the expression for w, and observing that these suppositions reduce w to $\int_{-\infty}^{\infty}\sin^2 h_{\prime}p\,.\frac{dp}{p^2}$ we get

$$\int_{-\infty}^{\infty}\left(\frac{\sin h_{\prime}p}{p}\right)^2 dp=\pi h_{\prime},$$

a result already employed. This result would of course have been obtained more readily by differentiating with respect to $h_{\prime}$.

14. The preceding investigation will apply, with a very trifling modification, to Sir David Brewster's experiment, in which the retarding plate, instead of being placed in front of the object-glass of a telescope, is held close to the eye. In this case the eye itself takes the place of the telescope; and if we suppose the whole refraction to take place at the surface of the cornea, which will not be far from the truth, we must replace f by the diameter of the eye, and ψ by the angular extent of the portion of the spectrum considered, diminished in the ratio of m to 1, m being the refractive index of the cornea. When a telescope is used in this experiment, the retarding plate being still held close to the eye, it is still the naked eye, and not the telescope, which must be assimilated to the telescope considered in the investigation; the only difference is that ψ must be taken to refer to the magnified, and not the unmagnified spectrum.

Let the axis of x be always reckoned positive in the direction in which the blue end of the spectrum is seen, so that in the image formed at the focus of the object-glass or on the retina, according as the retarding plate is placed in front of the object-glass or in front of the eye, the blue is to the negative side of the red. Although the plate has been supposed at the positive side, there will thus be no loss of generality, for should the plate be at the negative side it will only be requisite to change the sign of ϱ.

First, suppose ϱ to decrease algebraically in passing from the red to the blue. This will be the case in Sir David Brewster's experiment when the retarding plate is held at the side on which the red is seen. It will be the case in Professor Powell's experiment when the first of the arrangements mentioned in art. 2 is employed, and the value of N in the table of differences mentioned in art. 5 is positive, or when the second arrangement is employed and N is negative. In this case ϖ is negative, and therefore $g' < -(h+k)$, and therefore (15.) is the expression for the intensity. This expression indicates a uniform intensity, so that there are no bands at all.

Secondly, suppose ϱ to increase algebraically in passing from the red to the blue. This will be the case in Sir David Brewster's experiment when the retarding plate is held at the side on which the blue is seen. It will be the case in Professor Powell's experiment when the first arrangement is employed and N is negative, or when the second arrangement is employed and N is positive. In this case ϖ is positive; and since ϖ varies as the thickness of the plate, g' may be made to assume any value from $-(4g+h+k)$ to $+\infty$ by altering the thickness of the plate. Hence, provided the thickness lie within certain limits, the expression for the intensity will be (16.) or (17.). Since these expressions have the same form as (1.), the magnitude only of the coefficient of $\cos \varrho'$, as compared with the constant term, being different, it is evident that the number of bands and the places of the minima are given correctly by the imperfect theory considered in Section I.

15. The plate being placed as in the preceding paragraph, suppose first that the breadths h, k of the interfering streams are equal, and that the streams are contiguous,

so that $g=0$. Then the expression (17.) may be dispensed with, since it only holds good when $g'=0$, in which case it agrees with (16.). Let T_0 be the value of the thickness T for which $g'=0$. Then $T=0$ corresponds to $g'=-(h+k)$, $T=T_0$ to $g'=0$, and $T=2T_0$ to $g'=h+k$; and for values of T equidistant from T_0, the values of g' are equal in magnitude but of opposite signs. Hence, provided T be less than $2T_0$, there are dark and bright bands formed, the vividness of the bands being so much the greater as T is more nearly equal to T_0, for which particular value the minima are absolutely black.

Secondly, suppose the breadths h, k of the two streams to be equal as before, but suppose the streams separated by an interval $2g$; then the only difference is that $g'=-(h+k)$ corresponds to a positive value, T_2 suppose, of T. If T be less than T_2, or greater than $2T_0-T_2$, there are no bands; but if T lie between T_2 and $2T_0-T_2$ bands are formed, which are most vivid when $T=T_0$, in which case the minima are perfectly black.

Thirdly, suppose the breadths h, k of the interfering streams unequal, and suppose, as before, that the streams are separated by an interval $2g$; then $g'=-(h+k)$ corresponds to a positive value, T_2 suppose, of T: $g'=-(h\backsim k)$ corresponds to another positive value, T_1 suppose, of T, T_1 lying between T_2 and T_0, T_0 being, as before, the value of T which gives $g'=0$. As T increases from T_0, g' becomes positive and increases from 0, and becomes equal to $h\backsim k$ when $T=2T_0-T_1$, and to $h+k$ when $T=2T_0-T_2$. When $T<T_2$ there are no bands. As T increases to T_1 bands become visible, and increase in vividness till $T=T_1$, when the ratio of the minimum intensity to the maximum becomes that of $h-k$ to $h+3k$, or of $k-h$ to $k+3h$, according as h or k is the greater of the two, h, k. As T increases to $2T_0-T_1$, the vividness of the bands remains unchanged; and as T increases from $2T_0-T_1$ to $2T_0-T_2$, the vividness decreases by the same steps as it before increased. When $T=2T_0-T_2$, the bands cease to exist, and no bands are formed for a greater value of T.

Although in discussing the intensity of the bands the aperture has been supposed to remain fixed, and the thickness of the plate to alter, it is evident that we might have supposed the thickness of the plate to remain the same and the aperture to alter. Since $\varpi \propto T$, the vividness of the bands, as measured by the ratio of the maximum to the minimum intensity, will remain the same when T varies as the aperture. This consideration, combined with the previous discussion, renders unnecessary the discussion of the effect of altering the aperture. It will be observed, that, as a general rule, fine bands require a comparatively broad aperture in order that they may be well-formed, while broad bands require a narrow aperture.

16. The particular thickness T_0 may be conveniently called the *best thickness*. This term is to a certain extent conventional, since when h and k are unequal the thickness may range from T_1 to $2T_0-T_1$ without any change being produced in the vividness of the bands. The best thickness is determined by the equation

$$\varpi=-\frac{d\varrho}{d\xi}=\frac{\pi}{\lambda f}(4g+h+k).$$

Now in passing from one band to its consecutive, ϱ changes by 2π, and ξ by e, if e be the linear breadth of a band; and for this small change of ξ we may suppose the changes of ϱ and ξ proportional, or but $-\frac{d\varrho}{d\xi}=\frac{2\pi}{e}$. Hence the best aperture for a given thickness is that for which

$$4g+h+k=\frac{2\lambda f}{e}.$$

If $g=0$ and $k=h$, this equation becomes $h=\frac{\lambda f}{e}$.

The difference of distances of a point in the plane xy whose coordinates are ξ, 0 from the centres of the portions of the object-glass which are covered by the interfering streams, is nearly

$$\frac{\xi}{f}\left(g+\frac{1}{2}h\right)-\frac{\xi}{f}\left(-g-\frac{1}{2}k\right),\ \text{or}\ \frac{\xi}{2f}(4g+h+k);$$

and if δ be the change of ξ when this difference changes by λ,

$$4g+h+k=\frac{2\lambda f}{\delta}.$$

Hence, when the thickness of the plate is equal to the best thickness, $e=\delta$, or the interval between the bands seen in the spectrum is equal to the interval between the bands formed by the interference of two streams of light, of the colour considered, coming from a luminous line seen in focus, and entering the object-glass through two very narrow slits parallel to the axis of y, and situated in the middle of the two interfering streams respectively. This affords a ready mode of remembering and calculating the best thickness of plate for a given aperture, or the best aperture for a given thickness of plate.

17. According to the preceding explanation, no bands would be formed in Sir DAVID BREWSTER's experiment when the plate was held on the side of the spectrum on which the red was seen. Mr. AIRY has endeavoured to explain the existence of bands under such circumstances*. Mr. AIRY appears to speak doubtfully of his explanation, and in fact to offer it as little more than a conjecture to account for an observed phenomenon. In the experiments of Mr. TALBOT and Mr. AIRY, bands appear to have been seen when the retarding plate was held at the red side of the spectrum; whereas Sir DAVID BREWSTER has stated that he has repeatedly looked for the bands under these circumstances and has never been able to find the least trace of them; and he considers the bands seen by Mr. TALBOT and Mr. AIRY in this case to be of the nature of NEWTON's rings. While so much uncertainty exists as to the experimental circumstances under which the bands are seen when the retarding plate is held at the red side of the spectrum, if indeed they are seen at all, it does not seem to be desirable to enter into speculations as to the cause of their existence.

* Philosophical Transactions for 1841, Part I. p. 6.

XVII. *An Experimental Inquiry undertaken with the view of ascertaining whether any, and what signs of current Electricity are manifested during the organic process of Secretion in living animals, being an attempt to apply some of the discoveries of* FARADAY *to Physiology**. *By* H. F. BAXTER, *Esq.*

Communicated by Sir BENJAMIN BRODIE, *Bart., F.R.S.*

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THE conjecture and experiment of WOLLASTON†, the cautious opinion of PROUT‡, the experiments of DONNÉ§ and of MATTEUCCI‖, are so well known that a mere allusion to them may be deemed sufficient. BECQUEREL¶, in speaking of MATTEUCCI's experiment, adds, "Si de nouvelles expériences confirment les résultats, il faudrait en conclure réellement que les organes qui sécrètent un liquide acide ou alcalin, ont des facultés électriques analogues à celles de la pile."

POUILLET** inserted one of the platinum extremities of a galvanometer into the jugular vein of a rabbit, the other into the carotid artery, without any effect being produced on the needle. MÜLLER†† says, "With the galvanometer no electric current can be discovered in the blood. I perceived no variation of the magnetic needle of the multiplier, even when I inserted one wire into an artery of a living animal, the other into a vein."

The galvanometer‡‡ that was used in these experiments is one of the common and ordinary construction, astatic, consisting of but few coils; the needles suspended by silk-worm silk, and the electrodes attached by screws. As the object was to ascer-

* It would be unjust to BECQUEREL not to state, that great and important assistance was obtained during the progress of the inquiry from his valuable work, entitled "Traité de l'Électricité," respecting one class of phenomena especially, viz. the action of an acid and an alkali upon each other when separated by a membrane or any other porous diaphragm. But, at the very commencement of the inquiry, it soon became apparent that no real progress could be reasonably expected until clearer ideas had been obtained as to the origin of the power in the voltaic circle. This led to the study of those admirable memoirs of FARADAY. The inquiry and the study of those memoirs went hand-in-hand. The opinions *supposed* were those from FARADAY; the experiments, tests of those opinions. The endeavour to master the *meaning* of that important, comprehensive, and expressive phrase, "*an axis of power having contrary forces, exactly equal in amount, in contrary directions*," was the mental difficulty; in *that* consists the requisite mental labour. It is not too much to add, that without those memoirs the inquiry would not have been continued.

† Philosophical Magazine, vol. xxxiii. p. 488. ‡ On Stomach and Urinary Diseases, 3rd edit. p. xxv.

§ BECQUEREL, Traité de l'Électricité, tom. i. p. 327. ‖ Ibid. tom. iv. p. 300. ¶ Ibid. p. 301.

** Journal de Physiologie, tom. v. p. 5. †† MÜLLER's Physiology, translated by BALY, vol. i. p. 148, 2nd edit.

‡‡ Made by NEWMAN of Regent Street. There is one advantage attending the use of a galvanometer that is not very delicate in its construction in these experiments; the vibrations of the needle soon subside, and consequently more information can be obtained from a single experiment.

tain, if possible, the existence of the current, an accurate description of its delicacy will not be necessary. It is sufficiently so to indicate the existence of a current during the combination of an acid with an alkali, such as very dilute solutions of nitric acid and of potash, when separated by a membrane.

In several unsuccessful experiments, the following electrodes were used:—Two platinum wires, $\frac{1}{24}$th of an inch in thickness and eleven inches in length, were pointed at one extremity, to be easily inserted into a blood-vessel or an organ, and coated with sealing-wax for about two inches, leaving the extreme point bare to the extent of one-fourth of an inch, the other extremities being attached to the galvanometer. But during the inquiry, in consequence of the stiffness of the wires forming the electrodes occasioning a motion of the whole instrument, when making and breaking contact, the following alteration was made in the arrangement. Thick copper wires ten inches in length were connected with the galvanometer by the screws, and each of the free extremities so bent as to rest in a separate wooden cup containing mercury. A platinum plate, an inch square, was attached to one extremity of each of the platinum electrodes to increase the extent of surface, and being placed in the wooden cups, a communication was formed with the galvanometer by means of the mercury, the pointed extremities serving, as before, to be inserted into the different parts of the animal. Great care was taken to ascertain that the different contacts were perfect, and no result upon the needle occurred from the whole arrangement, when a circuit was formed with a weak solution of salt, or water, previous to each experiment.

It will be unnecessary to relate the experiments, thirteen in number, upon cats, kittens, a guinea-pig and rabbits, in which an endeavour was made to ascertain whether the effect of a diverted current might not be obtained by inserting the electrodes into the portal vein alone, supposing that the stomach and liver formed poles similar to those of a galvanic circle; or whether a current might not be obtained by inserting them into the portal and hepatic veins. These failures, combined with theoretical reasoning, led to the supposition that the effect sought for existed in a different quarter. The inquiry will therefore commence with relating the last of the unsuccessful experiments.

Experiment 1.—Rabbit, six weeks old. (*a*). One electrode inserted into the vena porta, the other into the vena cava, at the entrance of the hepatic veins; no effect. (*b*). Caput coli, and a vein coming from the same part; a slight effect appeared: the electrodes were cleaned and reinserted, but the same effect did not occur. (*c*). Stomach and liver. (*d*). Stomach and vena porta. (*e*). Stomach and gall-bladder; no effect.

In these and the following experiments the electrodes were cleaned after the formation of a previous circuit, whenever the substances adhering to them might influence the result. Repeating the experiments of MATTEUCCI and of DONNÉ, only once did the effect occur, and that but slight, in the guinea-pig. The following fact may perhaps account for it.

Instead of the plates dipping into the mercury, the points were used for this purpose, and the surface of the plates served to form the free extremities of the electrodes.

The stomach was laid open, one of the plates pressed on the mucous membrane, the other upon the surface of the liver; a decided effect was now produced upon the needle, and by making and breaking contact so as to catch the vibrations of the needle, made to increase to 15° or 20°. The points were used as before; no effect. When the surface of the plate of one electrode was connected with the mucous membrane of the stomach, and the point of the other electrode with the liver, the effect upon the needle was greater than when the pointed extremity of the electrode was connected with the stomach, and the surface of the plate with the surface of the liver; in the latter instance the effect was very slight; but in neither instance was the effect so great as when both plates were used. The direction of the current, as indicated by the needle, was from the stomach along the metallic conductor to the liver.

The liver was removed and placed upon the intestines, the stomach remaining; the same actions occurred, but not to the same extent.

Stomach almost empty.

Considering that the failure in former experiments arose from the want of surface at the free extremities of the electrodes, the sealing-wax was removed.

Experiment 2.—Rabbit. The pointed extremity of one electrode was inserted into the caput coli, and pressed against the mucous membrane, the inferior mesenteric vein was wounded, and the plate of the other electrode dipped into the blood; the latter positive 5°, and made to increase by making and breaking contact.

An incision was made into the stomach, and the plate of one electrode inserted into it; the plate of the other lightly pressed on the surface of the liver; stomach positive 10°, and made to increase by making and breaking contact.

In cleaning one of the electrodes it broke, about half an inch from the plate. The plate of the electrode was in contact with the mucous membrane of the ilium, and the pointed extremity of the broken electrode inserted into a vein coming from the same part; blood slightly positive.

A platinum wire (No. 18), six inches in length, was flattened out at one extremity so as to present a surface three-fourths of an inch in length, and one-sixth of an inch in breadth. This will be called electrode A (E A). The former electrode with the plate attached, electrode B (E B). The broken electrode, having the broken extremity flattened out to the extent of one-fourth of an inch in length and to one-eighth of an inch in breadth, electrode C (E C).

In describing the circuits, when the pointed extremity is in contact with the animal, it will be stated the *p* of E A, &c., otherwise the broad extremity is used.

Experiment 3.—Rabbit. Mucous membrane of the ilium and vein from the same part; the *p* of E A in contact with the former to the extent of an inch and a half, the *p* of E B with the latter; blood positive 4°; when E B was used a greater effect, and made to increase; no effect ensued when the *p* of E B was inserted into a different vein; returning to the former vein, the action upon the needle occurred; should the blood however from the two veins become mixed, then an effect upon the needle ensued, more especially if E B be used. No effect when E B was placed

on the outside of the gut over E A where the blood had not been, whereas with blood flowing from the vein, they may be separated to the extent of an inch or more, the effect occurring. Stomach and liver; the *p* of E A inserted into the former to the extent of an inch, E B lightly pressed on the surface of the latter; stomach positive 8°; an attempt was made to insert the *p* of E B into a vein coming from the stomach, but failed from the difficulty in obtaining a vein of sufficient size; when it was inserted into the portal vein, no effect. E C inserted into the caput coli; stomach slightly positive.

Experiment 4.—Cat. Prussic acid dropped on the nose. Mucous membrane of the ilium, and a vein from the same part; the *p* of E A in contact with the former to the extent of an inch and a half, the *p* of E B with the latter; blood positive 5°, and made to increase: a vein coming from a different part was punctured, no effect: with the former vein the effect ensued, but not when it had become empty.

Mucous membrane of the stomach and blood from the same organ; the *p* of E A was in contact with the great curvature of the stomach to the extent of an inch, a vein corresponding to the lesser curvature was punctured with the *p* of E B; no effect: the portal vein was punctured at its commencement; as the blood flowed out a slight effect occurred, blood positive: E B was lightly pressed on the surface of the liver; the latter slightly positive.

Mucous membrane of the upper part of the colon, and vein from the same part; *p* of E A in contact with the former to the extent of an inch and a half, the *p* of E B with the latter; blood positive: E B pressed on the surface of the liver; the latter positive. The liver and intestines covered with blood.

The *p* of E A inserted into the stomach, E C into the colon; no effect.

Stomach and small intestines empty.

Experiment 5.—Cat. Killed as the last. In opening the abdomen the liver was wounded. The mucous membrane of the upper part of the small intestine, where active digestion was going on, was in contact, to the extent of an inch, with the *p* of E A; an artery going to the same part was punctured with the *p* of E B; no effect: when the vein was wounded, a decided result; blood positive. The inside and outside of the gut were formed into a circuit; no effect.

The mucous membrane of the stomach, in contact with the *p* of E A, to the extent of an inch, E B with blood flowing from the vena porta; blood slightly positive: placed on the liver; the same result, but not when the point was used. The liver covered with blood from the wound.

The pelvis and renal vein of the same kidney were formed into a circuit, with the *p* of E C and of E B; vibrations occurred, but impossible to obtain any decided effect by making and breaking contact.

Between the mucous membrane of the colon and vein; blood positive.

Stomach half-full of food.

The following experiment was to ascertain how far the different solid and fluid substances in contact with the electrodes might interfere with the result.

Experiment 6.—E B covered with some blood from the cat, was dipped with E A into water. A similar circuit was formed with fresh blood and water containing fæcal matter from the colon, the electrodes were close to each other. A portion of the colon of the cat, with its contents, was removed and placed in a weak solution of potash; E A and E B were dipped, one into the solution, the other into the gut, and then changed; in no instance did any effect occur. Some very dilute nitric acid was mixed with the contents of the colon and the same circuit formed; contents of the colon positive 3°. Can the difference, which is observed between cats and rabbits, as to the direction of the current when the stomach and liver are formed into a circuit, be attributed to the action of the poison, although no poison had been taken into the stomach?

Experiments 7 and 8 were upon rabbits; in one, prussic acid was used as in *experiment* 5; in the other, after a circuit had been formed between the mucous membrane of the stomach and blood flowing from the aorta, an opening was made and about ten drops of prussic acid poured into the stomach. The results only need be detailed. The mucous membrane of the stomach was positive to the following parts: surface of liver, mesentery, blood flowing from the vena porta and aorta, gall-bladder, mucous membrane of the duodenum and of the caput coli.

The blood flowing from the mucous membrane of the ilium and of the caput coli was positive to these parts from 3° to 5°.

The surface of the liver and the mesentery were slightly positive, when E B was in contact with them, to the mucous membrane of the ilium and caput coli.

In the following circuits no result was obtained:—Mucous membrane of the stomach and vena porta, the pointed extremity being inserted into the latter. Gall-bladder and vena porta. Mucous membrane of the ilium and that of the caput coli. Mucous membrane of the ilium and vein coming from the same part; this occurred when the aorta had been wounded, no blood flowed and the intestine was pale.

The contents of the stomach were positive to those of the colon when formed into a circuit out of the body; the effect was not so great as when the circuit was formed between the stomach and caput coli, the parts remaining *in situ*. In one experiment, the stomach, liver and intestines were removed from the body: the stomach was positive to every other part; it was detached, emptied of its contents, and washed, the same circuits then formed, but no effect; some of the food replaced, the former effects were reproduced, but not so decided. The caput coli was slightly negative to the liver and mesentery. With the small intestines no effect. In both experiments the contents of the stomach reddened litmus paper, and formed a coarse, dry, compact mass; those of the caput coli a soft homogeneous mass, but produced no effect either on litmus or turmeric paper. From these two experiments, the difference of effect between cats and rabbits, when the stomach and liver are formed into circuits, cannot be attributed to the action of the poison.

At the present stage of the inquiry we may be justified in drawing the following

inference:—*When the electrodes of a galvanometer are brought into communication, one with the mucous membrane of the alimentary canal, the other with the blood flowing from the same part, a deviation of the needle takes place, indicating that the secreted product and the blood are in opposite electric states.*

The two following *experiments* 9 and 10, upon rabbits, were undertaken to ascertain how far they would confirm this inference. To avoid unnecessary repetition, it may be stated, that the same results occurred as in experiments 7 and 8; and, in addition, the following circuits were formed.

Renal vein and pelvis of the same kidney; renal vein of the opposite kidney and urinary bladder: in both instances vibrations only were produced.

Mucous membrane of the stomach and bile flowing from the gall-bladder; the former positive.

Food from stomach and blood; former positive.

No result was obtained in the following circuits:—Gall-bladder and vena cava at the entrance of the hepatic veins. Gall-bladder and vena porta. Mucous membrane of the ilium and superior mesenteric artery. Stomach and gall-bladder (the pointed extremity being inserted into its whole length). Mucous membrane of the stomach and vena cava at the entrance of the hepatic veins. Mucous membrane of the stomach and vena porta. Contents of caput coli and blood. Mucous membrane of the ilium and blood on its outer surface.

In one experiment the liver was much diseased, dark-coloured, and half the usual size; the rabbit appeared weak and unable to support itself.

The parts in the last experiment were covered over with a towel and allowed to remain undisturbed for eight hours. It will be unnecessary to state the circuits that were then formed, with their results, and likewise those in a rabbit, which died from natural causes, twenty-two, twenty-five, and thirty-nine hours after death, as the following experiment containing the principal facts worthy of notice will be related at full length.

Experiment 11.—Rabbit six hours after death, which arose from natural causes. The surface of the viscera and contents of the abdomen were remarkable for their appearance, being so similar to that of a healthy and live rabbit. The veins being full of fluid blood, a favourable opportunity occurred of ascertaining whether the same effects would be produced as in the living animal.

The following circuits were formed:—The blood flowing from the ilium, an empty portion of the descending colon, and caput coli and the mucous membrane of these parts; in the latter only did any effect occur, the mucous membrane slightly positive.

Contents of the caput coli produced no effect on litmus paper.

Mucous membrane of the stomach and the following parts, surface of the liver, mesentery and small intestines; each of the latter slightly positive.

Food in the stomach acid: the moisture covering the different viscera restored the slightly reddened litmus, but produced no effect upon turmeric paper.

E A inserted into the centre of the mass of food in the stomach, E B lightly pressed on the upper external surface over E A; E B slightly positive: E B placed on the liver or intestines; E A slightly positive.

E B and the broken plate of E C were placed, one on the upper surface of the stomach, the other on the mesentery; no effect. Some food was taken out of the stomach and placed on the intestines, the broken plate on the mesentery, E B on the food; E B positive. The same arrangement was made with some of the contents of the caput coli; the latter slightly positive.

The following circuits were formed: food from the stomach and contents of the caput coli; the former positive: some food from the centre of the mass in the stomach, and some in contact with the mucous surface; the latter positive. The mass in the stomach was coarse, dry and compact; that in the caput coli of a finer and softer consistence.

The parts were covered over with a towel for twelve hours.

E B placed on the upper external surface of the stomach, E A inserted into the caput coli; E B positive. E A remaining, E B on the small intestines; effect very slight.

E C inserted into the centre of the contents of the stomach, E B lightly pressed on the upper external surface; E C positive. The stomach was moved over and E B placed on that which had been its inferior surface; E B positive: E B was placed on the part where the stomach had been lying; E B positive.

The different parts had become more moist.

No doubt can exist as to the cause of the production of the effects obtained in the last experiment, viz. the difference of the fluids and solids in contact with the electrodes. The proposition, that, when two heterogeneous fluids are separated from each other by a membrane, or any other porous diaphragm, that which performs the part of an acid takes positive electricity, the other, that of an alkali, takes negative electricity, has been well established by Becquerel*. The results observed in the last experiment beautifully illustrate it; and at the same time show how impossible it is to say, *à priori*, what would be the effect upon the needle when the electrodes are inserted into different parts of a dead animal. The mere circumstance of placing the electrodes in contact with different portions of the contents of the stomach produces an effect or not upon the needle; the state of the parts as to moisture, and the transudation of the secreted fluids or contents through the walls of the different viscera; the contents of the colon and of the stomach considered as a mass, forming one conducting body; the difference with respect to the extent of surface of the electrodes; each of these circumstances may be readily seen to be influential and unnecessary to particularize. There may be two or three acting points between the electrodes, determining the current in opposite directions, or assisting each other, the action upon the needle being the result of a differential, or of a combined current, according to circumstances.

Shall we be justified in referring the effects observed in experiments 2, 3, 4, 5, 7

* Traité de l'Électricité, tom. iii. p. 387. v. part. ii. p. 192.

and 8 to the same cause? Let us take the simple fact, that, when the mucous membrane of the small intestine and the blood flowing from it are formed into a circuit, a deviation of the needle amounting to 3°, 4° or 5° is obtained, the intestine being empty and therefore uninfluenced by the circumstances we have just now mentioned; why did the effect not occur in experiment 11? the conditions appeared to be precisely similar; or in experiments 7 and 10, when the artery was wounded; or in experiments 3 and 4, when blood flowing from a different part formed the circuit; or in experiments 3, 5, 9, when a circuit was formed between the inside and outside of the gut? Did not the failure arise in these instances in consequence of the absence of one necessary condition, the flowing of the blood from the same part, the transmission of the carrying particles from one electrode to the other, as shown in experiments 2, 3, 4, 5 and 7?

Without entering into any discussion as to the mode in which the effect may be supposed to be brought about in the living animal, and the difference observed when the stomach and other parts were formed into circuits in rabbits, and when the same circuits were formed in cats, we may be justified in adding the following to our former inference, viz. *that the effect is produced during the organic action of the part, it ceasing after the death of the animal.*

Instead of endeavouring to refute the notion that the stomach and liver form poles similar to those of a galvanic circle, let us briefly allude to the experiment and conjecture of WOLLASTON. Was not that experiment to *illustrate*, rather than to prove his conjecture? and are not these experiments identical with that conjecture? He evidently saw, mentally speaking, the *meaning* of that phrase "AN AXIS OF POWER," &c. This inquiry has been undertaken with the advantage both of FARADAY's labours and the use of the galvanometer. WOLLASTON's conjecture and experiment have existed for forty years.

The following is a brief recapitulation of the general conclusions which may be deduced from the foregoing experiments and reasonings:—

1. When the electrodes of a galvanometer are brought into communication, one with the mucous membrane of the alimentary canal, the other with the blood flowing from the same part, a deviation of the needle takes place, indicating that the secreted product and the blood are in opposite electric states.

2. That the effect occurs during the life of the animal, it ceasing after its death.

3. That the effect may be considered as arising from the decomposition of the blood; *i. e.* the changes which occur during the formation of the secreted product and venous blood.

4. That these changes are effected by the *organic action* of the part.

The author begs to acknowledge the great kindness that he has received from Sir B. C. BRODIE, Bart., and from Dr. TODD.

12 *New Burlington Street.*

Explanation of the Woodcut.

Fig. 1. Wooden mercurial cup.

Fig. 2. Copper wire to form the communication between the galvanometer and mercury.

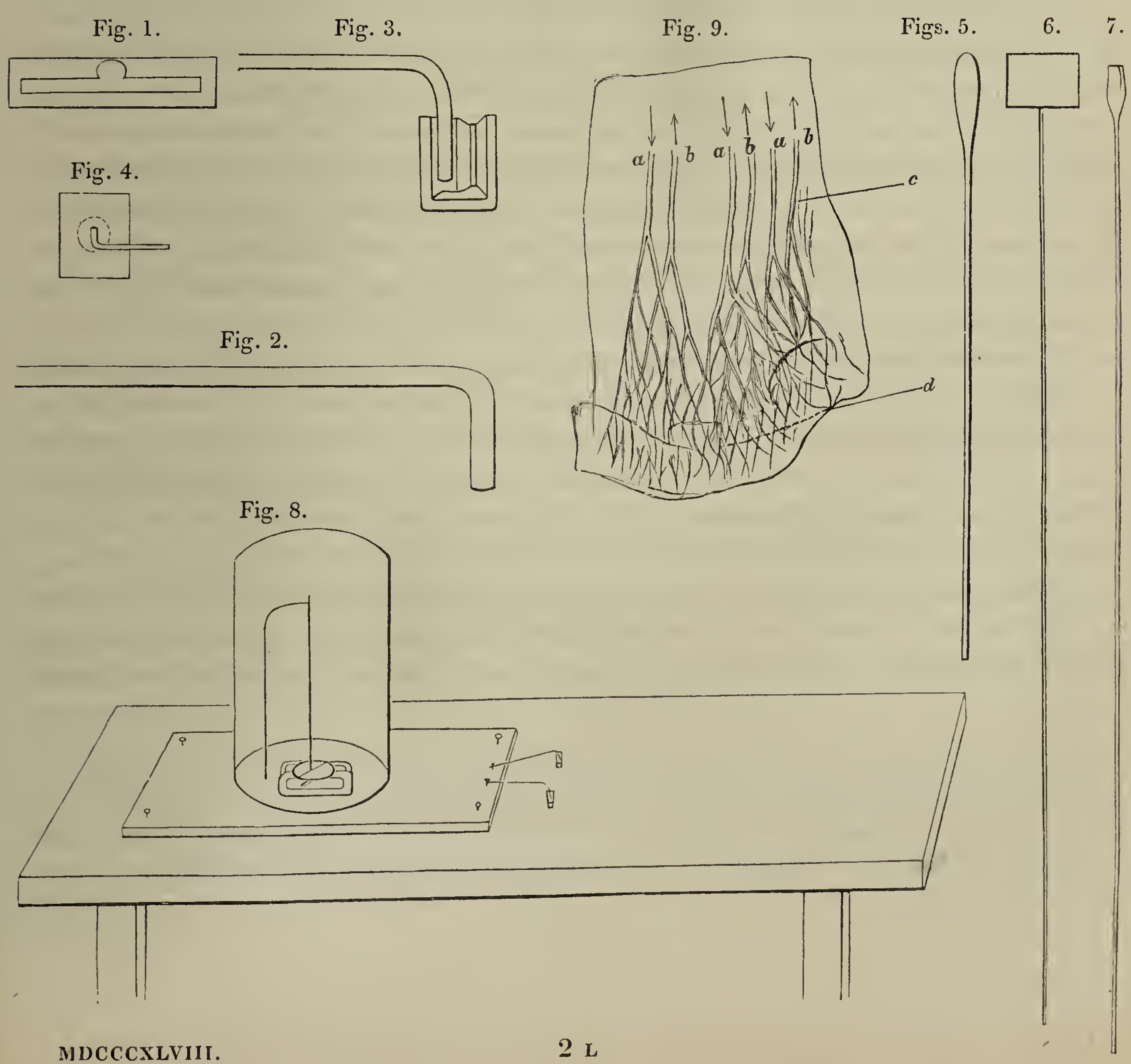
Fig. 3. A section of the mercurial cup, showing the mode in which the thick copper wire was connected with it.

Fig. 4. Shows the manner in which the plate of platinum was soldered on to the platinum electrode, pure gold being used; all the part within the dotted line, and for a short distance up the wire, was coated with shell-lac to prevent the action of the mercury upon it.

Fig. 5. Electrode A. Fig. 6. Electrode B. Fig. 7. Electrode C.

Fig. 8. The arrangement with the galvanometer previous to each experiment.

Fig. 9. *a a a*, arteries; *b b b*, veins; the arrows indicate the course of the blood: *c*, point of the electrode in contact with the wounded vein; the dotted line of *d* the electrode in contact with the mucous surface of the small intestine.



XVIII. *On the Direction assumed by Plants. By Professor* Macaire *of Geneva. Communicated by* P. M. Roget, *M.D., Sec. R.S.*

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§ 1. *On the Curling-up of Tendrils.*

THE plants with tendrils are very numerous. According to Mr. Palm there are about five hundred, divided into seventeen families. Of these, one hundred and sixty have a ligneous stem, eighty-three are perennial herbs, and one hundred and seventeen are annuals.

My experiments on the mode of curling-up of these organs were made on the tendrils of the *Tamus communis**, a plant of the family of the Asparageæ. The tendrils of this plant seem to be a thread-like degeneration of the footstalk of a leaf, whose place they occupy on the stem of the plant. They are at first straight, and are implanted perpendicularly on the stem, so as to form almost a right angle with it; the extreme end of the tendril only has a slight tendency to bend towards the stem. When the tendril of the Tamus is touched by any solid body whatever on a point of its surface not too far from the extremity, it contracts itself from the outside inwards, forming at first a hook and then a curl, so as to embrace the body closely if that body be circular; if angular, the knot is only tight on the angles, and bulges out on the surfaces. When a first knot is tied, the end of the tendril continues to roll itself up in a coil, though not in contact with the body in that part, and the coil slides over the external object, coming nearer and nearer to it so as to embrace it several times: in the mean while, the other end of the tendril continues also to contract itself. In this way as many as seven or eight knots are formed. I have frequently seen three tied before my eyes within the space of a quarter of an hour on a metallic wire, small branches of wood, a pencil, my finger, &c. The contact of any solid body whatever is sufficient to produce this effect; so much so, that although the tendril is evidently destined by nature to support the creeper to which it belongs, by means of the surrounding plants, yet if it chances to meet a part of the very same plant of Tamus of which it is itself a portion, the contact causes it immediately to roll itself up around that portion.

* Since this paper was read the author has been informed that *Tamus communis* is a plant without tendrils. The plant on which he made the experiments here referred to is a common weed in the gardens in Switzerland. Being without the means in this country of identifying it, he must supply the information on a future occasion, only adding that *Smilax aspera*, another of the Asparageæ, has been suggested to him as being probably the plant.

The tendril of the Tamus is very smooth, and its surface contains neither resinous nor glutinous matter, nor hair of any kind. When slightly rubbed between the fingers, it does not contract itself. To obtain its curling up, it is necessary that the contact should be only on one side. If the solid body round which the tendril had begun to coil be removed, it continues to contract itself in the air, but without fastening the knot on itself, the empty ring remaining constantly open. If, after a little time, an object of suitable size and form be introduced in this empty ring, it contracts anew and the knots tie themselves firmly on this new body.

If the tendril be left without internal support after its contraction, it does not turn again, nor resume its primitive direction in a straight line; on the contrary, the contractions are soon extended over the whole of the tendril so as to give it the appearance of a corkscrew.

The same thing happens to those tendrils the extremities of which are attached to a supporting body. This arrangement has the effect of preventing the tearing of the tendril when the plant is shaken by the wind, by giving it the shape and elastic properties of an helicoidal spring (*ressort à boudin*). I placed a small portion of a branch in contact with a tendril of Tamus; when it had begun to contract itself and the first knot had been tied, I let the branch go and it remained suspended. Not only did the tendril support the weight of the branch, but it continued to roll itself up around it, raising it more and more by each knot. Ten rings were thus formed around the slip, regularly arranged in a spiral by the side of each other. The branch was entirely covered over by them, and as there was no room for more, the tendril continued to contract itself in the air towards its base, and to form empty rings in the form of a corkscrew, having nearly the same dimensions as those on the branch.

When a body, such as an iron rod, too heavy to be supported, is placed in this way, the knot formed becomes loose and the rod drops. If the tendril rolls itself round a body that is soft and not elastic, such as a piece of packthread, it presses it tightly enough to render its diameter visibly less in the part where the knots are tied. This pressure may even be rendered sensible to the touch if the knot be suffered to form round the finger, and it goes on increasing to a certain extent.

When the tendril lays hold of an elastic body having a conical shape, such as the flat part of a leaf rolled up in a funnel, the knots slide over the leaf as they are formed and suffer it to escape. When a tendril of Tamus has begun to curl near its extremity and to fasten itself round any object, if the upper portion of the same tendril chance to meet with another exciting body, another part of the same branch for instance, it may curl over again in a spiral at this point and tie its knots there. The same thing may happen a third time; and in this way may be seen in the same tendril two or three portions closely wound round an object, while the remaining part of the tendril is loose and detached. The contraction of the tendrils of the Tamus always takes place in the same direction, and the curling is turned inwards, whether there be or be not an object round which it may occur: and more than this, when a round

body is put in contact with the external surface of a tendril in its straight form, it does not curl round the body which excites the movement, but on the contrary from the outside inwards, although there is nothing there for it to embrace.

This tendency to curl only on one side has induced me to examine the tendrils, to see whether I could discover by means of the microscope a special anatomical structure that might explain this peculiarity.

I have made observations with the microscope of Amici and a magnifying power of 800 times on very thin slices of the tendril, taken some from the external, and some from the internal side. When the specimens were cut out of tendrils already contracted in a spiral form, the only visible difference was that the cells on the internal side appeared very little elongated and almost square; while, on the contrary, they were much narrower and longer on the opposite side. But this appearance was evidently mechanical and produced by the contraction itself; for when I examined slices cut in all directions from tendrils still in their straight form and not contracted, I found the anatomical structure to be precisely the same in all, and indeed similar to that of those leafstalks of the same plant which are not susceptible of being curled up.

I caused a straight tendril of Tamus to curl by the excitement occasioned by the touch of a pencil. Two knots were closely tied within the space of ten minutes. I then cut out the tendril near its base, so as to interrupt its communication with the plant; the tendril, though kept in the light and in a warm and damp atmosphere, did not continue to curl itself.

When a straight tendril of Tamus is placed in a tumbler full of water, so as not to touch its edges, it does not experience any contraction, but is just as ready as before to curl up by the first contact of any solid body.

I have immersed a tendril many times over in a solution of gum-arabic, leaving it to dry between each successive immersion, so as to form round it a thin coat; the tendril so covered had not in any degree lost its power of contraction by the contact of a solid body, and the coat of gum did not seem to prevent the close adherence of the knots.

Liquid ammonia in a diluted state has no influence on tendrils. Alcohol and eau de Cologne seem to possess a slight exciting influence on the contraction of tendrils, which are thereby bent in the form of a bow, but do not continue to curl, and they appear not to experience any bad effects. When immersed, with the same precautions, in sulphuric acid diluted with water, the tendril contracts itself immediately and energetically. It curls up in a spiral form, though there be no object present round which it might roll itself. The knots, at first very loose, continue to tighten up to a certain point, although there always remains an empty space in the middle of the rings. After some time, the part touched by the acid becomes disorganized and dries up.

Diluted nitric acid has the same effect as sulphuric acid, but the contraction is

less rapid. The fumes that escape from the acid, without any actual contact with the liquid, cause the tendrils to contract and curl in the air, though in a less prompt and energetic manner than when immersed in it. The parts of the tendril that have been steeped in nitric acid, are afterwards found withered and dead. The tendrils, on the contrary, that have curled up by simple exposure to the nitrous vapours do not wither.

A solution of corrosive sublimate seems slightly to excite the contraction of the tendrils of the Tamus; but after a few hours the tendril so treated withers and dies.

I immersed in prussic acid, prepared by Scheele's process, a tendril of Tamus in its straight and vigorous state, carefully preserving it from contact with the vessel. I kept it there for two minutes and then took it out. The tendril did not seem to have experienced any alteration; but when I placed in contact with it a foreign body to excite it to curl up, no contraction could be produced; and although the tendril was to all appearance fresh and healthy, it could not be made to curl round. When a tendril that has been previously excited and has begun to curl itself in a spiral form is plunged into prussic acid, it loses the power of continuing its contraction, and the knots that were commenced do not become tied. Two days after such an experiment a tendril was found precisely in the same state in which it had been left, although the curling had begun on a branch of proper size, and the tendril appeared endowed with complete vegetative power. I tried if I could destroy with ammonia this narcotic influence of prussic acid, but without any decided results. A fresh and healthy tendril of Tamus was excited till it began to curl, and then immersed for two minutes in prussie acid. The knot in its incipient state was placed on a branch of precisely the same size as that by which it had been excited, and I observed every hour what took place. During five days that the experiment was followed up, no change was produced: the knot begun did not become tighter, nor was any other knot formed. The tendril remained to all appearance perfectly healthy; but the prussic acid seemed to have destroyed in it all power of contracting farther. These experiments require to be continued and multiplied; but they warrant the conclusion, that whatever be the cause of the irritability of the tendrils, their curling-up cannot be explained, as Knight thought, by the unequal action of the light on both sides of the tendril; nor as Decandolle admitted, by the obstacle afforded to vegetation by the contact of the leafstalk with the body on the side where it touches it. The rapidity of contraction in the tendrils of the Tamus cannot be accounted for by so slow a process. This irritability is a vital property inherent in the organ itself, but which ceases when it is separated from the parent plant, and which, as I have already shown to be the case with those plants called sensitive, is excited, modified, and even suspended or destroyed by the influence of poisons, either of the vegetable or mineral kingdom.

§ 2. *On the Inclination of Stems towards the Light.*

Every one knows that the branches and green parts of plants have an invariable tendency to direct themselves towards the most luminous quarter of the place in which they grow. The experiments of M. Teissier, who caused plants to vegetate in a cellar where they received air by an opening that did not give light, and light by a window which did not admit air, have shown that it was really light and not air that the plants were in search of. Decandolle imagined that he could account for this phenomenon by the inequality of the development of a plant that receives light only on one side. The lighted side, he says, decomposes more carbonic acid, solidifies a larger portion of carbon in its tissue, and becomes more solid; at the same time it exhales more water, and consequently contains more of the inorganic matter left behind, which contributes to harden it still further. The other side remains softer, the fibres are more elongated, and the consequence is the bending of the branch towards the side which is least elongated, that is to say, towards the light. This illustrious botanist has extended the same reasoning to the curling of tendrils; but I have already attempted to show that it could not be applied to this rapid physiological action in those of the Tamus.

It was necessary to investigate if there exists a special attraction exercised by light on the green parts of a plant, and in particular if a plant so placed as to be able to move, could be brought to change its position under the influence of light. My experiments on that head have been made under two circumstances of a different nature.

The subjects of the first were some special plants of the family of the Nayadeæ, the *Lemna minor* and *Lemna polyrhiza* (duck-weed), which naturally float on the water. These plants form a kind of anomaly in the vegetable kingdom, in as far as they consist of a single floating leaf, having submerged roots and producing at their margin new leaflets that often detach themselves from the mother plant, like the polypus, to form a separate individual; they are in consequence viviparous. I have nevertheless ascertained by experiment that they receive from the action of light the same influence as other plants; thus, if left on the water in complete darkness, the new leaves that shoot appear blanched and yellowish. If immersed entirely in spring-water and exposed to the rays of the sun, they remain green and vigorous, and emit a great quantity of bubbles of oxygen gas that gathers by degrees in the upper part of the bell.

I arranged an oval and elongated vessel so that, being half-full of water, one-half of it was kept in darkness by means of a diaphragm which was placed in the middle, and kept one line above the surface of the water. The part of the vessel that was intended to be kept dark, was carefully covered over with many sheets of black paper: the other half of the vessel was freely exposed to the influence of light. The apparatus was arranged in a locality perfectly free from disturbance and from

the agitation of the wind. A great number of floating plants of Lemna were successively placed in the dark part of the vessel. Though they could freely move on the water by the smallest impulse, I never saw them come near the lighted part or move into it, even when the experiment was continued long enough to allow the symptoms of blanching, already mentioned, to manifest themselves in the Lemnæ. No motion towards the light was ever perceived. It is to be understood that, for such a result, it is necessary that there should not be any possibility of shaking or vibration in the liquid, and a first experiment, undertaken without sufficient precautions in that respect, gave results by which I was at first deceived. My doubts on this result were excited by the following experiment. In a tumbler completely covered with thick black paper, I put some water on which I placed a few floating plants of Lemna. A narrow slip was cut out in the paper on the side opposite to the light, so as to bring a single ray across the vessel. The apparatus, covered over with a thick book, was left in a place not exposed to vibration. The Lemnas did not change the position which they had been placed in, and, far from arranging themselves, as I expected, in a line along the luminous ray, they did not move at all towards the illuminated space. Varied in different ways, the experiments with the Lemnas have always given negative results to the supposition that they have a real attraction exercised on them by light.

I have endeavoured to repeat the experiments on other plants. I have ascertained that seeds of peas, French beans, mustard, &c. put on floating pieces of cork in a vessel filled with water, not only did germinate, but that the plant produced could develope itself without any other care than replacing the evaporated or absorbed water. It could thus furnish a complete vegetative course, expand its leaves, make its flowers blow, and even bring its fruit to maturity; only the stems are very slender and the leaves smaller than usual.

The germination of seeds took place pretty nearly in the same way, whether they were exposed to light or kept in complete obscurity. As it had been said that light exercises an influence on the direction of roots, I made an experiment with four peas, placed on floating pieces of cork. The first (No. 1) was left to germinate and grew in a common glass tumbler; the second (No. 2) in a tumbler covered with black paper, but leaving a thin stream of light to enter by a slit in the paper; the third (No. 3) in a glass coloured blue, and admitting only a thin stream of blue light; and the last (No. 4) was kept in complete obscurity. The seed in the free light developed itself well, and in twenty-four days exhibited a strong stem of a fine green colour, five inches long and well-stocked with leaves. Nothing remarkable occurred in the roots.

No. 2, receiving only a ray of common light, germinated well and reached its full growth. After eighteen days its stem was four and a half inches in length. The root was very long, six inches at least, and was covered with small branching roots, which had all been formed on the side of the opening that admitted light, and had directed

themselves towards it. I turned the plant over so as to place the side of the large root deprived of small radicular threads opposite to the luminous slit. Five days afterwards, radicular fibrils had sprouted on that side; and those which had developed themselves before, had indeed not perished, but had ceased to grow. The experiment was completed after thirty-four days. The root was then a foot long, was covered on all sides with radicular threads, and had turned itself into a spiral form. The stem was slender, very little coloured, and had a length of ten inches and a half.

No. 3, receiving the blue rays, had, after eighteen days, a stem only three inches long, and the root measured four inches. The radicular threads were all on the side opposite to the slit giving admission to the blue light. I changed the position of the plant, and placed the radicular threads on the luminous side. Thirty-four days from the beginning of the experiment, radicular threads had sprouted on the side of the root where there were none before, and the root had also taken a spiral shape. It was seven inches long; the stem, nine inches long, was very slender and almost white.

No. 4, in the dark, germinated after five days, but a few radicular threads alone developed themselves without a stem.

These trials were repeated on peas and mustard-seeds, and gave results of the same nature, as far as concerns the curling of the roots in a spiral shape under such circumstances, and the tendency of the white light to favour, and of the blue light to hinder, the growth of radicular fibrils.

The experiments relating to the direction of the stems were made pretty nearly in the same way as with the Lemnæ; only the vessel in which the floats were placed was much larger, and the diaphragm was not so near the surface of the water. The consequence was that the darkness of one-half of the vessel went on increasing from the diaphragm towards the extremity of the covered portion, and though very intense at that last part, was nevertheless not complete. As soon as the seeds had germinated, they were placed on the cork floats, in which I had bored a hole through which the root emerged into the water. The float was placed in the dark part of the vessel, far enough from its side to avoid the capillary attraction, and it could be moved by the gentlest impulse. It is hardly necessary to say that the apparatus was kept perfectly free from shaking or from the action of the wind.

On the 14th of May, a pea in germination was placed on the float in the dark part of the vessel: it grew very slowly, and on the 22nd began to exhibit a very short and almost white stem. This stem by degrees became longer and longer, spreading itself along the surface of the water, and on the 8th of June it had reached the diaphragm. It formed then a long stem, etiolated, slender and white, running along the surface of the water; its length was two feet. The cork float had not moved forwards a single line towards the luminous aperture; only the weight of the plant throwing itself, as it were, all on one side, had by degrees bent down the cork float, but on the same spot it before occupied. When once the stem of the pea had

passed under the diaphragm and had reached the illuminated part of the vessel, it grew in an erect position, became green, and began to develope leaves.

This experiment, very often repeated with peas, French beans, &c., always gave similar results. Never did any attraction of the whole plant towards the light manifest itself; and however long a stem it might be necessary to produce in order to reach the diaphragm, that was always the means to which the plant had recourse.

In order to vary the mode of experiment, I placed on a cork float in the obscure part, a plant of French beans already developed, green and strong, and the roots of which were immersed in the water. The green stem took little or no ulterior development; but from the neck of the root there grew out another stem, white and etiolated, that spread itself along the water to reach the diaphragm and the light portion of the vessel; there it grew erect and gave forth its leaves. The float had not changed its place, and, as before, had only been bent down by the weight. The same experiments repeated with green and vigorous peas, gave precisely the same results.

Germinated seeds of mustard placed on very light floats in a tumbler half-full of water and surrounded with black paper, so as to admit only a luminous ray, were put in the dark portion, but very near the aperture. One of the plants grew and sprouted a stem that went all round the tumbler to come and spread its leaves in that part of the vessel where the luminous aperture was, and once there, the plant did not pass beyond that point. It grew erect in that part, though the light was too weak to render it entirely green. No motion was perceptible in the float; and it is very remarkable that, being close to the luminous aperture, the plant in search of the light had taken this long circuit rather than communicate to the float the slight motion that would have placed it in possession of it. I venture to conclude from these experiments, that the direction of plants towards the light is not the result of an attraction, properly so called, which, similar to the physical attractions, could carry over the attracted body towards the agent.

The ingenious supposition of Decandolle of a mechanical bending, due to a greater solidification of the fibres on the light side than on the other, appears to me not to be more applicable to the direction of stems towards the light, than to the curling of tendrils; for, by the arrangement of the apparatus, the lightest side was not always the same, and the stems advanced straight towards the light without incurvation or bending.

There remains to examine the hypothesis of Dutrochet, who admits that there exists in the stems and branches of plants a system of cells progressively increasing in size from the centre to the circumference in the central parts of the stem, and from the circumference inwards in the cortical portion. He supposes that these cells are gorged with sap by the influence of endosmose, and that this endosmose tends to give a bending in contrary directions to the two systems, so that the stem is inflected in the direction that predominates. He admits too the existence of a fibrous system

placed in the central cellular organization, and supposes that its fibres are also decreasing in size and are inflected by being filled with gas or oxygen endosmose. The incurvation of this tissue always takes place inwards, and that of the cellular system outwards. According to Dutrochet, the influence of light has a tendency to diminish the filling up of the cells and impair their power of incurvation, and, on the contrary, to increase the production of oxygen gas that creates a bending in the opposite direction in the fibrous tissue. He has grounded this theory on some curious experiments. For instance, by cutting in a longitudinal direction a stem of *Medicago sativa* bent towards the light, he saw that the part of the stem which had been in the light increased its curvature, while the other, on the contrary, became first straight and then bent in an opposite direction. There was then, according to him, an antagonism between the two slips, and this fact was sufficient to overthrow Decandolle's theory, in which, as the exterior slip is supposed to be the only active one, its incurvation ought to have been preserved or even increased in the primitive direction. I have repeated the same experiment with the same result on many stems or footstalks, among others on the footstalk of the leaf of *Calla æthiopica*. But in trying to repeat the sections in all directions, I observed that the bending of the slips was always towards the bark, or outwards, and that the position of the stem with regard to the light had nothing at all to do with it. This phenomenon is due to the contraction or the resistance to any elongation of the exterior cuticle.

Thus, when a segment of stem or footstalk is immersed in water with its cuticle entire, the liquid, penetrating into the cells, fills the cellular tissue, renders it turgid, and there results an elongation of the central parts; but as the fibres of the cuticle cannot follow this elongation and therefore resist it, the segment of stem or stalk is inflected towards the resisting side, or the cuticle, and the incurvation becomes manifest outwards. If, leaving the central cellular system entire, I only removed the cuticle, the immersion of the slip in water, though it brought on the turgid state, did not create incurvation, which yet ought to have appeared if it were, as Dutrochet supposed, a necessary result of the cells decreasing in size. The same consequence followed when the segment was immersed in water after the cuticle had been slit in two or three places, taking care that the incision did not go beyond it; the imbibition took place, the cellular system was swollen and elongated, but the stem remained straight because the cuticle no longer presented any resistance.

The cuticle alone put into water curls by the shortening of the fibres pretty nearly like a wet rope, so that, far from lengthening it, the liquid has a tendency to shorten it.

Besides, Dutrochet's hypothesis that light has a tendency to diminish the endosmose by the vacuum produced by the exhalation in the cells, appears not to agree with the supposition of the same author, by which he attributes the ascension of the sap to endosmose. He admits that it is through the cellular membrane of plants and by endosmose from cell to cell that it produces the ascensional march of sap, a

liquid which he supposes to be inferior in density to that inclosed in the cells. In the endosmose experiments it is usual to take liquids rather different in their specific gravities; and it seems that if we admit that the sap contained in the cells of the upper part of the plant, where it has been already elaborated, may have a specific gravity sufficiently greater than the surrounding water to let it come in by endosmose, it is not so easy to understand this difference of density from cell to cell as being sufficient not only to allow endosmose to take place, and still less to produce the enormous ascensional power which is admitted to exist in the sap. It occurred to me that the effects of endosmose in such a case might be ascertained by means of experiments.

I took three glass tubes of unequal diameters, such as could be put one inside the other. One of the apertures of each tube was closed with a piece of bladder, and after a solution of sugar of 1·13 specific gravity had been introduced into them, they were placed the one within the other: the last or largest tube was dipped in pure water. The height of the liquid in each tube was carefully ascertained, and the endosmose then observed. The liquid in the largest tube did, as usual, increase in bulk by the replenishing endosmose of the water outside of it, and the effect went on gradually increasing for several days. But during this time, the liquid of the two other tubes did not experience any change of level, the difference between the respective densities of the solutions they contained and the liquid of the larger tube not being sufficiently great to produce endosmose. This experiment, many times repeated, both with alcohol and with solutions of gum or sugar, always gave the same result, and seems to show that a succession of cells would not act otherwise. The transmission of a liquid from one to the other by way of endosmose either would not take place, or at all events would be too slow to account in any manner for the rapid ascent of the sap.

In repeating Dutrochet's experiments on the turgid state that the cellular tissue of plants takes in water, and employing for that effect coloured liquids, it occurred to me, that the absorbed liquor was not inside the cells themselves, but in the intercellular spaces, which accords with the rapidity of the result, which it would be impossible to understand if the liquid were to proceed from cell to cell and through their walls.

It occurred to me also, that if endosmose was the only or principal agent of the ascension of the sap in plants, that is to say of vegetation, this phenomenon ought to be influenced by the two indispensable agents of vegetation, heat and light. As there is something rather mysterious in the phenomenon of endosmose, it was not impossible that such might be the fact. Dutrochet, from an experiment on the cæcum of a fowl, concluded that a rise in the temperature increased both the rapidity and the quantity of endosmose. In his experiments on acids he even announces having discovered that the temperature changes the direction of endosmose, which goes from the water towards the acid, or from the acid towards the water,

according as the thermometer is low or high. At all events he does not generalize this singular result, which, according to him, belongs only to the acids; but as a great many saps have an acid reaction, the result ought to be, that, according to the state of the temperature in those cases, the sap should enter the tree or flow out of it: this seems to be in opposition to all known facts.

In all the experiments I have tried, and they have been numerous, I have never been able to ascertain that heat had any apparent influence in promoting endosmose. It is obviously necessary to deduct the effect produced by the dilatation of the liquid by the caloric, or wait until it has returned to its previous temperature, before measuring its bulk. With this precaution, the quantity of water introduced into tubes perfectly similar and containing the same solution of sugar, was found to be exactly equal in the same time, though one of the tubes was maintained at a temperature of 65° C. (149° Fahr.) and the other at 10° C. (50° Fahr.). The ascent of the liquids by endosmose proceeded at exactly the same rate when the tube was kept at a constant low temperature as when it was much more heated, and then brought back again to the usual temperature of the atmosphere. Heat does not appear to me to have any influence in increasing either the quantity or the rapidity of endosmose.

Similar experiments were made, preserving for the endosmose apparatus the same temperature, but exposing one to the influence of light and keeping the other in complete darkness. The tubes and the liquids were absolutely similar, and there was no difference whatever found in the rapidity or quantity of the ascent of the liquid in the tube. When the same endosmose apparatus was exposed alternately to light and to darkness, the same decreasing rate of the endosmose was observed that would be seen in an apparatus constantly kept in darkness or in the light, and no special influence of light could be discovered. Thus, to give an instance of one of the numerous experiments performed, the mean rising of the liquid in an endosmose tube has been in lines,—

		Lines.
During one hour	in the light	3
	in darkness	1·9
	in light	1·8
	in dark	1·8
	in light	1·7
	in dark	1·7
	in light	1·2
	in dark	1
	in light	0·7, &c.

Many times repeated, and with various liquids, these experiments have always given the same results; and I think I am entitled to conclude from them, that heat and light have no apparent influence on the endosmose phenomenon.

There appears to me, from all these facts, very little probability that endosmose

actually performs in vegetation, and in particular in the direction of stems, the important part that has been attributed to it.

§ 3. *On the Direction of Leaves.*

It is known to every one that the leaves of plants have a tendency to assume the same constant position with respect to their two surfaces. One of these surfaces is generally of a deeper green, smooth and glistening*, the nerves of the leaf being very little if at all prominent; this is the surface naturally turned to the sky, and it is on that account called the upper, or superior surface. The other is of a paler green colour, full of little asperities or covered with short hair; it has little or no varnish, and, as it is naturally turned towards the earth, it is called the under, or inferior surface. As far as regards the anatomical structure, M. Adolphe Brongniart has found out that the under surface of leaves contains a greater quantity of pneumatic cavities communicating with the external air by the apertures of stomata than the upper surface. It is to this accumulation of air that they owe, it seems, their whitish colour; and some leaves, like those of French beans, when placed in water under a vacuum, allow this air to disengage itself through the stomata, and, as it is replaced by water, the inferior surface of the leaf becomes of the same green colour as the upper surface. A long immersion in water without having recourse to a vacuum causes the air gradually to replace that liquid, and gives an uniform colour to the two surfaces of the leaf. In some leaves, in those of the Grasses for instance, there is hardly any difference between the two surfaces. In aquatic leaves, as in those of the *Nymphœa*, the air that exists in the pneumatic cavities appears, according to Dutrochet, to come in and out of them by the vessels of the footstalk. It seems that this air contains a little less oxygen gas than the atmosphere. Dutrochet makes out that the oxygen contained in the air of plants goes on diminishing from the leaves, where it is in the ratio of 16 per cent., to the roots, where it is only 8 per cent. His experiments have been made on *Nymphœa lutea*.

The tendency of leaves to present their varnished surface to the sky and their pale surface to the earth, though very general, is not altogether without exceptions. Thus, the misletoe presents its leaves in all directions. The *Ruscus aculeatus* and some grasses turn their unvarnished surfaces upwards. It is the most deep-coloured surface that is directed towards the sky; and if it were to the presence of air that they are indebted for their paleness, it is the surface most provided with pneumatic cavities that would be directed towards the earth.

Among naturalists, Bonnet is the first who has examined the phenomenon of the direction of leaves. He has shown that if the position of a leaf be changed, so as to present to the sky its under surface, the leaf turns itself over spontaneously more or less quickly. This turning over is more rapid in young leaves than in old ones,

* I have ascertained that this varnish is produced by a thin coat of wax, or rather myricine soluble in sulphuric ether.

but towards the end of autumn it ceases to manifest itself. As Bonnet believed that the leaves turn themselves over in the dark as well as in the light, he imagined that this phenomenon is the consequence of the under surface of leaves containing fibres liable to contract themselves by humidity, and their upper surface fibres that contract themselves by the influence of heat. He went so far as to construct, in support of his hypothesis, artificial leaves, of which the upper surface was of parchment and the lower of linen, and he states that the heat and the damp produced in them the same motions as in natural leaves. According to him, the dampness of the earth determines the under surface of the leaves to turn themselves on that side, and the heat of the sun acts in the same way on the upper surface. This opinion was the more singular in Bonnet, from his having observed that the leaves turn themselves over in water as well as in the air, and from his believing that this change of position in the two surfaces of leaves is the result of the contortion or the inflection of the footstalk.

Dutrochet indeed attributes to the influence of light the turning over of leaves, but he considers the flat part of the leaf as passive, and the footstalk as the only agent in its motion. He quotes besides no other experiment in proof of the influence of light but those of Bonnet, and especially the one in which some leaves of a cherry-tree, placed under the shadow of a table so as to receive light laterally, put themselves in a vertical position with the end downwards.

He gives for the chief ground of his opinion respecting the action of light in this phenomenon, the fact that having placed a *Convolvulus* in the circumference of a wheel rapidly revolving, the leaves after eighteen hours presented their upper surface to the centre of rotation and their under one to the circumference. He concludes from this experiment that leaves present naturally their upper surface to the light; but, as Decandolle justly observes, it would have been more natural to conclude from it that they present it to gravitation. He recapitulates his opinion in these words:—"I have attempted without success to discover how light acts in determining the torsion of the footstalk when the leaf is turned over. It seems evident that in this circumstance the influence of light is exercised on the flat part of the reversed leaf, and that this influence is transmitted to the footstalk whose contortion it produces; but I do not see here what is the connection between cause and effect. The turning over of a leaf depends on two different causes:—first, the disposition of the footstalk to raise itself towards the sky when it has been accidentally bent towards the earth; secondly, the disposition of the footstalk to bend towards the light, but only when it presents its upper surface to it; the flat part of the leaf is entirely passive."

I know no more recent paper on this subject, which appeared to me to be far from being sufficiently elucidated, and I have therefore endeavoured to add something to what was already known.

I thought it was necessary first to ascertain clearly whether light was really the

agent that caused the turning over of leaves, and afterwards to study the mode of its action on the two surfaces of those organs.

After I had verified most of the experiments of Bonnet on the turning over of the leaves of a great number of plants, as far as relates to the rapidity of the phenomenon, the frequency of its reproduction on the same leaves, &c., in leaving the plants exposed to the light, I placed many plants in such a manner that their inverted leaves remained in complete darkness. Contrary to Bonnet's observations, I found that the leaves in that case did not regularly turn over. It often happened, for instance in a branch of lilac, in a plant of *Polemonium cœruleum,* that the footstalks and the flat part of the leaves so kept in darkness, turn about first in one direction, then in another; but this happened equally to the leaves left in their natural position. This appears to me to be the consequence of the state of uneasiness in which the privation of light puts the leaves; they seem to move about as if in search of this agent, and this cause, as well as the elastic reaction of the footstalk, when drawn from its natural position by torsion, which process I have always avoided, may have led the observer into error.

With the conviction that light is the only agent whose influence causes the turning over of leaves, I made the following experiment. In cold weather (45° Fahr.) I placed a vase of geranium in such a manner that one of its leaves, young, though fully developed, without altering in the least its position or that of the vase, should be entirely covered from the light on its upper surface, by a double screen of black paper that did not touch it, while, on the contrary, the under surface was strongly lighted by a mirror placed below at the proper inclination. Three days after, the flat part of the leaf had begun to turn over; and after six days, the leaf was entirely inverted, so as to present its upper or varnished surface towards the mirror. The apparatus was kept in the same state during a fortnight, and the leaf continued in the same position, the upper surface downwards towards the mirror, and the under upwards towards the screen. The mirror and the screen were then simultaneously removed, and the leaf was left presenting to the diffused light its under surface that was uppermost. After a few days the leaf turned itself over again, and the torsion took place by a motion of the flat part on the point where it is inserted in the footstalk. This experiment has been repeated many times on various plants with the same results, and seems to me conclusive.

When the two surfaces of leaves are exposed to light at the same time, they do not change their position, but seem to suffer; if the under surface be turned towards the mirror, while at the same time the upper surface being shadowed by other leaves, does not receive as large an amount of direct light as the other surface receives of reflected light, the flat part of the leaf contracts itself so as to render it almost globular. The edges of the upper surface are thus presented to the mirror and cover over the under surface.

Some geranium leaves were covered with a screen of black paper attached to their

upper surface by a little gum or a pin. In that case, all the marginal parts of the flat part of the leaf separate themselves from the screen, bend downwards, and the leaf takes a globular form round the point tied to the screen. It succeeds in that manner to expose a part at least of its upper surface to the diffused light below the shade of the screen. When the same experiment is repeated with a mirror placed so as to cast light upon the under surface of the screened leaf, the bending downwards of the marginal portions of the flat part is still more marked and rapid; the mirror giving a stronger light to those parts of the upper surface thus brought to receive it.

When the screen is large enough to allow of no light reaching the upper surface of the leaf, even after the bending of its flat part in a globular form, and no mirror is used to supply the deficiency, the leaf turns yellow and dies.

Light, then, is the direct and indispensable agent in the turning over of inverted leaves, and causing them to assume their natural direction. Its influence is the more rapid, all other circumstances being alike, the greater the difference between the two surfaces of the leaf experimented upon. Thus the leaves of French beans, or of raspberry-bushes, where the two surfaces are of very different colours, turn over completely when inverted, and frequently in less than two hours. In the lilac, where the two surfaces of the leaves are very similar to one another, the turning over is slow and does not seem complete; the flat part of the inverted leaf assuming often a spiral form.

Bonnet and Dutrochet agree in admitting that the turning over of leaves always takes place by a flexion or torsion of the footstalk. The last-named author has even said that a footstalk alone could raise itself erect by the action of light as well as if it were part of a whole leaf. My own observations have convinced me that the flat part of the leaf, or even a separate portion of it, can turn itself over.

In some leaves, for instance in those of the lilac, the *Polemonium cœruleum*, it is always in the flat part alone of the leaf that the motion takes place, and the leaf turns itself into a spiral form to come back by degrees to a regular position. In others, on the contrary, the motion takes place in the footstalk. This is the case in the geranium, the French bean, the raspberry-bush, the horse-chestnut, the plane-tree, the judas-tree, &c. In this latter I have seen the turning over of the leaf take place without any change of position in the footstalk, but by a sort of rotation of the flat part on that portion of the footstalk where it is inserted, and where there is a natural swelling, which swelling was much increased by the change. The footstalk then formed an obtuse angle with the flat part of the leaf, instead of a very acute angle as usual.

In this experiment, as in all the others, the branches had been inflected and tied in a convenient position for the light, but without touching the footstalks, in order to avoid the reaction of torsion.

A vase of geranium, the leaves of which had all directed their upper surfaces towards a window, had been reversed so as to present to the window the under surface of its

leaves. All the young leaves turned themselves over by moving on the footstalk, either bending it downwards or turning it over at its base. Out of the thirteen leaves which had turned themselves over, nine repeated the same process, chiefly by a torsion on the base of the footstalk: the others dried up. On reversing the plant a third time, the leaves did not turn themselves over any more, but became dry. The experiments were made during the autumn, the least favourable season for them; for in the spring the same leaf turns itself over as many as fifteen times. When, therefore, the turning over of most leaves can be freely effected, it takes place in most cases by the motion of the footstalks; but even in plants where it is effected by this means, it can take place in the flat part of the leaf itself. Thus, I placed a geranium leaf, covered with a black paper screen fastened on its upper surface, so as to have it illuminated underneath by means of a mirror. It is evident that if the footstalk had been bent or inflected, the position of the leaf with respect to light could not have been more favourable than it was. The screen would have been placed between the mirror and the upper surface, just as it was between it and the sky; and the light of day would have fallen on the under surface then turned upwards, as the light of the mirror fell. In this dilemma, the flat part of the leaf bent itself downwards, turning from the screen as far as the pin which held it would allow; and the edges, continuing their motion so as to unite below the screen, gave a globular shape to the leaf, and thus exposed a considerable portion of the upper surface to the reflected light, and shaded from it the under surface.

In the same experiment, a young leaf which was accidentally shaded on its upper surface by the screen, though at some distance, bent its footstalk to avoid it. Thus on the same plant, and at the same moment, each leaf took a different, but the most appropriate mode to place itself on the best possible position with regard to the light. The same experiments tried on the *Polemonium cœruleum* gave very similar results.

Many other experiments exhibited the faculty possessed by the flat part of leaves to turn itself over independently of the footstalk. Thus, after having ascertained that, as had been already seen by Bonnet, leaves turn themselves over when immersed in water, just as they do in the air, I placed in a glass full of water some leaves of geranium, French beans, &c., of which the footstalks were passed through a hole bored in a stick, and in such a way as to leave outside the flat part alone of the leaf. When its under surface was presented to the light, the flat part of the leaf turned itself over in curling up. The same thing happened with leaves whose footstalks had been entirely removed, and replaced by a small wooden pin fastened in the hole of the stick to make the flat part of the leaf steady. A leaf deprived of its footstalk and freely suspended in water so as to expose to light its under surface only, had curled itself up and had taken a form somewhat globular, so as to cause a great portion of its upper surface to receive the light.

These experiments seem to show that the flat part of the leaves is far from being passive in their turning over, as Dutrochet had supposed. The turning over takes

place either by the action of the footstalk, or by the motion of the flat part, according as each of these modes will most conduce to the final result. Thus, when I immersed in water an entire branch of geranium so as to expose the under surface of its leaves to the light, all the young leaves turned themselves over in three days by moving on the point of insertion of the flat part of the leaf into the footstalk, and without the curling-up of the leaf. It has been the same when the footstalk has been kept on a leaf fixed in the bored stick, so as to allow the footstalk perfect freedom of motion in placing it in the hole.

I have endeavoured to ascertain whether this action of light on the surface of leaves, which so evidently exists, would be so powerful as to excite in them a real physical attraction or repulsion. With this view I placed on a moveable cork-float some leaves of the raspberry-shrub (*Rubus idæus*), which are among those whose turning over is most energetic. I placed the float on a glass full of water, and arranged the leaves so that their under surface alone was exposed to light, all the other sides of the apparatus being carefully darkened by means of black paper screens. The whole was kept free from all vibration, and an index showed the position of the float by means of a graduated circle. The leaves turned themselves over by a torsion of the footstalk at the point where the flat part is inserted into it, but there was not the least motion in the float.

The same experiment was repeated many times with leaves of French beans, maple (*Acer pseudoplatanus*), clover (*Trifolium pratense*), &c., and gave the same results.

In a leaf of geranium placed under the same circumstances, the turning over took place by a bending of the footstalk, which thus brought down the flat part of the leaf so as to present the upper surface to the light. In reversing the apparatus, and replacing the leaf in an inverted position, the footstalk raised itself up and regained its primitive situation. There was no motion whatever in the float. The same result was obtained with a leaf of *Camellia japonica*.

When the flat part of the leaf was alone the subject of experiment, it curled up without the float changing its position.

When the plant itself was left floating freely in the water, properly counterpoised, the leaves placed in an inverted position with respect to the light turned themselves over by a motion of the footstalks; but the body of the plant itself remained stationary.

It would appear, therefore, that there is neither attraction nor repulsion, in the material and physical meaning of the word, between light and the surface of leaves.

In compound leaves, the turning over sometimes takes place on the common footstalk, as in the raspberry-shrub, and sometimes on the particular footstalk of each leaflet, which turns itself either on the point situated near the insertion of the footstalk to the flat part of the leaf, as in clover, or else on the point of insertion to the common footstalk, as in the horse-chestnut. The removal of one or more leaflets does not prevent the remaining portion turning itself over when placed towards the light in an inverted position.

I wished to ascertain which of the rays of light had the most influential action in producing the turning over of leaves. In order to make absolutely conclusive experiments on this subject, it would have been necessary to operate by means of the rays of light analysed by the prism; but I had not at my disposal the apparatus of a heliostat, &c., which would have been necessary, and had I possessed it, it would probably have been difficult to arrange the experiment properly; I was consequently obliged to confine myself to the use of coloured glasses. But with the view of ascertaining what degree of confidence they deserved, I analysed with a flint-glass prism a ray of light transmitted through each of the coloured glasses I made use of. I could thus easily find out their degree of purity. The following are the results of this trial.

Glass coloured red by protoxide of copper—spectrum perfectly pure.

Glass coloured blue by cobalt—spectrum contains a little red.

Glass coloured green by chrome—spectrum pure contains very little yellow.

Glass coloured yellow by silver—spectrum contains many orange rays.

Glass coloured violet by manganese—a very fine spectrum, but it contains red rays and a few others, chiefly blue.

For the purpose of observing the effect of coloured glasses on the turning over of leaves, I made use of only three colours,—blue, red and violet.

Three leaves of the same size and age, taken from the same geranium, were placed in an inverted position towards the light; the first behind a blue glass, the second behind a red, and the third behind a violet one. In all these experiments, and they were repeated a great many times, the leaves turned themselves over in the violet and blue rays, and remained motionless in the red. The most remarkable effect was in the blue.

It now remained to investigate the influence of light acting on the two surfaces of leaves with relation to the physiological functions of these organs, so important in vegetative life. These functions are chiefly the exhalation and the decomposition of carbonic acid. I propose now to examine the influence of light under these two heads.

§ 4. *Action of Light on Exhalation.*

Every one knows that plants exhale in the atmosphere a large quantity of water, and that this function is chiefly ascribed to the leaves. To these organs, indeed, the exhalation of water, properly so called, seems to belong; the other parts of the plant losing only an infinitely smaller portion of water, apparently the result of the common evaporation which every humid body experiences in the air. DECANDOLLE calls this last-mentioned loss of water *deperdition*, to distinguish it from *exhalation* properly so called.

SENEBIER has alleged that the vegetative exhalation is nothing, or nearly so, in the dark. This may be true when the temperature is very low and the atmosphere very damp; but I have ascertained, in accordance with DUTROCHET's results, that in summer

and in ordinary circumstances, exhalation continues during the night, though in much less quantity than during the day.

I have attempted to determine what difference could be perceived in the exhalation of leaves when the one or the other of their surfaces was exposed to light. I began by ascertaining that, when the leaves of a great many different plants are thus exposed in the air or in the sun's rays at the same temperature, the loss of weight they experienced in the two cases is proportional during the same time; at first, however, rather greater when their under surface is exposed to light than when it is the upper or varnished surface. Thus, in two hours, horse-chestnut leaves lost water in the ratio of 13·6 per cent., when the under surface was exposed to light, to 11·2 per cent. when it was the upper one; with pear-tree leaves, the ratio was 8·45 to 7·75 per cent. But the more the leaf dries up, the quicker the difference diminishes; and when no more real exhalation exists, but there is only deperdition of water, it becomes nothing, and the leaves lose, during an equal time and at the same temperature, a proportional weight of water, whatever be the surface exposed to light.

To ascertain the variations in the exhalation properly so called, I weighed the leaves recently gathered, either with their footstalks only, or with the branch on which they grew; I immersed them afterwards by the footstalk or branch in a bottle filled with water of which the weight had been accurately taken. One of the surfaces of the leaf was then exposed to diffused light and the other covered with a screen. After a given time, I weighed accurately the leaf and the bottle, and the loss gave the amount of exhalation. The same leaf was then inverted and placed in a contrary direction, so that its other surface was exposed to light during a space of time precisely equal to the first. The loss in weight disclosed to me the quantity of the exhaled water. The temperature was carefully observed, though I had ascertained by direct experiment, so far agreeing with Senebier's, that heat had very little influence on the exhalation itself. I could besides reverse many times during the day the experiment on the same leaf, and thus obtain alternately for its two surfaces the same circumstances of heat and light. For leaves that have a great tendency to turn over, those of the raspberry-bush, for instance, it was somewhat difficult to maintain them without contorsion of the flat part of the leaf with their under surface exposed to light. With leaves of this description the experiment on that surface could not be continued above two hours.

I have made numerous experiments on a great many species of leaves, in all temperatures and in all weathers, but it would be tedious to give the particulars of them. I shall only report the general results.

1st. A leaf immersed in water by its footstalk increases at first in weight a little more during the same time, when its under surface is exposed to light than when it is the upper one that is so. This result is the consequence of the absorption being a little greater in the first case; but at the end of the experiments the leaves have pretty nearly the same weight they had at the beginning.

2nd. As soon as sufficient time has elapsed to allow water to penetrate into the leaf, the loss by exhalation is, in all temperatures and in every atmospherical state, much greater when the under surface of leaves is exposed to light than when it is the varnished side. The proportion has varied a little in the experiments, and according to the nature of leaves, but in general the loss has been three times more considerable; for instance in the maple, the horse-chestnut, the pear-tree, the plane, &c. In some cases, the difference has been still greater; but sometimes it has been only double. It is obvious that to this enormous increase of the loss of water, which takes place in the inverted leaves, a loss already so considerable in the ordinary state, is to be mainly attributed the state of uneasiness, followed by withering and death, which is the consequence of this position when it is forced upon them; and we thus see one of the ends of nature in giving them the means of delivering themselves from this evil.

By means of the coloured glasses, I endeavoured to estimate the influence of each of the rays of the spectrum in the production of this phenomenon. Out of a great many series of experiments I shall select only one.

In two hours, a leaf of raspberry-bush, weighing twenty-three grains, placed in 300 grains of water, exhaled, with its under surface exposed to the light, in a temperature of 20° C. (68° F.),—

	grs.
In diffused light	4·3 water.
In the blue rays	6·3 water.
In the yellow rays	2·0 water.
In the green rays	2·0 water.
In the red rays	1·0 water.
In the dark	0·4 water.

This leaf, the next day, during an equal space of time, and in the same temperature, exhaled, with its upper surface exposed to the light,—

	grs.
In diffused light	2·2 water.
Under blue glass	2·8 water.
Under yellow glass	0·5 water.
Under red glass	0·4 water.
In the dark	0·0 water.

In all the experiments the exhalation was greater in the blue rays than in the others, whether it was the upper or the under surface of the leaf that was exposed to the influence of light. The blue rays excite a greater exhalation than the diffused light; but this light has more influence than the other rays. The red is that in which the exhalation is the smallest.

§ 5. *Action of Light on the Decomposition of Carbonic Acid.*

It is known that this phenomenon, which in plants effects the assimilation of carbon, and perhaps also of oxygen gas, takes place only under the influence of the

solar light on the green parts of vegetables. Senebier has advanced that it is independent of the (*cuticle*) green organs, and takes place in the parenchymatous matter. This is true as long as the plant is not disorganized. Thus, for instance, a slice cut in the parenchyma of a leaf of *Rochœa falcata*, weighing ten grains, gave out in the sun, when immersed in water containing carbonic acid in solution, a certain number of bubbles of oxygenated air. The bubbles appeared to form themselves in the exterior cells, and seemed as if they could not disengage themselves without trouble; they remained attached to the fragment, and it was necessary to shake the vessel in order to make them come up. An equal weight of the parenchyma of the *Rochœa* having been pounded, so as to destroy all the cells, was exposed to the sun in an equal quantity of the carbonic acid solution and during the same time. No oxygenated air was produced. The same negative result was obtained when the expressed juice of this leaf was put in the sun with water, although it contained a great deal of the green matter that fell to the bottom of the tube. When I put into the water a slice of the leaf of Rochæa of the same dimensions as the first, but in which the cuticle had remained with a small layer of green matter, although this fragment weighed only five grains, or the half of the preceding one, I saw a regular series of bubbles disengage themselves from the stomata. I could measure in an equal length of time with the two tubes plunged in the same solution of carbonic acid, three times as much oxygenated air from the slice covered by its cuticle than from the one that was not so covered.

The same experiments repeated with a great many other plants gave similar results, and show,—first, that the green chromule alone is not endowed with the property of decomposing carbonic acid, and that this faculty is the consequence of a physiological action of the cells; secondly, that this decomposition is increased by the agency of the vessels and pores that exist on the cuticles of leaves.

In order to appreciate the influence of light on the production of oxygen gas when it acted on one or the other of the surfaces of leaves, I took two leaves of the same plant as equal as possible in weight and surface, and placed them in two similar bell-glasses, inverted over the same weak solution of carbonic acid or over spring-water. One of the leaves was placed in such a manner as to present to the sun its under surface, and the other its upper one. The other side of the bells was covered with black paper. The gases produced were carefully measured after the experiment, and the temperature was noted.

I have made a great many experiments on a great variety of leaves; and without giving the details in this place, I will only add, that when the leaves are in their natural position, that is to say, with their varnished surface exposed to the light, the bubbles of oxygen gas are produced much quicker and in far greater quantity than when the under surface is exposed to light. During the same length of time, two or three times as much gas is formed in the first case as in the second; and the difference is the more marked the longer the experiment is continued, for the produc-

tion of oxygen by the leaf whose under surface is exposed to the sun goes on constantly diminishing.

I devised another rather striking mode of making the same experiment. In one of my trials, I had been surprised to observe that the leaves of *Camellia japonica* did not, when exposed to the sun in spring-water, disengage oxygen gas by their stomata, but that bubbles of this gas went off through the footstalks. I ascertained afterwards that this fact had already been mentioned by Dutrochet. It afforded me the means of showing the difference of the action of light on the two surfaces of the same leaf.

To make this experiment, I place in two large tubes filled with spring-water, two Camellia leaves having an equal surface, and their footstalk directed upwards. One of the leaves has its upper, the other its lower surface exposed to light, and the opposite side is shaded. I leave the apparatus for at least five or six hours in the dark, after which I expose it to the diffused light at the temperature of 20° C. (68° F.); the direct rays of the sun not being necessary to the production of the phenomenon. After twenty minutes of exposure, there appear on the leaf, the upper surface of which is exposed to the light, numerous bubbles of a gas containing 85 to 90 per cent. of oxygen, disengaging themselves from the aërial vessels placed in the centre of the footstalk, and whose apertures are clearly visible. The bubbles are very small, and so numerous and so rapidly emitted, that it is absolutely impossible to count them; they chase one another, and are all gathered up at the top of the tube. Begun at half-past nine in the morning, for instance, the bubbling lasts with the same activity for an hour, after which it ceases. Towards the end, the current of bubbles is a little less rapid; but the smallest number I was able to count at twenty minutes past ten o'clock, was 120 bubbles per minute. In the leaf whose lower surface is exposed to light, the gas begins to disengage itself only after thirty minutes of exposure to light; the current of bubbles is much less rapid, and during the whole time of duration I could always easily count them. The greatest number produced was 145 in a minute; and while the other leaf still gave 120 of them per minute, this one gave only thirty. The bubbles ceased at the same moment in both leaves, and consequently lasted only three quarters of an hour in the leaf whose under surface was illuminated. I was not surprised, therefore, in measuring the quantities of disengaged gases, to find that the leaf of Camellia exposed to light in its natural position had given three times as much oxygen gas as the other whose position was inverted, all other circumstances being similar, and both being plunged in the same liquid. When the apparatus is again placed in darkness, a new accumulation of carbonic acid takes place in the cells of the leaves; and if they remain in it a sufficient time, the disengagement of bubbles of oxygen gas by exposure to light begins again, with the same differences as to time and proportion, according as the leaf is illuminated on its upper or lower surface. I have seen the same leaves producing the same phenomenon for seven or eight days in succession, by keeping them alternately in

the dark and in the light. The rays of the sun are not indispensable; and the experiment, which is easy to perform and pleasing to witness, succeeds very well in diffused light.

I have ascertained that some other leaves, among others those of the *Laurus thymus*, Portugal laurel, &c., give similar results.

In conclusion, I have endeavoured to show:—

1st. That light is the only agent in the turning over of leaves.

2nd. That it does not act by a physical attraction properly so called.

3rd. That the turning over of leaves takes place sometimes by a torsion of the footstalk, sometimes by a curling of the flat part of leaves.

4th. That the blue rays appear to be the most, and the red the least active in effecting the turning over of leaves.

5th. That the exhalation of leaves is much increased when their under surface is exposed to light.

6th. That the decomposition of carbonic acid and the disengagement of oxygen gas are, under the same circumstances, considerably diminished.

XIX. *Microscopical Examination of the Contents of the Hepatic Ducts, with conclusions founded thereon as to the Physiological signification of the Cells of Hepatic Parenchyma, and as to their Anatomical relation to the Radicles of the Hepatic Ducts.* By T. Wharton Jones, *F.R.S.*

Received May 11,—Read May 25, 1848.

1. In the contents of the larger branches of the hepatic duct, there are seen on microscopical examination,—1st, detached columnar epithelium; 2nd, free nuclei, some round, some oval, about $\frac{1}{3000}$th of an inch in diameter; 3rd, minute granules, free or in amorphous flakes, globules of oil, and fragments of cell-walls.

2. In the contents of smaller branches of the hepatic duct, I have repeatedly observed, in addition to the objects just enumerated, cells of a polygonal shape and about $\frac{1}{1100}$th of an inch in diameter, containing round nuclei about $\frac{1}{3000}$th of an inch in diameter, together with minute granules and globules of oil; cells, in short, identical with those of the parenchyma of the same liver, except that for the most part they were paler, on account of the contained granules and oil-globules being fewer and more minute. In some instances the cells were partially broken up*.

3. To assist us in interpreting the observation now related, and in coming to a conclusion therefrom as to the physiological signification of the cells of hepatic parenchyma, and as to their anatomical relation to the radicles of the hepatic duct, it will be useful to examine the contents of the duct of a gland the structure of which is well understood, such as the pancreas, and to compare them with the proper anatomical elements of the same gland.

4. The proper anatomical elements of the pancreas, it is to be called to mind are,—1st, vesicles composed of tunica propria opening into the radicles of the duct; 2nd, the endogenous cells of these vesicles, or the true secretory corpuscles. The endogenous or true secretory corpuscles are round granulous masses, about $\frac{1}{1600}$th of an inch in diameter, containing in their interior a round nucleus about $\frac{1}{3000}$th of an inch in diameter, and either altogether destitute of a cell-wall, or possessed of an imperfectly-formed one.

5. In the contents of the pancreatic duct, there are found on microscopical

* The human liver and the sheep's liver, principally the former, were the subjects of examination. The contents of the duct were taken up for examination by means of a small microscopical forceps passed from the larger branches in the transverse fissure of the liver towards the smaller imbedded in the substance of the organ. In this way I believe the accidental admixture of hepatic cells from a cut surface of the liver, with the contents of the ducts, was effectually guarded against.

examination,—1st, detached columnar epithelium ; 2nd, free round nuclei ; and 3rd, a quantity of minute granules, free or in amorphous flakes. Of these objects, the columnar epithelium is the same as that of the wall of the duct itself, and has evidently been detached from it ; the free nuclei are identical with those of the endogenous corpuscles of the glandular vesicles, and are evidently, together with the minute granules, the fragments of those corpuscles in process of resolution into pancreatic juice.

6. To apply what we have now learned of the nature of the objects contained in matter taken from the pancreatic duct to the elucidation of our subject. It is scarcely necessary, in the first place, to say that the fragments of columnar epithelium found in the matter from the hepatic ducts have been, like the columnar epithelium contained in the matter from the pancreatic duct, detached from the walls of the ducts themselves. Of the free nuclei, the round ones* are identical with those of the cells of hepatic parenchyma, and are, together with the granulous substance, globules of oil and fragments of cell-walls, evidently such as might be supposed to be the remains of broken-up hepatic cells and their contents in process of resolution into bile,—as evidently as the free nuclei and granulous substance found in the matter of the pancreatic duct are the debris of the endogenous corpuscles of the vesicles of the pancreas broken up and in process of resolution into pancreatic juice. The fact of the existence of hepatic cells in the smaller hepatic ducts above enunciated, sufficiently accounts for the presence of their broken-up remains in the ducts.

7. From what has now been stated, I believe I am warranted in concluding that the cells of hepatic parenchyma are the analogues of the endogenous cells or corpuscles of the glandular vesicles of the pancreas and other racemose glands, or of the glandular tubules of tubular glands, and are, like them, being constantly reproduced, cast off, received into the radicles of the ducts, broken up and resolved into the secreted matter.

8. But besides establishing this physiological proposition, the fact of the existence of hepatic cells in the smaller ducts of the liver, throws light on the anatomical relation of the hepatic cells to the radicles of the hepatic ducts,—a point the most essential in the minute anatomy of the liver, but one which has not as yet been decisively determined by direct anatomical demonstration, though different hypothetical explanations of it have been offered.

9. The different hypothetical explanations referred to may be reduced to two heads.

According to the explanations under the one head, the hepatic cells themselves stand in such a relation to the radicles of the biliary ducts that they pour their contents into them, either by opening separately like follicles at all points, or, after coalescing to form tubules, by these tubules opening into cœcal radicles of the biliary ducts.

* The oval nuclei, mentioned in § 1, resemble those of the columnar epithelium cells, and are probably derived from brcken-up cells of that structure.

According to the explanations under the other head, the masses of hepatic cells of which the parenchyma of the liver is composed are pervaded by intercellular passages leading directly into ducts, which from having a proper coat are recognisable as such. The hepatic cells, analogous to the endogenous cells of other glands, which form, like an epithelium, the immediate wall of the intercellular passages, become, in the recognisable ducts, superseded by a proper epithelium.

10. Of these hypothetical explanations, it is to be observed, that those under the first head are founded on the assumption that the hepatic cells correspond to glandular vesicles,—for it is glandular vesicles and not endogenous cells which open into ducts either separately or after having coalesced to form tubules,—an assumption already opposed to analogy and altogether unsupported by any fact, but now completely set aside by the facts and arguments which have been adduced in this paper. Whilst the explanations under the first head must thus be rejected, that under the second head, which assumes the hepatic cells to correspond to endogenous cells, and which was first suggested by Professor Henle of Heidelberg as the most probable, has by the same facts and arguments been proved to be correct in principle.

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